

57:020

Fluid Mechanics

Class Notes
Fall 2006

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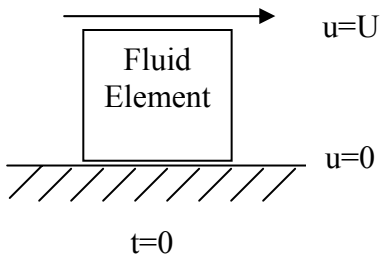
Chapter 1: Introduction and basic concepts

Fluids and the no-slip condition

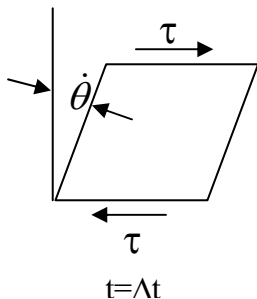
Fluid mechanics is the science of fluids either at rest (fluid statics) or in motion (fluid dynamics) and their effects on boundaries such as solid surfaces or interfaces with other fluids.

Definition of a fluid: a substance that deforms continuously when subjected to a shear stress

Consider a fluid between two parallel plates, which is subjected to a shear stress due to the impulsive motion of the upper plate



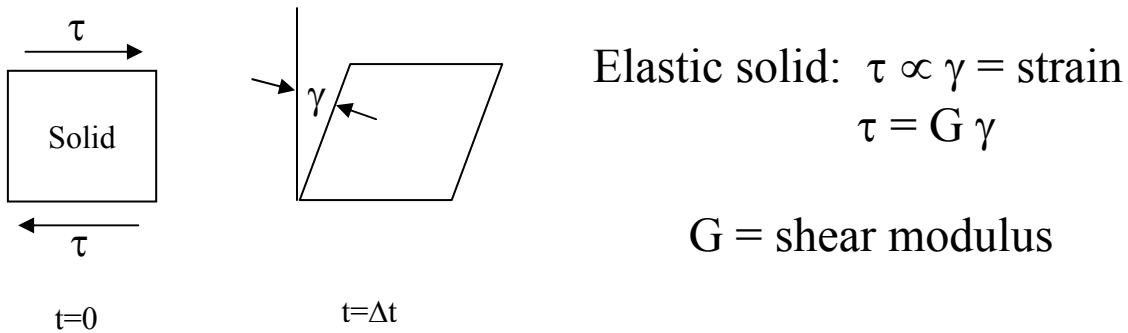
No slip condition: no relative motion between fluid and boundary, i.e., fluid in contact with lower plate is stationary, whereas fluid in contact with upper plate moves at speed U .



Fluid deforms, i.e., undergoes rate of strain $\dot{\theta}$ due to shear stress τ

Newtonian fluid: $\tau \propto \dot{\theta} = \text{rate of strain}$
 $\tau = \mu \dot{\theta}$
 $\mu = \text{coefficient of viscosity}$

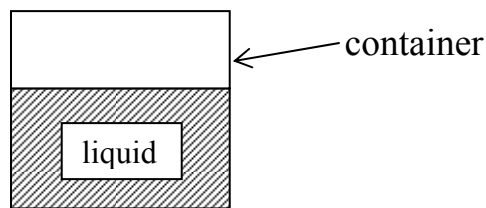
Such behavior is different from solids, which resist shear by static deformation (up to elastic limit of material)



Both liquids and gases behave as fluids

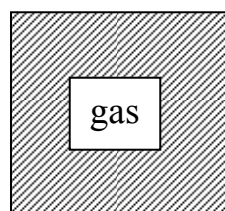
Liquids:

Closely spaced molecules with large intermolecular forces
 Retain volume and take shape of container



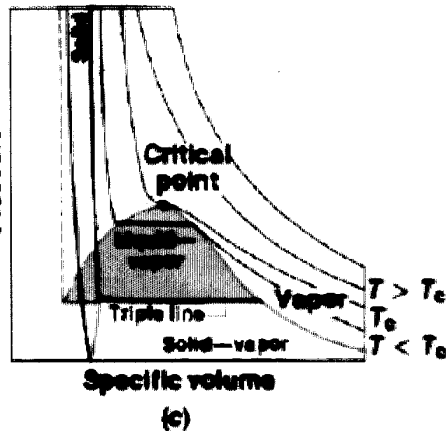
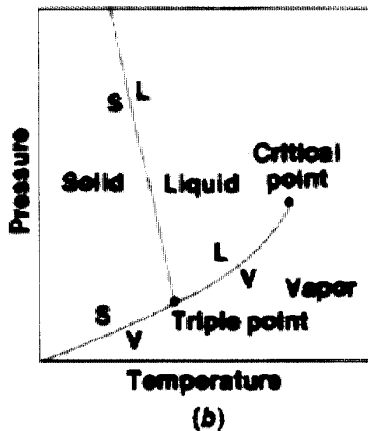
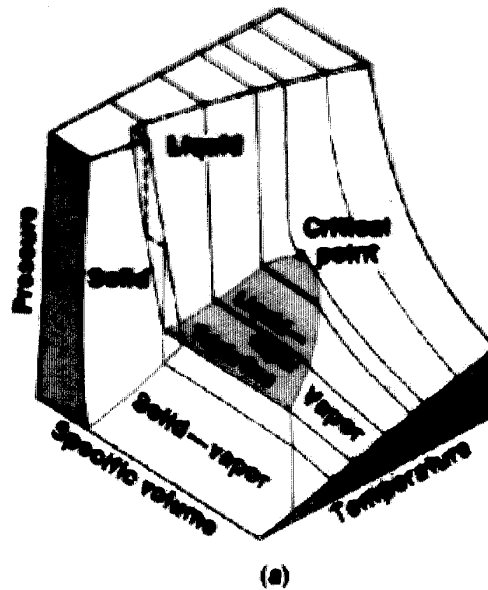
Gases:

Widely spaced molecules with small intermolecular forces
 Take volume and shape of container



Recall p-v-T diagram from thermodynamics:
single phase, two phase, triple point (point at which solid, liquid, and vapor are all in equilibrium), critical point (maximum pressure at which liquid and vapor are both in equilibrium).

Liquids, gases, and two-phase liquid-vapor behave as fluids.



Continuum Hypothesis

In this course, the assumption is made that the fluid behaves as a continuum, i.e., the number of molecules within the smallest region of interest (a point) are sufficient that all fluid properties are point functions (single valued at a point).

For example:

Consider definition of density ρ of a fluid

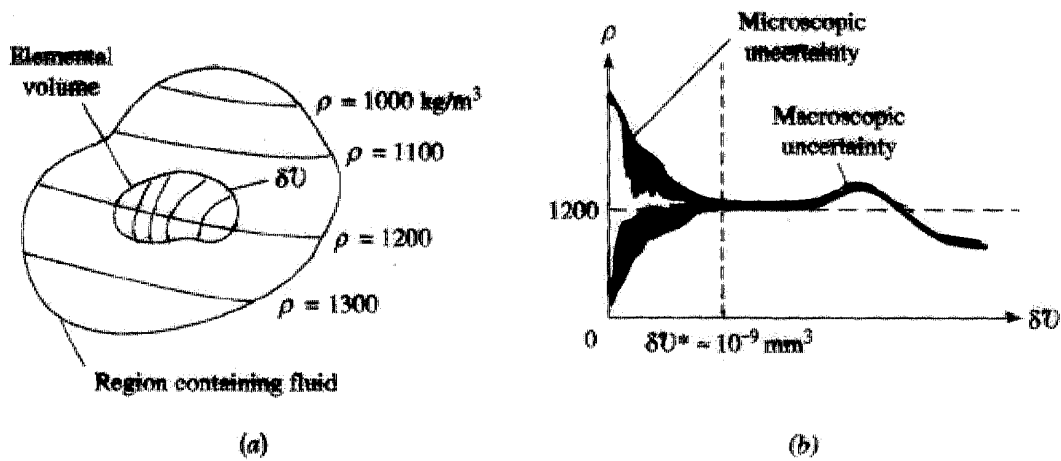
$$\rho(\underline{x}, t) = \lim_{\delta V \rightarrow \delta V^*} \frac{\delta M}{\delta V} \quad \begin{array}{l} \underline{x} = \text{position vector} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \\ t = \text{time} \end{array}$$

δV^* = limiting volume below which molecular variations may be important and above which macroscopic variations may be important

$\delta V^* \approx 10^{-9} \text{ mm}^3$ for all liquids and for gases at atmospheric pressure

10^{-9} mm^3 air (at standard conditions, 20°C and 1 atm) contains 3×10^7 molecules such that $\delta M / \delta V = \text{constant} = \rho$

Note that typical “smallest” measurement volumes are about $10^{-3} - 10^0 \text{ mm}^3 \gg \delta V^*$ and that the “scale” of macroscopic variations are very problem dependent



²One atmosphere equals $2116 \text{ lbf/ft}^2 = 101,300 \text{ Pa}$.

Exception: rarefied gas flow

Properties of Fluids

Fluids are characterized by their properties such as viscosity μ and density ρ , which we have already discussed with reference to definition of shear stress $\tau = \mu \dot{\theta}$ and the continuum hypothesis.

Properties can be both dimensional (i.e., expressed in either SI or BG units) or non-dimensional.

See: Appendix Figures B.1 and B.2, and Appendix Tables B.1, B.2, B.3, B.4, Appendix C and D, and tables 1.3, 1.4, 1.5, 1.6, 1.7, and 1.8.

Basic Units

System International and British Gravitational Systems

Primary Units	SI	BG
Mass M	kg	Slug=32.2lbm
Length L	m	ft
Time t	s	s
Temperature T	°C (°K)	°F (°R)

Temperature Conversion:

$$^{\circ}\text{K} = ^{\circ}\text{C} + 273$$

$$^{\circ}\text{R} = ^{\circ}\text{F} + 460$$

°K and °R are absolute scales, i.e., 0 at absolute zero.
 Freezing point of water is at 0°C and 32°F.

Secondary (derived) units	Dimension	SI	BG
velocity V	L/t	m/s	ft/s
acceleration a	L/t ²	m/s ²	ft/s ²
force F	ML/t ²	N (kg·m/s ²)	lbf
pressure p	F/L ²	Pa (N/m ²)	lbf/ft ²
density ρ	M/L ³	kg/m ³	slug/ft ³
internal energy u	FL/M	J/kg (N·m/kg)	BTU/lbm

Weight and Mass

F = Ma Newton's second law (valid for both solids and fluids)

Weight = force on object due to gravity

$$W = Mg \quad g = 9.81 \text{ m/s}^2 \\ = 32.2 \text{ ft/s}^2$$

$$\text{SI: } W \text{ (N)} = M \text{ (kg)} \cdot 9.81 \text{ m/s}^2$$

$$\text{BG: } W \text{ (lbf)} = \frac{M(\text{lbfm})}{g_c} \cdot 32.2 \text{ ft/s}^2 = M(\text{slug}) \cdot 32.2 \text{ ft/s}^2$$

$$g_c = 32.2 \frac{\text{lbfm} \cdot \text{ft}}{\text{s}^2 \cdot \text{lbf}} = 32.2 \frac{\text{lbfm}}{\text{slug}}, \text{ i.e., } 1 \text{ slug} = 32.2 \text{ lbfm}$$

$$1\text{N} = 1\text{kg} \cdot 1\text{m/s}^2$$

$$1\text{lbf} = 1 \text{ slug} \cdot 1\text{ft/s}^2$$

System; Extensive and Intensive Properties

System = fixed amount of matter
= mass M

Therefore, by definition

$$\frac{d(M)}{dt} = 0$$

Properties are further distinguished as being either extensive or intensive.

Extensive properties: depend on total mass of system,
e.g., M and W (upper case letters)

Intensive properties: independent of amount of mass of system, e.g., p (force/area, lower case letters) and ρ (mass/volume)

Properties Involving the Mass or Weight of the Fluid

Specific Weight, γ = gravitational force, i.e., weight per unit volume ∇
= W/∇
= mg/∇
= ρg N/m^3

(Note that specific properties are extensive properties per unit mass or volume)

Mass Density ρ = mass per unit volume
= M/∇ kg/m^3

Specific Gravity S = ratio of γ_{liquid} to γ_{water} at standard $T = 4^\circ\text{C}$
(or air at standard conditions for gases)
= $\gamma/\gamma_{\text{water}, 4^\circ\text{C}}$ dimensionless

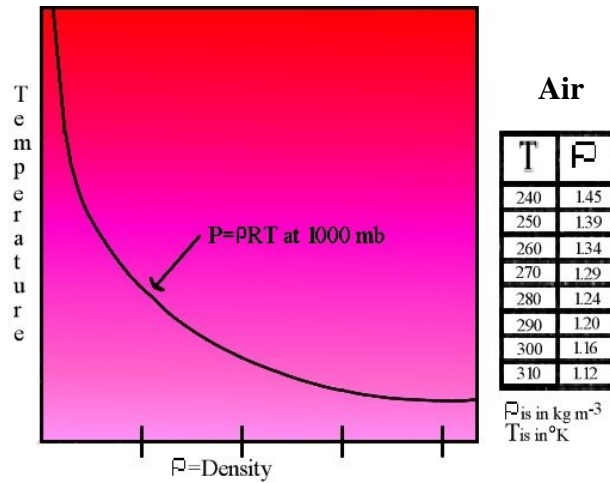
$\gamma_{\text{water}, 4^\circ\text{C}} = 9810 \text{ N/m}^3$ for $T = 4^\circ\text{C}$ and atmospheric pressure

Variation in Density

gases: $\rho = \rho(\text{gas}, T, p)$ equation of state (p - v - T)
= p/RT ideal gas

$R = R(\text{gas})$

$$R (\text{air}) = 287.05 \text{ N}\cdot\text{m}/\text{kg}\cdot^\circ\text{K}$$



liquids: $\rho \sim \text{constant}$

Liquid and temperature	Density (kg/m ³)	Density (slugs/ft ³)
Water 20°C (68°F)	998	1.94
Ethyl alcohol 20°C (68°F)	799	1.55
Glycerine 20°C (68°F)	1,260	2.45
Kerosene 20°C (68°F)	814	1.58
Mercury 20°C (68°F)	13,350	26.3
Sea water 10°C at 3.3% salinity	1,026	1.99
SAE 10W 38°C(100°F)	870	1.69
SAE 10W-30 38°C(100°F)	880	1.71
SAE 30 38°C(100°F)	880	1.71

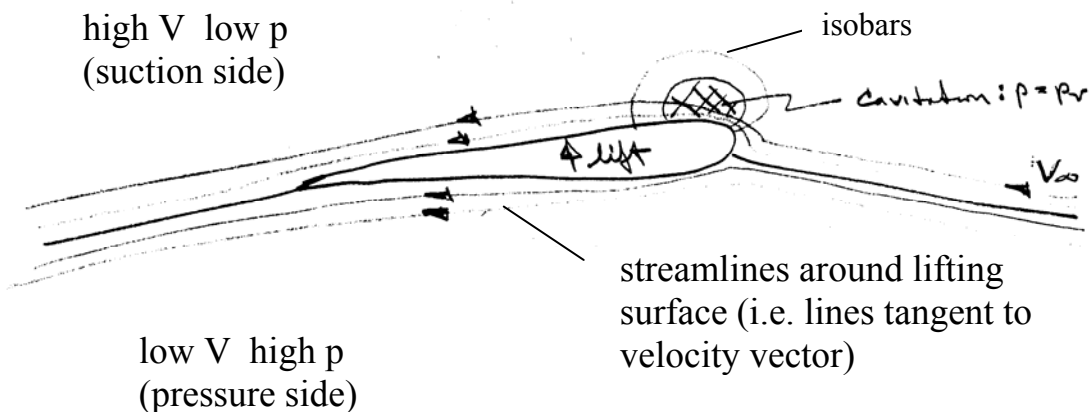
For greater accuracy can also use p-v-T diagram

$$\rho = \rho (\text{liquid}, T, p)$$

$$\begin{array}{ll} T \uparrow & \rho \downarrow \\ p \uparrow & \rho \uparrow \end{array}$$

Vapor Pressure and Cavitation

When the pressure of a liquid falls below the vapor pressure it evaporates, i.e., changes to a gas. If the pressure drop is due to temperature effects alone, the process is called boiling. If the pressure drop is due to fluid velocity, the process is called cavitation. Cavitation is common in regions of high velocity, i.e., low p such as on turbine blades and marine propellers.



$$\text{Cavitation number} = \frac{p - p_v}{\frac{1}{2} \rho V_\infty^2}$$

< 0 implies cavitation

Properties Involving the Flow of Heat

For flows involving heat transfer such as gas dynamics additional thermodynamic properties are important, e.g.

specific heats	c_p and c_v	J/kg·°K
specific internal energy	u	J/kg
specific enthalpy	$h = u + p/\rho$	J/kg

Elasticity (i.e., compressibility)

Increasing/decreasing pressure corresponds to contraction/expansion of a fluid. The amount of deformation is called elasticity.

$$dp = -E_{\forall} \frac{d\forall}{\forall} \qquad dp > 0 \Rightarrow \frac{d\forall}{\forall} < 0$$

∴ minus sign used

$$E_{\forall} = -\frac{dp}{d\forall/\forall} = \frac{dp}{d\rho/\rho} = \frac{N}{m^2} \qquad E_{\forall} = \rho \frac{dp}{d\rho}$$

Alternate form: $M = \rho\forall$
 $dM = \rho d\forall + \forall d\rho = 0$ (by definition)

$$-\frac{d\forall}{\forall} = \frac{d\rho}{\rho}$$

Liquids are in general incompressible, e.g.

$$E_{\nabla} = 2.2 \text{ GN/m}^2 \quad \text{water}$$

i.e. $\Delta \nabla = .05\%$ for $\Delta p = 1 \text{ MN/m}^2$
 (G=Giga= 10^9 M=Mega= 10^6 k=kilo= 10^3)

Gases are in general compressible, e.g. for ideal gas at
 $T = \text{constant}$ (isothermal)

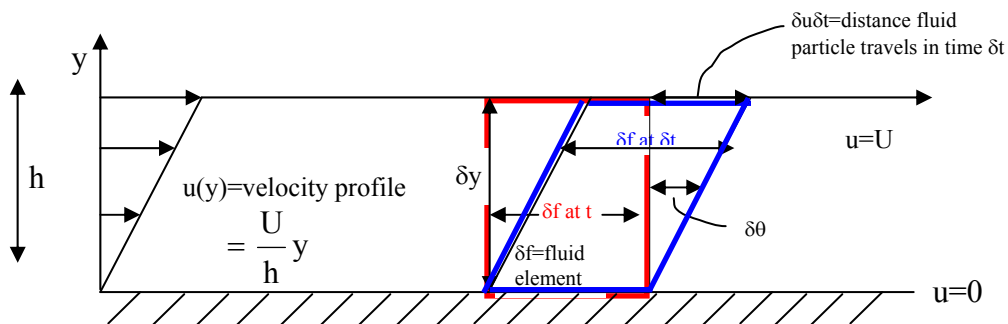
$$\frac{dp}{d\rho} = RT$$

$$E_{\nabla} = \rho RT = p$$

Viscosity

Recall definition of a fluid (substance that deforms continuously when subjected to a shear stress) and Newtonian fluid shear / rate-of-strain relationship ($\tau = \mu \dot{\theta}$).

Reconsider flow between fixed and moving parallel plates (Couette flow)



$$\text{Newtonian fluid: } \tau = \mu \dot{\theta} = \mu \frac{\delta\theta}{\delta t}$$

$$\tan \delta\theta = \frac{\delta u \delta t}{\delta y} \quad \text{or} \quad \delta\theta = \frac{\delta u \delta t}{\delta y} \quad \text{for small } \delta\theta$$

$$\text{therefore } \delta\dot{\theta} = \frac{\delta u}{\delta y} \quad \text{i.e., } \dot{\theta} = \frac{du}{dy} = \text{velocity gradient}$$

$$\text{and } \tau = \mu \frac{du}{dy}$$

Exact solution for Couette flow is a linear velocity profile

$$u(y) = \frac{U}{h} y \quad \text{Note: } u(0) = 0 \text{ and } u(h) = U$$

$$\tau = \mu \frac{U}{h} = \text{constant} \quad \text{i.e., satisfies no-slip boundary condition}$$

where

U/h = velocity gradient = rate of strain

μ = coefficient of viscosity = proportionality constant for Newtonian fluid

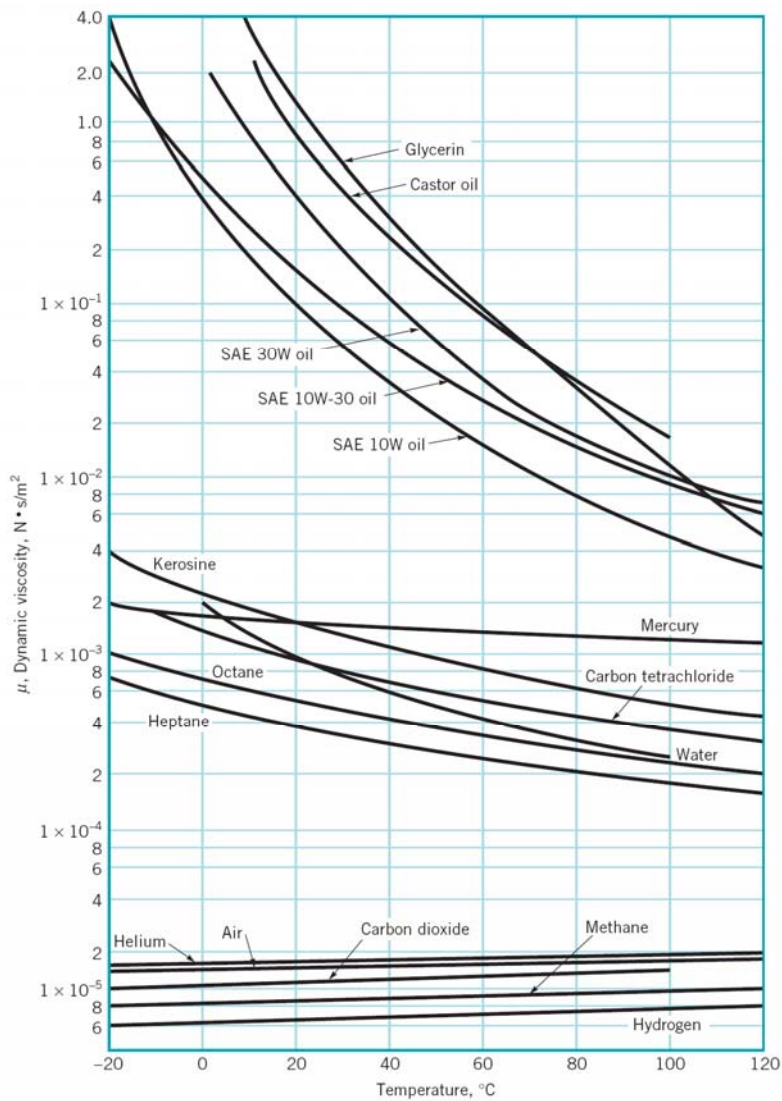
$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\frac{\text{N}}{\text{m}^2}}{\frac{\text{m}}{\text{s}}/\text{m}} = \frac{\text{Ns}}{\text{m}^2}$$

$$\nu = \frac{\mu}{\rho} = \frac{\text{m}^2}{\text{s}} = \text{kinematic viscosity}$$

$$\mu = \mu(\text{fluid}; T, p) = \mu(\text{gas}; T)$$

gas and liquid $\mu \uparrow p \uparrow$, but small $\Delta\mu$

gas:	$\mu \uparrow$	$T \uparrow$	} Due to structural differences, more molecular activity, decreased cohesive forces for gases
liquid:	$\mu \downarrow$	$T \uparrow$	

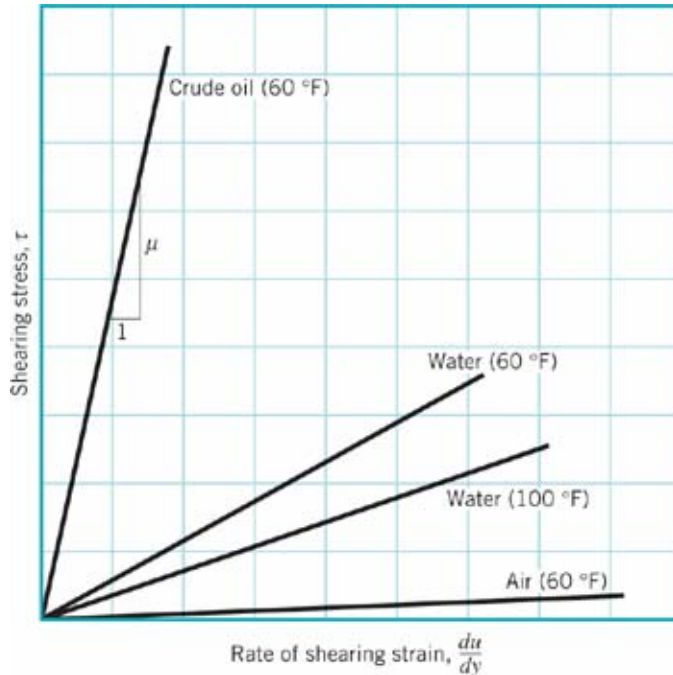


Newtonian vs. Non-Newtonian Fluids

Dilatant: $\tau \uparrow \quad dV/dy \uparrow$

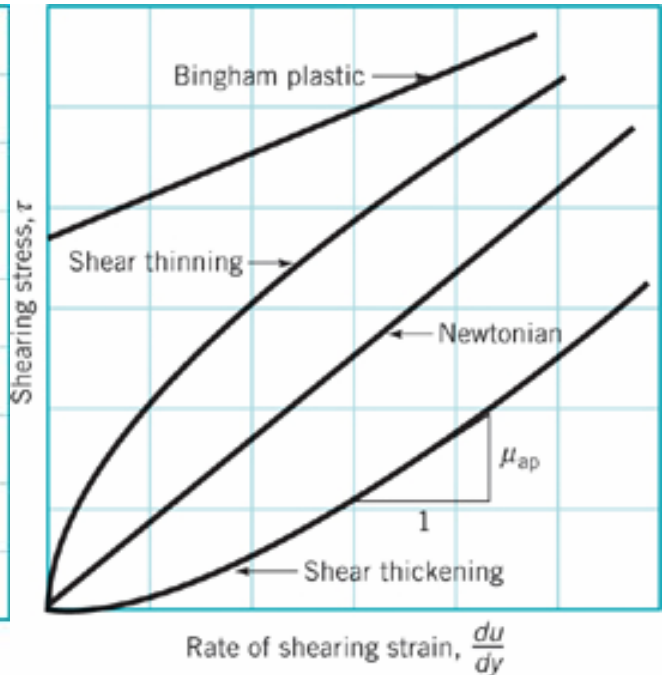
Newtonian: $\tau \propto dV/dy$

Pseudo plastic: $\tau \downarrow \quad dV/dy \uparrow$



$$\tau \propto dV/dy$$

$\mu = \text{slope}$



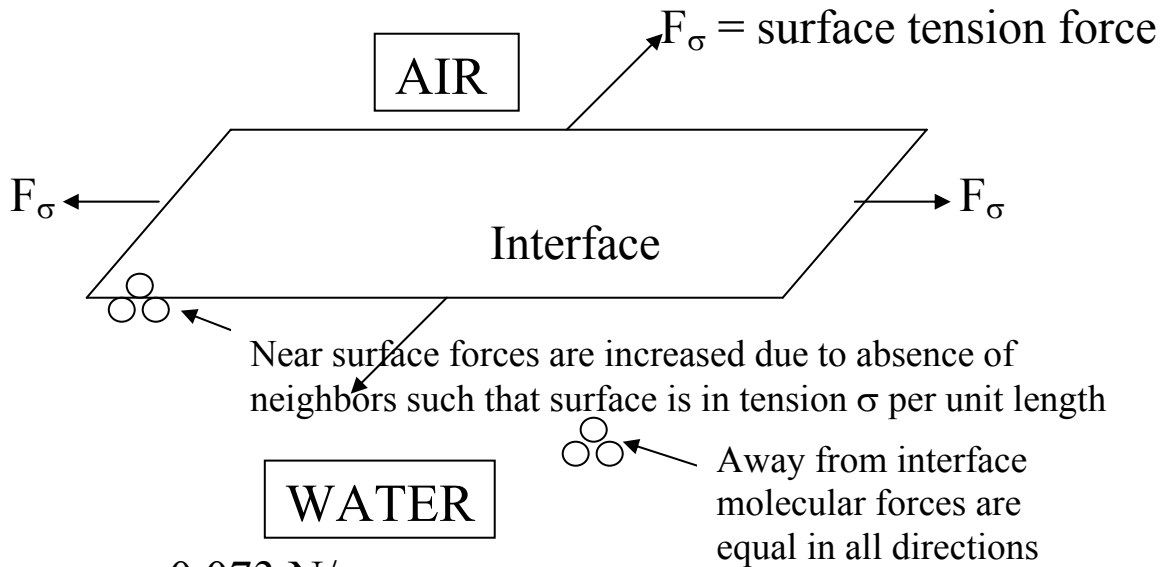
$$\tau \propto (dV/dy)^n$$

$n > 1$ slope increases
 with increasing τ
 (shear thickening)

$n < 1$ slope decreases
 with increasing τ
 (shear thinning),
 blood, paint, liquid plastic

Surface Tension and Capillary Effects

Two non-mixing fluids (e.g., a liquid and a gas) will form an interface. The molecules below the interface act on each other with forces equal in all directions, whereas the molecules near the surface act on each other with increased forces due to the absence of neighbors. That is, the interface acts like a stretched membrane

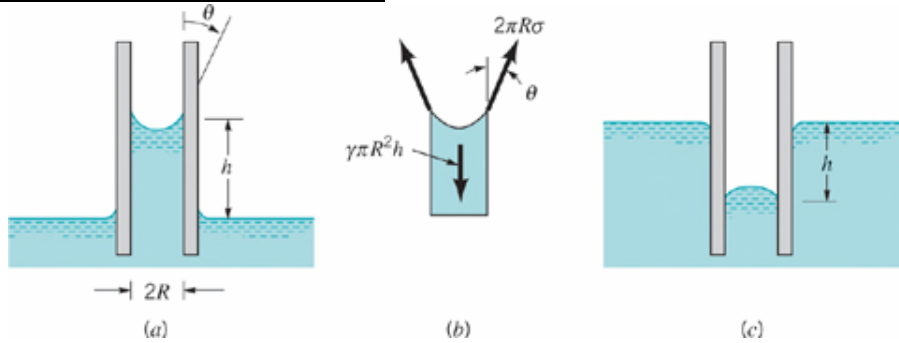


$$\sigma_{\text{air/water}} = 0.073 \text{ N/m}$$

$$F_{\sigma} = \sigma \times L = \text{line force with direction normal to the cut}$$

L = length of cut through the interface

Effects of surface tension:

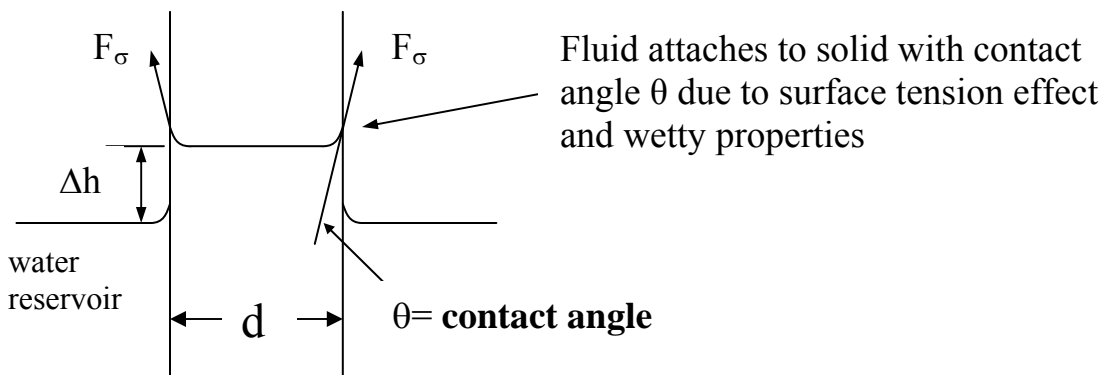


$\theta < 90^\circ$ Wetting

$\theta > 90^\circ$ Non-wetting

1. Capillary action in small tube $\Delta h = 4\sigma/\gamma d$
2. Pressure difference across curved interface
 $\Delta p = \sigma/R$ $R = \text{radius of curvature}$
3. Transformation of liquid jet into droplets
4. Binding of wetted granular material such as sand

Example



capillary tube $d = 1.6\text{mm} = 0.0016\text{m}$

$F_\sigma = \sigma \times L$, $L = \text{length of contact line between fluid \& solid}$

water reservoir at 20°C , $\sigma = 0.073\text{ N/m}$, $\gamma = 9790\text{ N/m}^3$

$$\Delta h = ?$$

$$\Sigma F_z = 0$$

$$F_{\sigma,z} - W = 0$$

$$\sigma \pi d \cos \theta - \rho g V = 0$$

$$\theta \sim 0^\circ \Rightarrow \cos \theta = 1$$

$$\rho g = \gamma$$

$$\sigma\pi d - \gamma\Delta h \frac{\pi d^2}{4} = 0$$

$$\nabla = \Delta h \frac{\pi d^2}{4} = \text{Volume of fluid above reservoir}$$

$$\Delta h = \frac{4\sigma}{\gamma d} = 18.6 \text{ mm}$$

A brief history of fluid mechanics

See text book section 1.10.

Fluid Mechanics and Flow Classification

Hydrodynamics: flow of fluids for which density is constant such as liquids and low-speed gases. If in addition fluid properties are constant, temperature and heat transfer effects are uncoupled such that they can be treated separately.

Examples: hydraulics, low-speed aerodynamics, ship hydrodynamics, liquid and low-speed gas pipe systems

Gas Dynamics: flow of fluids for which density is variable such as high-speed gases. Temperature and heat transfer effects are coupled and must be treated concurrently.

Examples: high-speed aerodynamics, gas turbines, high-speed gas pipe systems, upper atmosphere