57:020

Fluid Mechanics

Class Notes
Fall 2015

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CHAPTER 1: INTRODUCTION AND BASIC CONCEPTS

Fluids and the no-slip condition

Fluid mechanics is the science and technology of fluids either at rest (fluid statics) or in motion (fluid dynamics) and their effects on boundaries such as solid surfaces or interfaces with other fluids.

Definition of a fluid: A substance that deforms continuously when subjected to a shear stress

Consider a fluid between two parallel plates, which is subjected to a shear stress due to the impulsive motion of the upper plate
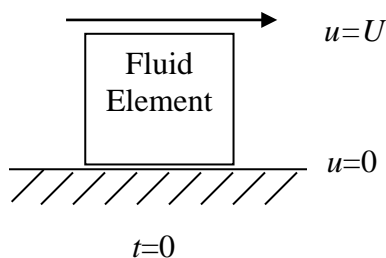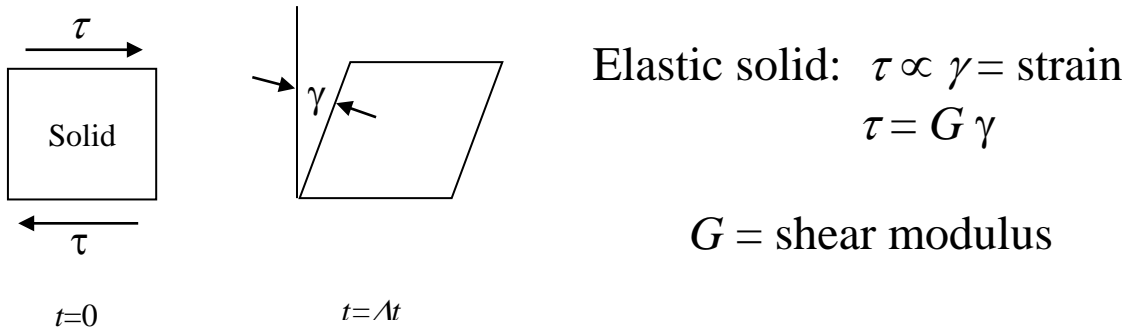
No slip condition: no relative motion between fluid and boundary, i.e., fluid in contact with lower plate is stationary, whereas fluid in contact with upper plate moves at speed U.
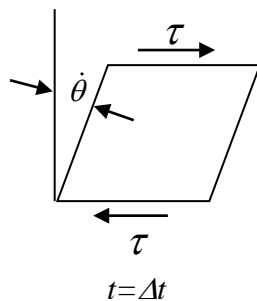
Fluid deforms, i.e., undergoes rate of strain $\dot\theta$ due to shear stress τ

Newtonian fluid: $\tau \propto \dot{\theta}$ = rate of strain

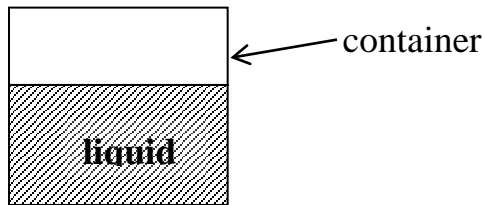$\qquad\qquad\qquad\quad \tau = \mu\,\dot{\theta}$

$\qquad\qquad\qquad\quad \mu$ = coefficient of viscosity

Such behavior is different from solids, which resist shear by static deformation (up to elastic limit of material)

Elastic solid: $\tau \propto \gamma$ = strain

$\qquad\qquad\qquad\quad \tau = G\,\gamma$

G = shear modulus

τ

Solid

τ

$t{=}0$ $t{=}\Lambda t$

γ

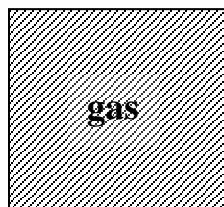## Both liquids and gases behave as fluids

Liquids:
Closely spaced molecules with large intermolecular forces
Retain volume and take shape of container

container

liquid

Gases:
Widely spaced molecules with small intermolecular forces
Take volume and shape of container

gas

Recall *p-v-T* diagram from thermodynamics:
single phase, two phase, triple point (point at which solid,
liquid, and vapor are all in equilibrium), critical point
(maximum pressure at which liquid and vapor are both in
equilibrium).

Liquids, gases, and two-phase liquid-vapor behave as flu-
ids.

(a)

(b)

(c)
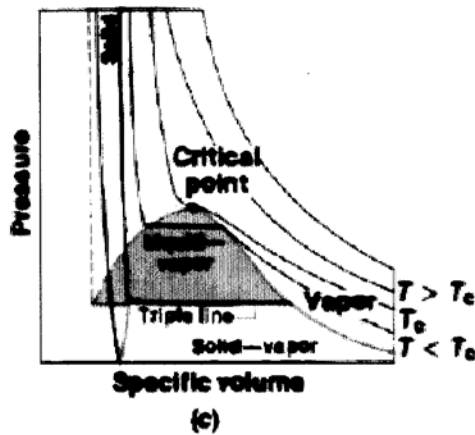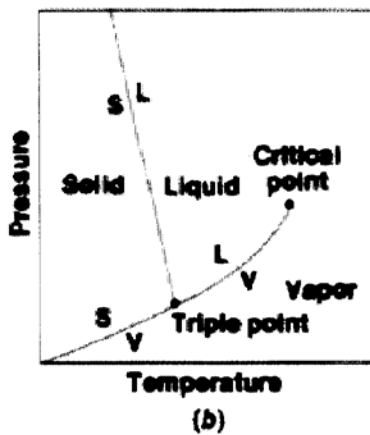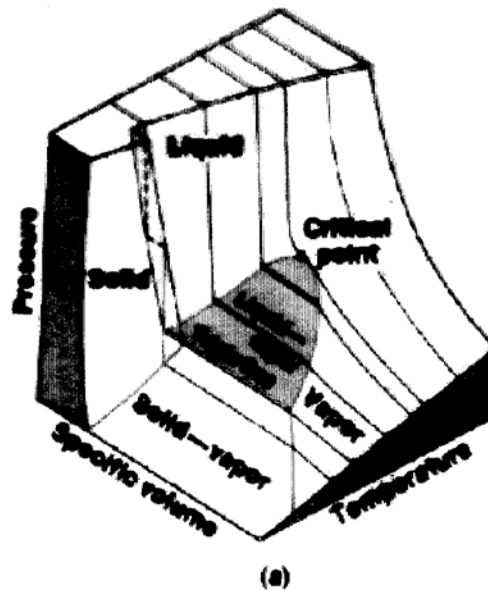
Continuum Hypothesis

In this course, the assumption is made that the fluid be-haves as a continuum, i.e., the number of molecules within the smallest region of interest (a point) are sufficient that all fluid properties are point functions (single valued at a point).

For example:
Consider definition of density ρ of a fluid

$$\rho\left(\underline{x},t\right) = \frac{\lim}{\delta V \to \delta V^*} \frac{\delta m}{\delta V} \qquad \underline{x} = \text{position vector} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$
$$t = \text{time}$$

$\delta V^* =$ limiting volume below which molecular variations may be important and above which macroscopic variations may be important

$\delta V^* \approx 10^{-9}$ mm^3 (or length scale of l$^* \approx 10^{-6}$ m) for all liq-uids and for gases at atmospheric pressure

10^{-9} mm^3 air (at standard conditions, 20°C and 1 atm) con-tains 3×10^7 molecules such that $\delta M / \delta V = $ constant $= \rho$

Note that typical "smallest" measurement volumes are about $10^{-3} - 10^0$ mm$^3 >> \delta V^*$ and that the "scale" of macro-scopic variations are very problem dependent

(a)   (b)
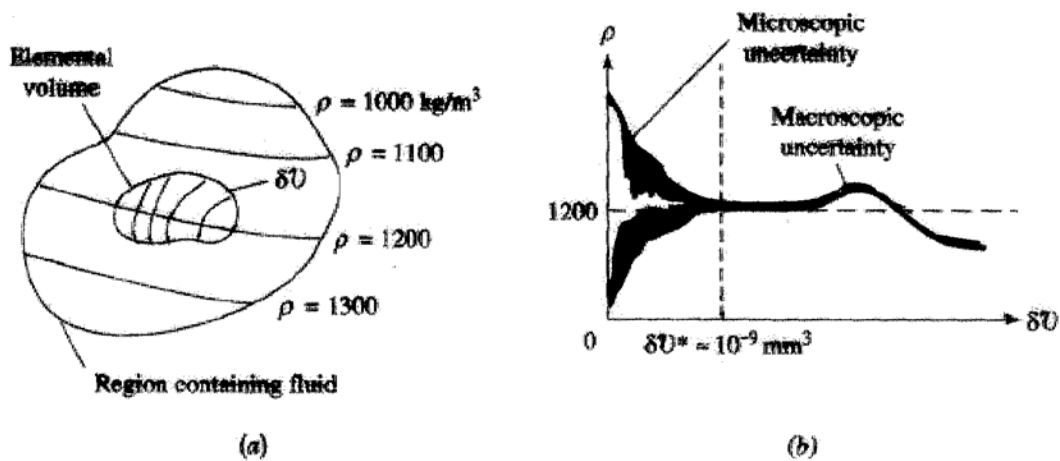
^{2}One atmosphere equals 2116 lbf/ft^2 = 101,300 Pa.

Exception: rarefied gas flow

$\delta \forall^*$ defines a point in the fluid, i.e., a fluid particle or infinitesimal material element used for deriving governing differential equations of fluid dynamics and at which all fluid properties are point functions:

$l^* = 10^{-6}$ m $\gg$ molecular length scales

$\quad \lambda$ = mean free path = 6×10^{-8} m

$\quad t_\lambda = 10^{-10}$ s = time between collisions

$l^* = 10^{-6}$ m $\ll$ fluid length scales $l = 10^{-4}$ m

For laminar flow: $l_{max} \approx$ smallest geometry scales of the flow

$\qquad\qquad\qquad U_{max} < U$ transition to turbulent flow

For turbulent flow: l_{max} and U_{max} determined by Kolmogorov scales at which viscous dissipation takes place, which for typical ship/airplane,

$\quad \eta \approx 2 \times 10^{-5}$ m (ship)/2.3×10^{-5} m (airplane)

$\quad u_\eta \approx 0.05$ m/s (ship)/1.64 m/s (airplane)

$\quad t_\eta \approx 4 \times 10^{-4}$ s (ship)/1.4×10^{-5} s (airplane)

Properties of Fluids

Fluids are characterized by their properties such as viscosity μ and density ρ, which we have already discussed with reference to definition of shear stress $\tau = \mu \dot{\theta}$ and the continuum hypothesis.

(1) **_Kinematic_**: Linear ($\underline{V}$) and angular ($\underline{\omega}/2$) velocity, rate of strain (ε_{ij}), Vorticity ($\underline{\omega}$), and acceleration ($\underline{a}$)

(2) **_Transport_**: Viscosity (μ), thermal conductivity (k), and mass diffusivity (D)

(3) **_Thermodynamic_**: Pressure (p), density (ρ), temperature (T), internal energy ($\hat{u}$), enthalpy ($h = \hat{u} + p/\rho$), specific heat (C_v, C_p, $\gamma = C_p/C_v$, etc.)

(4) **_Miscellaneous_**: Surface tension (σ), vapor pressure (p_v), etc.

Properties can be both dimensional (i.e., expressed in either SI or BG units) or non-dimensional:

Figure B.1 Dynamic (absolute) viscosity of common fluids as a function of temperature.

Figure B.2 Kinematic viscosity of common fluids (at atmospheric pressure) as a function of temperature.

Table B.1 Physical Properties of Water (BG Units)

Table B.2 Physical Properties of Water (SI Units)

Table B.3 Physical Properties of Air at Standard Atmospheric Pressure (BG Units)

Table B.4 Physical Properties of Air at Standard Atmospheric Pressure (SI Units)

Table 1.5 Approximate Physical Properties of Some Common Liquids (BG Units)

Table 1.6 Approximate Physical Properties of Some Common Liquids (SI Units)

Table 1.7 Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (BG Units)

Table 1.8 Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (SI Units)

Basic Units

System International and British Gravitational Systems

Primary Units	SI	BG
Mass M	kg	slug=32.2lbm
Length L	m	ft
Time t	s	s
Temperature T	°C (°K)	°F (°R)

Temperature Conversion:

$°K = °C + 273$

$°R = °F + 460$

°K and °R are absolute scales, i.e., 0 at absolute zero. Freezing point of water is at 0°C and 32°F.

Secondary (derived) units	Dimension	SI	BG
velocity $\underline{V}$	L/t	m/s	ft/s
acceleration $\underline{a}$	L/t^2	m/s^2	ft/s^2
force $\underline{F}$	ML/t^2	N (kg·m/s^2)	lbf
pressure p	F/L^2	Pa (N/m^2)	lbf/ft^2
density ρ	M/L^3	kg/m^3	slug/ft^3
internal energy u	FL/M	J/kg (N·m/kg)	BTU/lbm

Table 1.3 Conversion Factors from BG and EE Units to SI Units.
Table 1.4 Conversion Factors from SI Units to BG and EE Units.

Weight and Mass

$\underline{F} = m\underline{a}$ Newton's second law (valid for both solids and fluids)

Weight = force on object due to gravity

$\mathscr{W}^o = mg$ $g = 9.81$ m/s^2
                    $= 32.2$ ft/s^2

SI: $\mathscr{W}^o(\text{N}) = m \text{ (kg)} \cdot 9.81$ m/s^2

BG: $\mathscr{W}^o(\text{lbf}) = m(\text{slug}) \cdot 32.2\text{ft/ s}^2$

EE: $\mathscr{W}^o(\text{lbf}) = \dfrac{m(\text{lbm})}{g_c} \cdot 32.2$ ft/s^2

$g_c = 32.2 \dfrac{\text{lbm} \cdot \text{ft}}{\text{s}^2 \cdot \text{lbf}} = 32.2 \dfrac{\text{lbm}}{\text{slug}}$, i.e., 1 slug = 32.2 lbm

$1 \text{ N} = 1 \text{ kg} \cdot 1$ m/s^2

$1 \text{ lbf} = 1 \text{ slug} \cdot 1$ ft/s^2

System; Extensive and Intensive Properties

System = fixed amount of matter

= mass m

Therefore, by definition

$$\frac{d(m)}{dt} = 0$$

Properties are further distinguished as being either extensive or intensive.

Extensive properties: depend on total mass of system, e.g., m and $\mathcal{W}^o$

Intensive properties: independent of amount of mass of system, e.g., p (force/area) and ρ (mass/volume)

Properties Involving the Mass or Weight of the Fluid

Specific Weight, γ = gravitational force (i.e., weight) per
 unit volume $\forall$
 = $\mathcal{W}/\forall$
 = $mg/\forall$
 = ρg N/m^3

(Note that specific properties are extensive properties per unit mass or volume)

Mass Density ρ = mass per unit volume
 = $m/\forall$ kg/m^3

Specific Gravity S = ratio of γ_{liquid} to γ_{water} at standard $T = 4°C$
 = $\gamma/\gamma_{water,\ 4°C}$ dimensionless
(or γ_{air} at standard conditions for gases)

$\gamma_{water,\ 4°C}$ = 9810 N/m^3 for $T = 4°C$ and atmospheric pressure
γ_{air} = 12.01 N/m^3 at standard atmosphere (T = 15°C and p = 101.33 kPa)

Variation in Density

gases: $\rho = \rho\,(\text{gas}, T, p)$ equation of state $(p\text{-}v\text{-}T)$
$\qquad = p/RT \quad$ ideal gas
$\quad R = R\,(\text{gas})$

e.g. $R\,(\text{air}) = 287.05 \;\; N{\cdot}m/kg{\cdot}{}^{\circ}K$
$\qquad \rho\,(\text{air}) = 1.225 \; kg/m^3$ at Standard Atmosphere
$\qquad\qquad\qquad\qquad (T = 15{}^{\circ}C \text{ and } p = 101.33 \; kPa)$

liquids: $\rho \sim$ constant
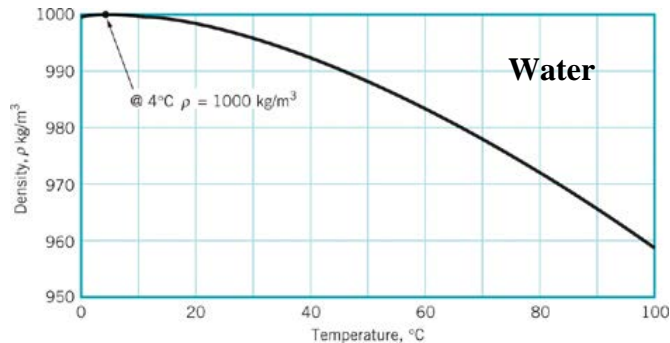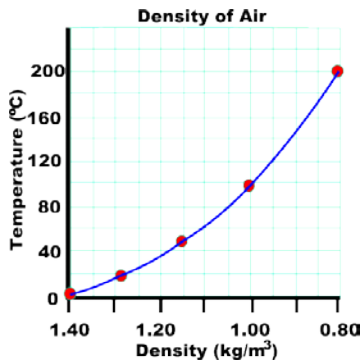
Note: For a change in temperature from 0 to 100°C, density changes about 29% for air while only about 4% for water.

Liquid and temperature	Density (kg/m^3)	Density (slugs/ft^3)
Water 20°C (68°F)	998	1.94
Ethyl alcohol 20°C (68°F)	799	1.55
Glycerine 20°C (68°F)	1,260	2.45
Kerosene 20°C (68°F)	814	1.58
Mercury 20°C (68°F)	13,350	26.3
Sea water 10°C at 3.3% salinity	1,026	1.99
SAE 10W 38°C(100°F)	870	1.69
SAE 10W-30 8°C(100°F)	880	1.71
SAE 30 38°C(100°F)	880	1.71

For greater accuracy can also use p-v-T diagram

$$\rho = \rho \, (\text{liquid}, \, T, \, p)$$

$T\uparrow$ $\qquad$ $\rho\downarrow$
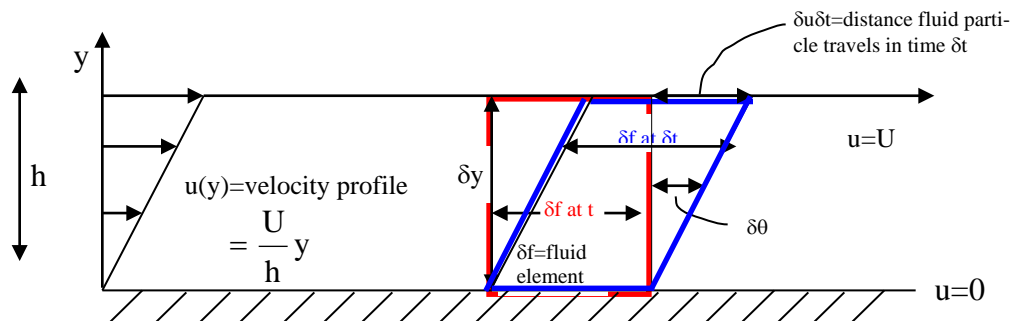$p\uparrow$ $\qquad$ $\rho\uparrow$

Properties Involving the Flow of Heat

For flows involving heat transfer such as gas dynamics additional thermodynamic properties are important, e.g.

specific heats $\qquad\qquad$ c_p and c_v $\qquad$ J/kg·°K

specific internal energy $\qquad$ $\hat{u}$ $\qquad\qquad$ J/kg

specific enthalpy $\qquad$ $h = \hat{u} + p/\rho$ $\quad$ J/kg

Viscosity

Recall definition of a fluid (substance that deforms continuously when subjected to a shear stress) and Newtonian fluid shear / rate-of-strain relationship: $\tau = \mu\dot{\theta}$.

Reconsider flow between fixed and moving parallel plates (Couette flow)

Newtonian fluid: $\tau = \mu\dot{\theta} \ = \ \mu\dfrac{\delta\theta}{\delta t}$

$\tan\delta\theta = \dfrac{\delta u \delta t}{\delta y}$ or $\delta\theta = \dfrac{\delta u \delta t}{\delta y}$ for small $\delta\theta$

therefore $\delta\dot{\theta} = \dfrac{\delta u}{\delta y}$ i.e., $\dot{\theta} = \dfrac{du}{dy}$ = velocity gradient

and $\tau = \mu\dfrac{du}{dy}$

Exact solution for Couette flow is a linear velocity profile

$$u(y) = \frac{U}{h}y$$ Note: $u(0) = 0$ and $u(h) = U$

i.e., satisfies no-slip boundary condition

$$\tau = \mu\frac{U}{h} = \text{constant}$$

where

U/h = velocity gradient = rate of strain

μ = coefficient of viscosity = proportionality constant for Newtonian fluid

$$\mu = \frac{\tau}{\dfrac{du}{dy}} = \frac{N/m^2}{\dfrac{m}{s}\bigg/m} = \frac{Ns}{m^2}$$

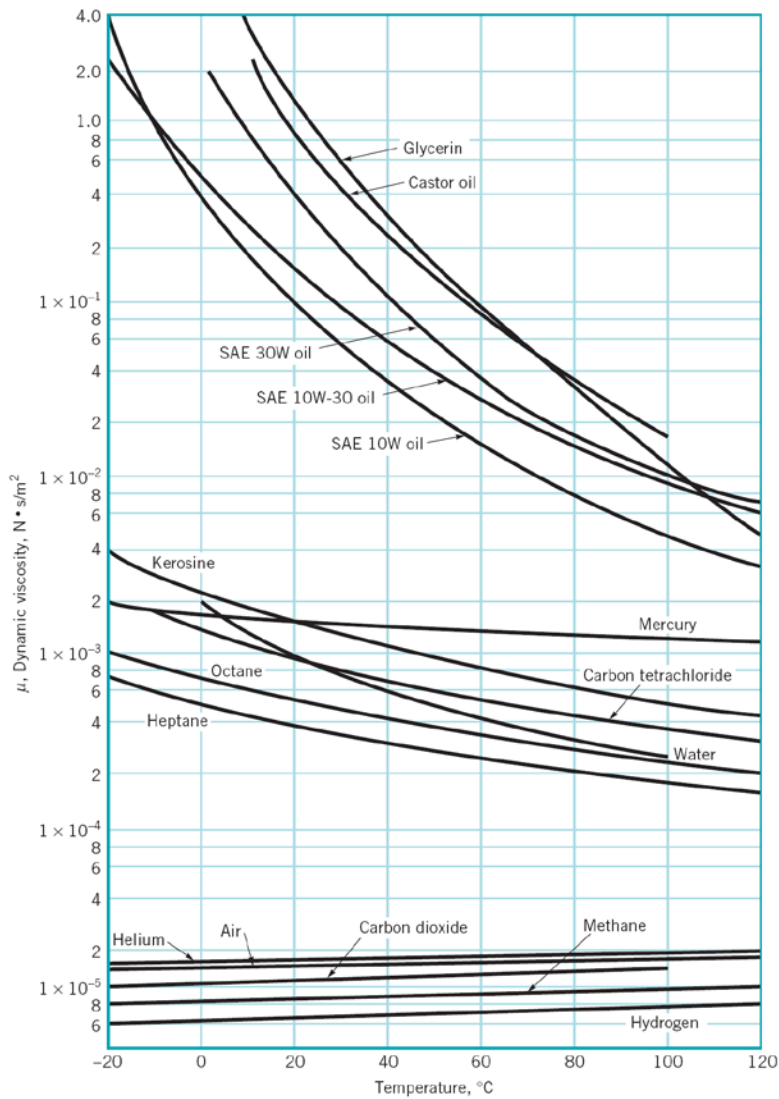$$\nu = \frac{\mu}{\rho} = \frac{m^2}{s} = \text{kinematic viscosity}$$

$$\mu = \mu(\text{fluid};T,p) = \mu(\text{gas/liquid};T)$$

gas and liquid $\mu\!\uparrow p\!\uparrow$, but small $\Delta\mu$

gas: $\mu\!\uparrow\ T\!\uparrow$ **Due to structural differences, more molecular**
liquid: $\mu\!\downarrow\ T\!\uparrow$ **activity for gases, decreased cohesive forces**
                                           for liquids

Newtonian vs. Non-Newtonian Fluids

Dilatant (Shear thickening): $\tau \uparrow$ $du/dy \uparrow$

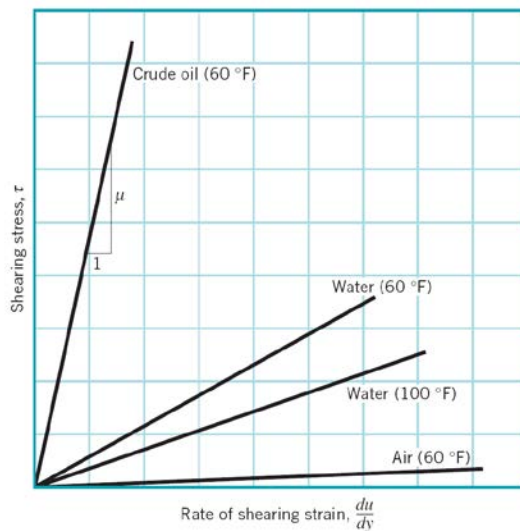Newtonian: $\tau \propto$ du/dy

Pseudo plastic (Shear thinning): $\tau \downarrow$ $du/dy \uparrow$

Bingham plastic: Requires τ before becomes fluid

Example: toothpaste, mayonnaise

Newtonian Fluids	Non-Newtonian Fluids

$$\tau \propto \frac{du}{dy}$$
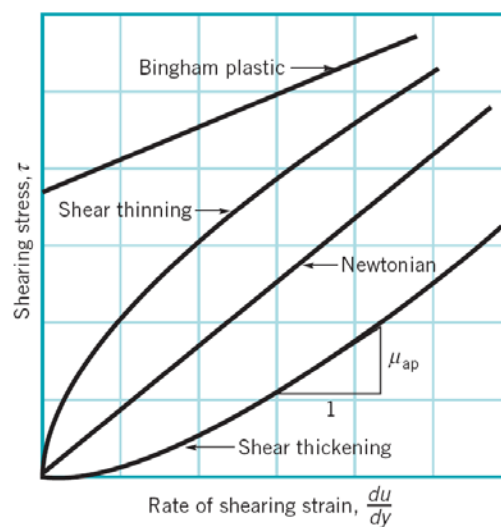
$$\tau \propto \left(\frac{du}{dy}\right)^n$$

$\mu = \text{slope}$

$n > 1$ (shear thickening) Slope increases with increasing τ; ex) cornstarch, quicksand

$n < 1$ (shear thinning) Slope decreases with increasing τ; ex) blood, paint, liquid plastic

Elasticity (i.e., compressibility)

Increasing/decreasing pressure corresponds to contraction/expansion of a fluid. The amount of deformation is called elasticity.

$$dp = -E_v \frac{d\forall}{\forall} \qquad\qquad dp > 0 \Rightarrow \frac{d\forall}{\forall} < 0$$

Increase pressure, decrease volume. $\therefore$ minus sign used

and by definition, $m = \rho\forall$

$$dm = \rho d\forall + \forall d\rho = 0$$
$$-\frac{d\forall}{\forall} = \frac{d\rho}{\rho}$$

Thus,

$$E_v = -\frac{dp}{d\forall/\forall} = \frac{dp}{d\rho/\rho} = \rho\frac{dp}{d\rho} \ [N/m^2]$$

Liquids are in general incompressible, e.g.

$$E_v = 2.2 \text{ GN/m}^2 \qquad\qquad \text{water}$$

i.e. $\Delta\forall = 0.05\%\forall$ for $\Delta p = 1\text{MN/m}^2$
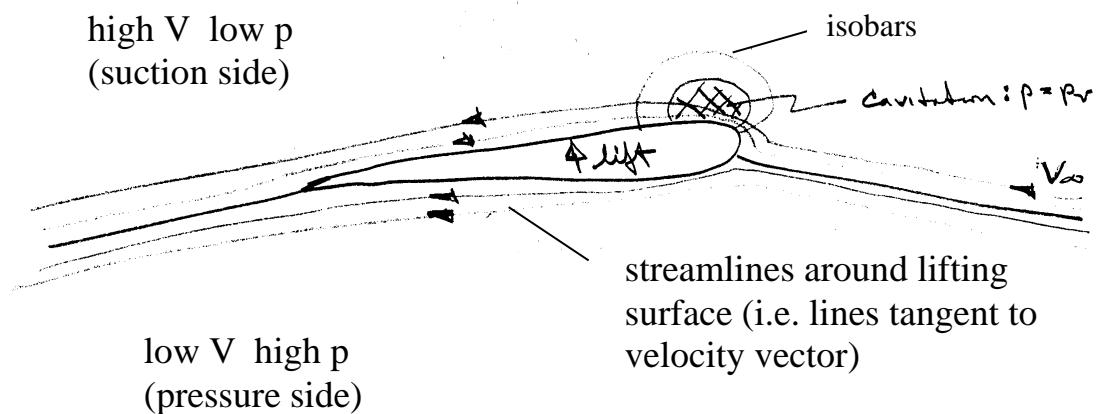
(G=Giga=10^9 M=Mega=10^6 k=kilo=10^3)

Gases are in general compressible, e.g. for ideal gas (i.e., $p = \rho RT$) at T = constant (isothermal)

$$\frac{dp}{d\rho} = RT$$

$$\boxed{E_v = \rho RT = p}$$

Vapor Pressure and Cavitation

When the pressure of a liquid falls below the vapor pressure it evaporates, i.e., changes to a gas. If the pressure drop is due to temperature effects alone, the process is called boiling. If the pressure drop is due to fluid velocity, the process is called cavitation. Cavitation is common in regions of high velocity, i.e., low p such as on turbine blades and marine propellers.

high V low p
(suction side)

isobars

Cavitation: $p = p_v$

V_∞

lift

streamlines around lifting surface (i.e. lines tangent to velocity vector)

low V high p
(pressure side)

Cavitation number, $C_a = \dfrac{p - p_v}{\dfrac{1}{2}\rho V_\infty^2}$

$C_a < 0$ implies cavitation

Surface Tension and Capillary Effects

At the interface of two immiscible fluids (e.g., a liquid and a gas), forces develop to cause the surface to behave as if it were a stretched membrane. Molecules in the interior attract each other equally, whereas molecules along the surface are subject to a net force due to the absence of neighbor molecules. The intensity of the molecular attraction per unit length along any line in the surface is call the surface tension and is designated by the Greek symbol σ.
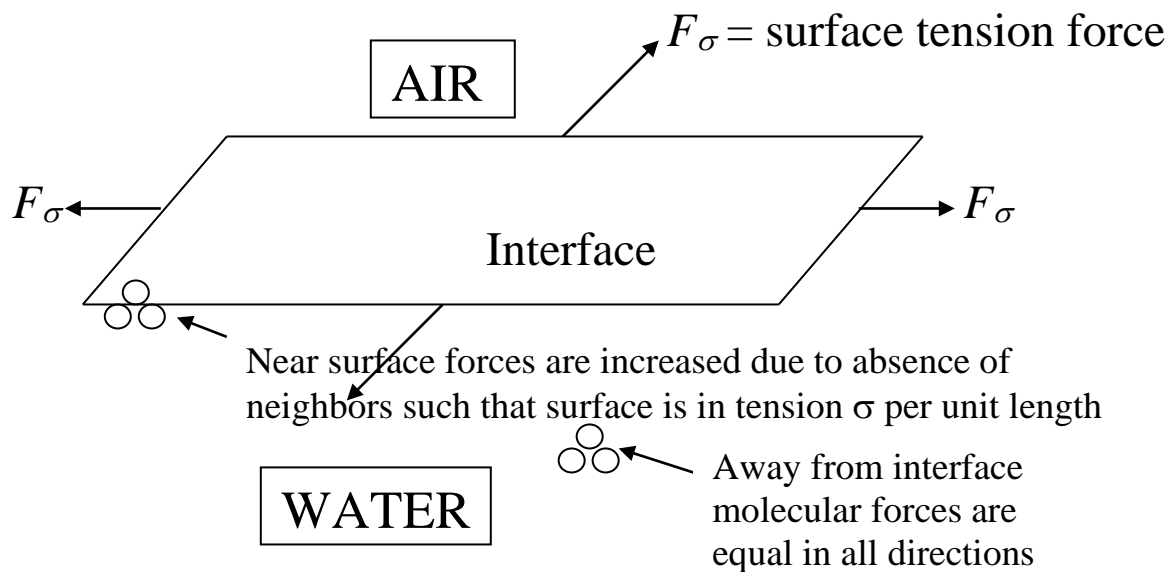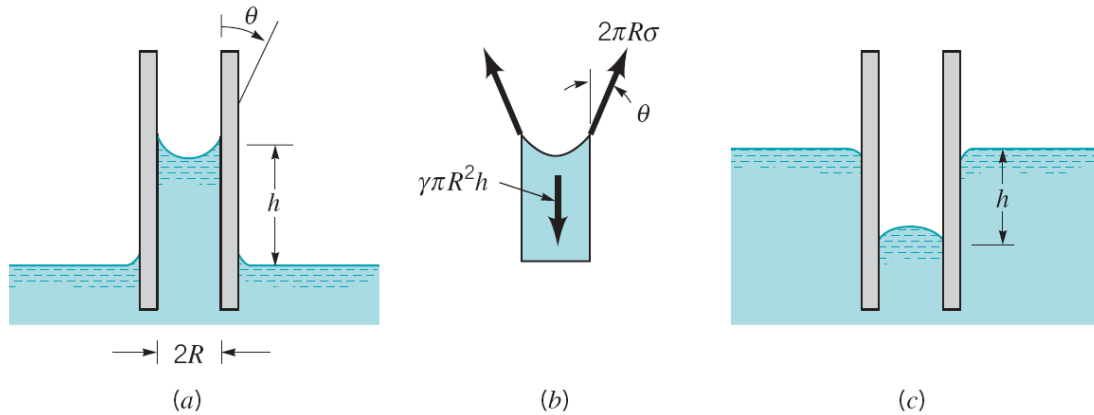

$\sigma_{air/water} = 0.073$ N/m

$F_\sigma = \sigma \times L =$ line force with direction normal to the cut

$L =$ length of cut through the interface

Effects of surface tension:

Contact angle:

$\theta < 90º$, Wetting $\theta > 90º$, Non-wetting
e.g., Water, $\theta \approx 0°$ e.g., Mercury, $\theta \approx 130°$

1. Capillary action in small tube

$$\Delta h = 4\sigma/\gamma d$$
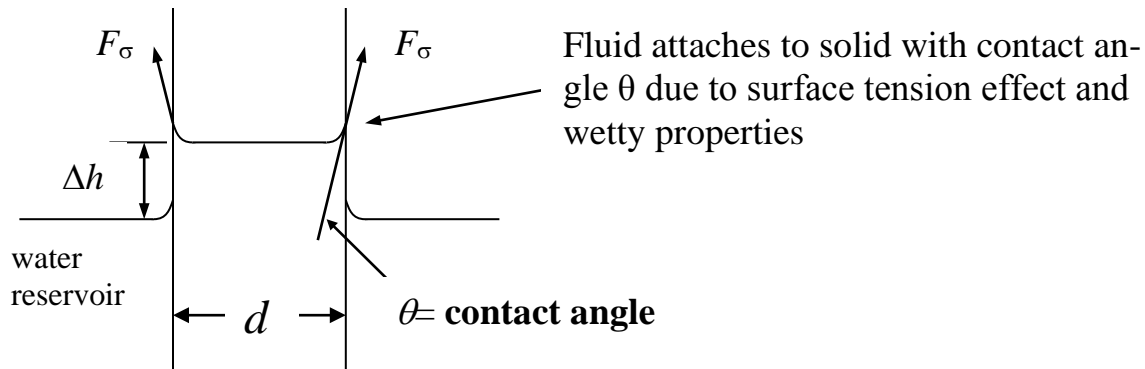
2. Pressure difference across curved interface

$$\Delta p = \sigma/R \qquad\qquad R = \text{radius of curvature}$$

3. Transformation of liquid jet into droplets

4. Binding of wetted granular material such as sand

5. Capillary waves: surface tension acts as restoring force
   resulting in interfacial waves called capillary waves

Capillary tube

F_σ F_σ Fluid attaches to solid with contact angle θ due to surface tension effect and wetty properties

Δh

water reservoir   ← d → θ= **contact angle**

Example:

Capillary tube $d = 1.6mm = 0.0016m$

$F_\sigma = \sigma \times L$, L=length of contact line between fluid & solid

(i.e., $L = \pi D$ = circumference)

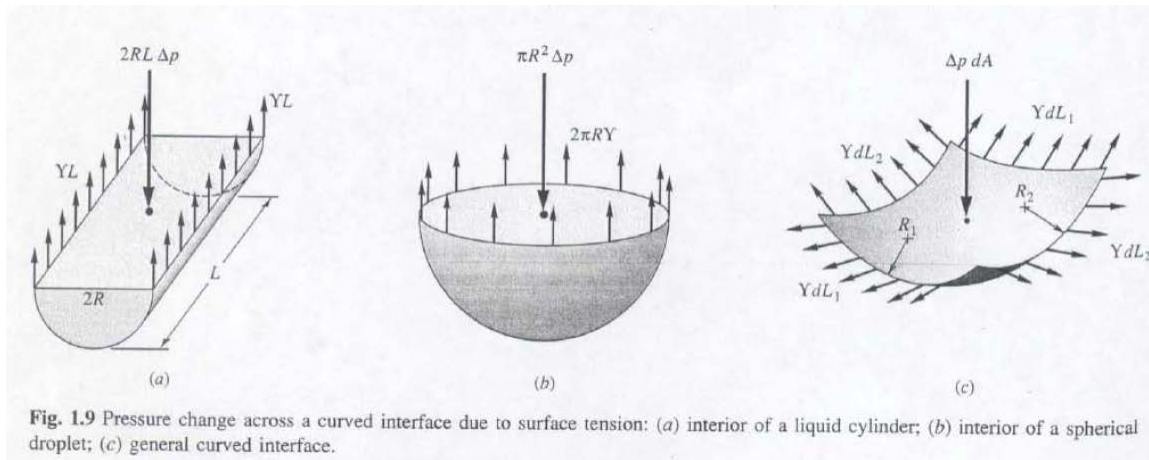water reservoir at 20° C, σ = 0.073 N/m, γ = 9790 N/m³

$\Delta h = ?$

$\Sigma F_z = 0$

$F_{\sigma,z} - \cancel{W} = 0$

$\sigma \pi d \cos\theta - \rho g \cancel{\forall} = 0 \qquad \theta \sim 0° \Rightarrow \cos\theta = 1$

$\qquad\qquad\qquad\qquad\qquad\qquad \rho g = \gamma$

$\sigma \pi d - \gamma \Delta h \dfrac{\pi d^2}{4} = 0 \qquad\qquad \forall = \Delta h \dfrac{\pi d^2}{4}$ =Volume of fluid above reservoir

$$\Delta h = \frac{4\sigma}{\gamma d} = 18.6mm$$

Pressure jump across curved interfaces

Fig. 1.9 Pressure change across a curved interface due to surface tension: (a) interior of a liquid cylinder; (b) interior of a spherical droplet; (c) general curved interface.

(a) Cylindrical interface
   Force Balance:

$$2\sigma L = 2\,RL\cdot(p_i - p_o)$$

$$\Delta p = \sigma/R$$

$p_i > p_o$, i.e. pressure is larger on concave vs. convex side of interface

(b) Spherical interface (Droplets)

$$\pi\cdot2R\sigma = \pi R^2 \Delta p \quad \rightarrow \quad \Delta p = 2\sigma/R$$

(c) General interface

$$\Delta p = \sigma(R_1^{-1} + R_2^{-1})$$

$R_{1,2}$ = principal radii of curvature

A brief history of fluid mechanics

See textbook section 1.10. (page 27)

Fluid Mechanics and Flow Classification

Hydrodynamics: flow of fluids for which density is constant such as liquids and low-speed gases. If in addition fluid properties are constant, temperature and heat transfer effects are uncoupled such that they can be treated separately.

Examples: hydraulics, low-speed aerodynamics, ship hydrodynamics, liquid and low-speed gas pipe systems

Gas Dynamics: flow of fluids for which density is variable such as high-speed gases. Temperature and heat transfer effects are coupled and must be treated concurrently.

Examples: high-speed aerodynamics, gas turbines, high-speed gas pipe systems, upper atmosphere