Review for Test 3

Chapter 9 Surface Resistance

- $C_D = C_{D_p} + C_f$

  form drag (C_{D_p}) skin-friction drag (C_f)

- Some exact solutions to the Navier-Stokes equations:
  Couette flow (without and with ∇p and incline)
  and flow down an inclined plane

- Laminar boundary layer

$$\rho\left(uu_x + vu_y\right) = -p_x + \mu u_{yy}$$
$$p = p_e$$
$$u_x + v_y = 0$$

- Blasius solution for laminar flow over a flat plate

- Turbulent boundary layer

  (1) Stability and transition

  (2) Decomposition of turbulent flows: mean-flow boundary layer equations

$$\rho\left(uu_x + vu_y\right) = -p_x + \mu u_{yy} - \frac{\partial}{\partial y}\left(\overline{u'v'}\right)$$
$$p = p_e$$
$$u_x + v_y = 0$$

  (3) Semi theoretical considerations of turbulent flow

  (a) Eddy-viscosity theory

$$-\rho\overline{u'v'} = \mu_t \frac{\partial u}{\partial y}$$

$$\mu_t = \rho l^2 \left|\frac{\partial u}{\partial y}\right|$$

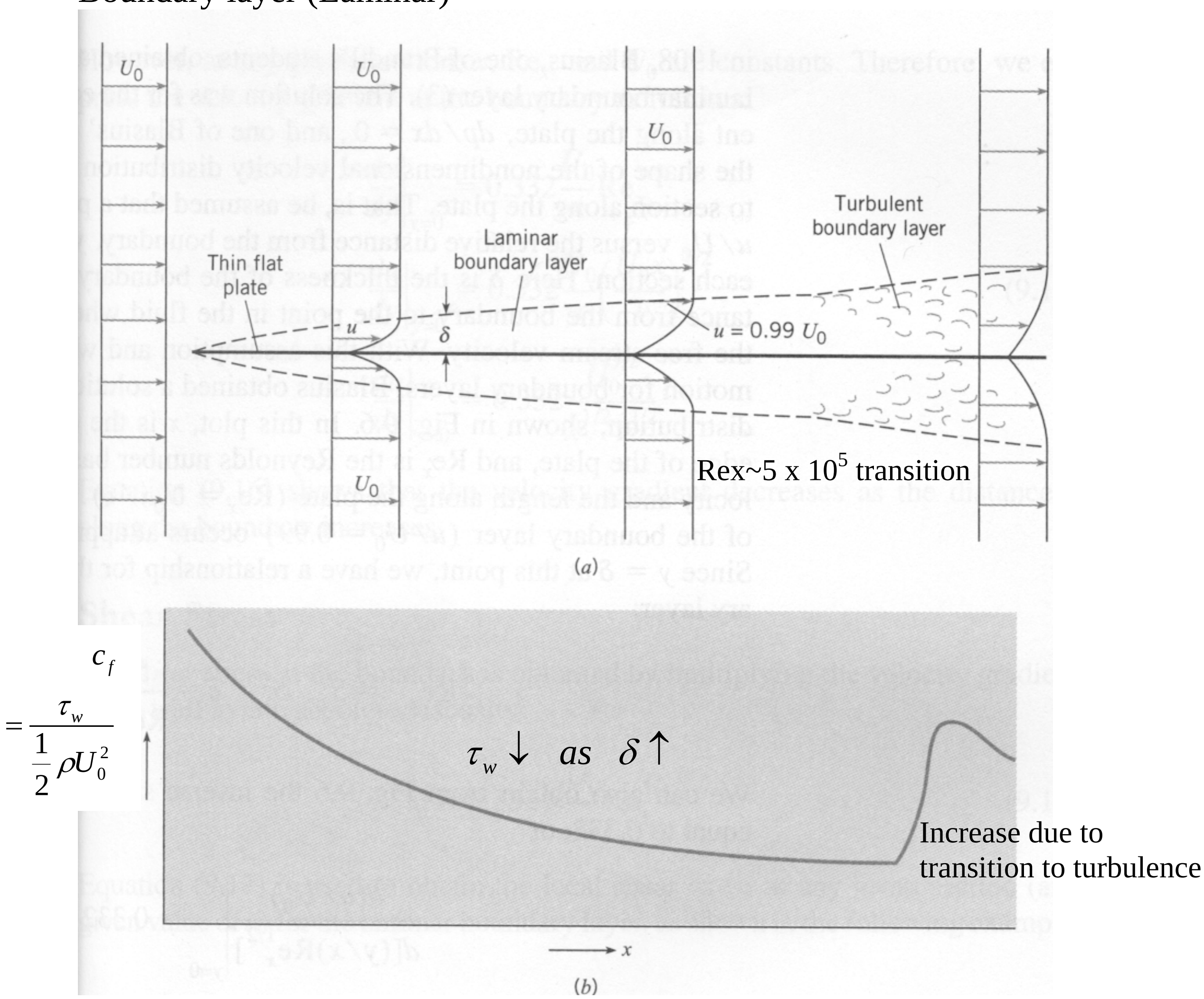
(b) Velocity profile correlations

Inner layer: $u^+ = y^+$

Overlap layer: $u^+ = \frac{1}{\kappa}\ln y^+ + B$

Outer layer: $\frac{U_e - u}{u^*} = f\left(y/\delta\right)$

- Boundary layer (Laminar)



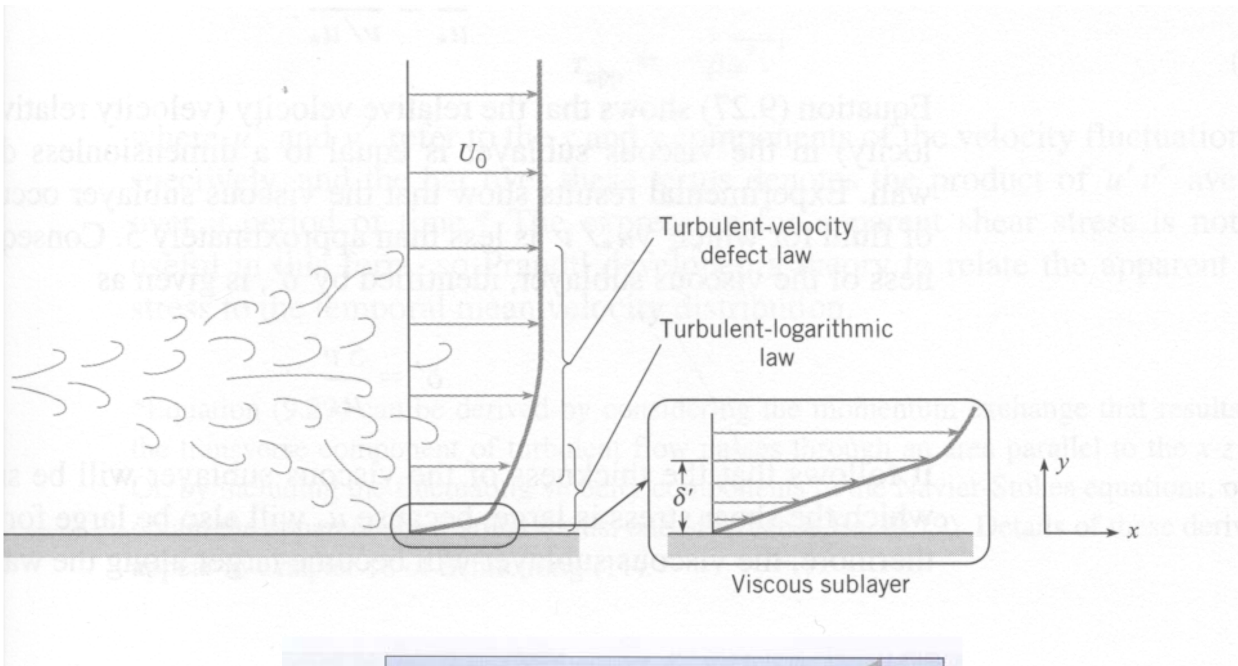
- Laminar flow solution by Blasius (1908):

$$\delta/x = 5/\sqrt{\mathrm{Re}_x}$$

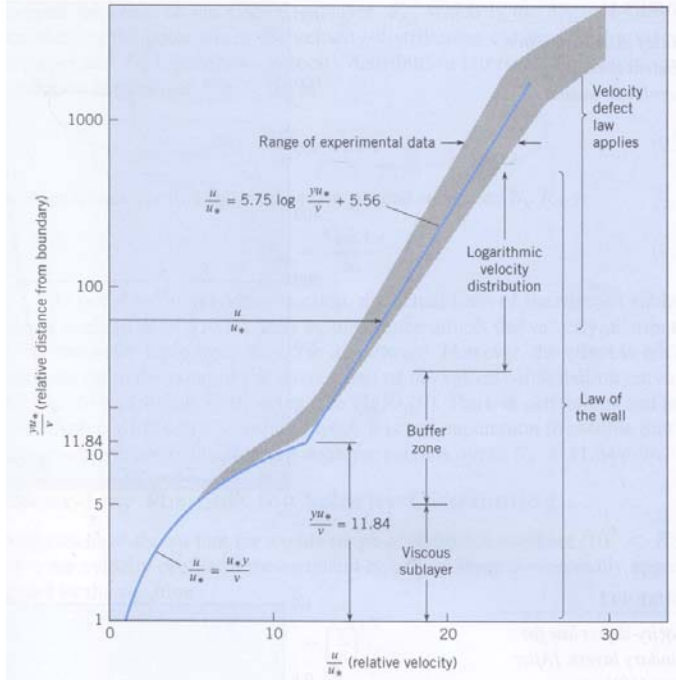
$$c_f = 0.664/\sqrt{\mathrm{Re}_x}$$ local wall-shear stress coefficient

$$C_f = B\int_0^L c_f\,dx = 1.33/\sqrt{\mathrm{Re}_L}$$ shear stress coefficient $= \dfrac{F_S}{\frac{1}{2}\rho U_0^2 BL}$

- Boundary layer (turbulent)



$$\frac{u}{U_0} \sim \left(\frac{y}{\delta}\right)^{1/7}$$



- Application of the momentum equation yields

$$\tau_w = \rho U_0^2 \frac{d\theta}{dx}$$

i.e., $\frac{c_f}{2} = \frac{d\theta}{dx}$ where $\theta =$ momentum thickness $= \int_0^\delta \frac{u}{U_0}\left(1-\frac{u}{U_0}\right)dy$

$=$ loss of momentum due to boundary layer

For $\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$, $\mathrm{Re} < 10^7$

$$\Rightarrow \theta = \frac{2}{72}\delta = \theta(\delta)$$

$$\frac{c_f}{2} = \frac{d\theta}{dx} \Rightarrow \frac{\tau_w}{\rho} = \frac{2}{72}U_0^2 \frac{d\delta}{dx}$$

From power-law fit to log-law evaluated at δ

$$\Rightarrow \frac{u}{u^*} = 8.74\left(\frac{\delta u^*}{\nu}\right)^{1/7} \qquad u^* = \sqrt{\tau_w/\rho}$$

Combining these equations and integrating yields

Boundary layer thickness $\frac{\delta}{x} = 0.37/\mathrm{Re}_x^{1/5}$

$\delta \propto x^{4/5}$

Laminar $\delta \propto x^{1/2}$

Faster growth than laminar, almost linear

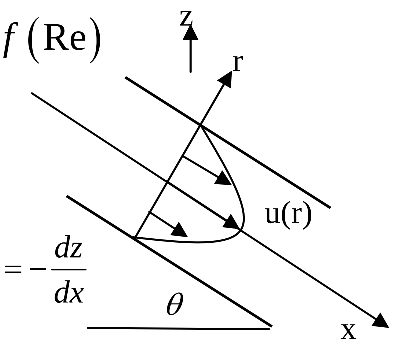
Local shear friction: $c_f = \frac{\tau_w}{\rho U_0^2/2} = \frac{0.058}{\mathrm{Re}_x^{1/5}}$

Skin friction drag: $C_f = \frac{1}{L}\int_0^L c_f\, dx = \frac{F_S}{\left(\rho U_0^2/2\right)BL} = \frac{0.074}{\mathrm{Re}_L^{1/5}}$

See text for some higher Re formulas

Chapter 10 Flow in Conduit

- Entrance vs. fully developed flow: $L_e/D = f(\mathrm{Re})$
- Laminar vs. turbulent flow: $\mathrm{Re}_{crit} \approx 2000$



$\sin\theta = -\frac{dz}{dx}$

Summary: Laminar pipe flow solution

$$u(r) = \frac{r^2 - r_0^2}{4\mu}\frac{d}{dx}(p+\gamma z),$$

$u(r) > 0$ for $\frac{d}{dx}(p+\gamma z) < 0$, i.e., favorable pressure gradient

$$\tau = -\frac{r}{2}\frac{d}{dx}(p+\gamma z)$$

$$\tau_0 = -\frac{r_0}{2}\frac{d}{dx}(p+\gamma z)$$

$$V = \frac{Q}{A} = -\frac{r_0^2}{8\mu}\frac{d}{dx}(p+\gamma z)$$

$$V = \frac{-D^2}{32\mu}\frac{[p_2+\gamma z_2 - p_1 - \gamma z_1]}{L}$$

$$\frac{p_1}{\gamma} + z_1 + \cancel{\frac{V_1^2}{2g}} = \frac{p_2}{\gamma} + z_2 + \cancel{\frac{V_2^2}{2g}} + h_L$$

$$-\gamma h_L = (p_2 + \gamma z_2) - (p_1 + \gamma z_1) \Rightarrow \frac{-\gamma h_L}{L} = \frac{d}{dx}(p+\gamma z)$$

$$h_L = \frac{32\mu L V}{\gamma D^2} = f\frac{L}{D}\frac{V^2}{2g}$$

$$\tau_0 = \frac{D}{4}\frac{\gamma h_L}{L} \Rightarrow h_L = \frac{4L\tau_0}{D\gamma} = 4L\frac{1}{8}\rho V^2 f/D\gamma = f\frac{L}{D}\frac{V^2}{2g}$$

$$\therefore \quad f = \frac{64}{\mathrm{Re}} \qquad \mathrm{Re} = \frac{VD}{\nu}$$

In general $f = f(\mathrm{Re}, k_s/D)$, $\quad k_s/D = roughness$

Summary: Turbulent pipe flow

Friction factor:

$1/\sqrt{f} = 1.14 - 2\log\left(k_s/D + 9.35/\mathrm{Re}\sqrt{f}\right)$, as shown in Moody diagram

$k_s/D =$ roughness parameter

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = h_1 - h_2,\ h_1 = \frac{p_1}{\gamma} + z_1,\ h_2 = \frac{p_2}{\gamma} + z_2$$

Types of problems:

1. Determine head loss

$$f = f\left(\text{Re}, k_s/D\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = h_1 - h_2$$

2. Determine flow rate

$$V = \underbrace{\left[\frac{2gh_L}{LD}\right]^{1/2}}_{known\ \ from\ \ data} f^{-1/2}$$

guess $f \Rightarrow V \Rightarrow \text{Re} \Rightarrow f$, repeat to converge

3. Determine pipe size

$$D = \underbrace{\left[\frac{8LQ^2}{\pi^2 gh_L}\right]^{1/5}}_{known\ \ from\ \ data} f^{1/5}$$

guess $f \Rightarrow D \Rightarrow \text{Re}\,\&\, k_s/D \Rightarrow f$, repeat to converge

Alternate method is experiment formulas in text

$$f = \frac{0.25}{\left[\log\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2}$$

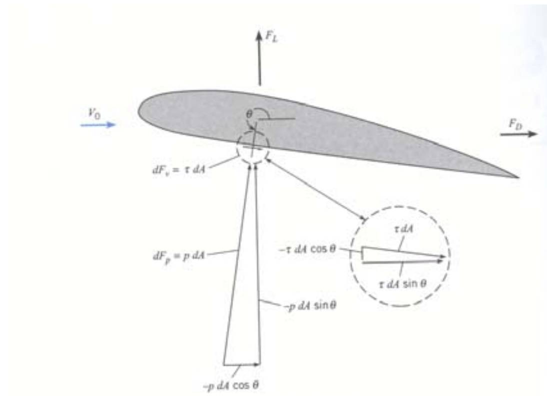
$$Q = -2.22D^{5/2}\sqrt{gh_L/L}\,\log\left(\frac{k_s}{3.7D} + \frac{1.78\nu}{D^{3/2}\sqrt{gh_L/L}}\right)$$

$$D = 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_L}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

Minor losses:

$$\frac{p_1}{\gamma}+z_1+\alpha_1\frac{V_1^2}{2g}+h_p=\frac{p_2}{\gamma}+z_2+\alpha_2\frac{V_2^2}{2g}+h_t+h_L+\sum_i h_{m_i}$$

$$h_{m_i}=\left(K\frac{V^2}{2g}\right)_i$$



Chapter 11 External flows

- $$F_D=\int_A \underbrace{\left(-p\cos\theta+\tau\sin\theta\right)dA}_{dF_D=\left(-p\mathbf{n}+\tau\mathbf{t}\right)\cdot\mathbf{t}dA}$$

$$C_D=\frac{F_D}{\frac{1}{2}\rho U_0^2 A_p}=C_{D_{Form}}+C_{D_{Skin\ Friction}}$$

Often must be obtained experimentally (C_D)

Projection (A_p)

$$C_D=C_D\left(geometry,\mathrm{Re},\frac{k_s}{L},Ma,etc.\right)$$

- Influence of geometry:
  - angular vs. curved bodies
  - streamlining
  - $C_{D\ Form}$ vs. $C_{D\ SF}$ as $f\left(geometry\right)$

- Roughness

- Vortex shedding: Strouhal No. $=S=\dfrac{nd}{U_0}=S\left(\mathrm{Re}\right)$

- Terminal velocity: $\sum\mathbf{F}=0\Rightarrow\mathbf{a}=0\quad define\quad V_{TERMIN}$

- $C_D\left(Ma\right)$ use near $Ma\sim 1$ due to shock waves

- Lift