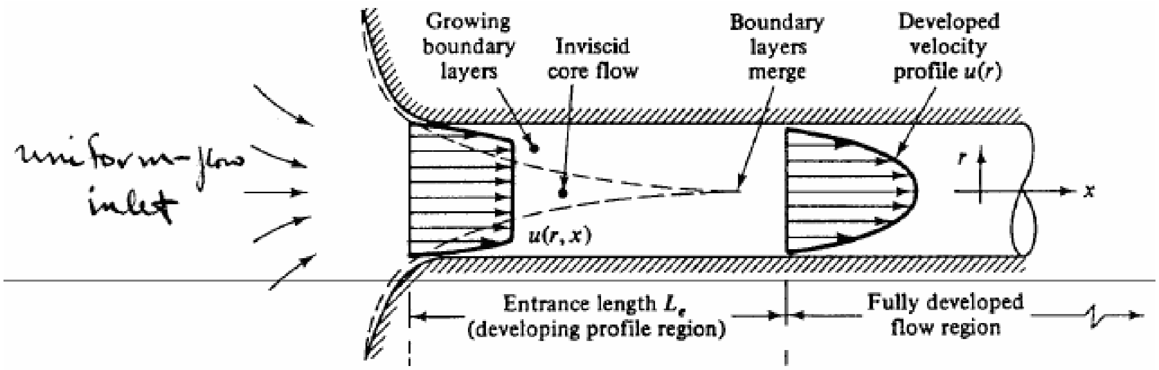
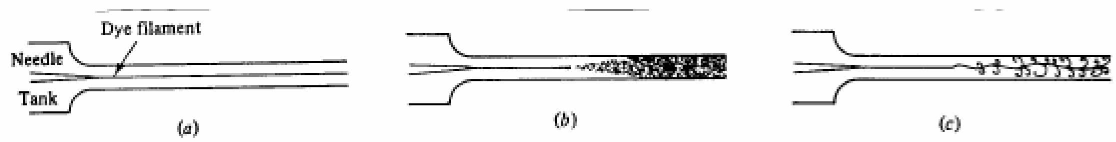
Review for Chapter 8 and Chapter 9

Chapter 8: Flow in Conduits



Laminar flow: $Re_{crit} \sim 2000$, i.e., for $Re < Re_{crit}$ laminar

$Re > Re_{crit}$ turbulent



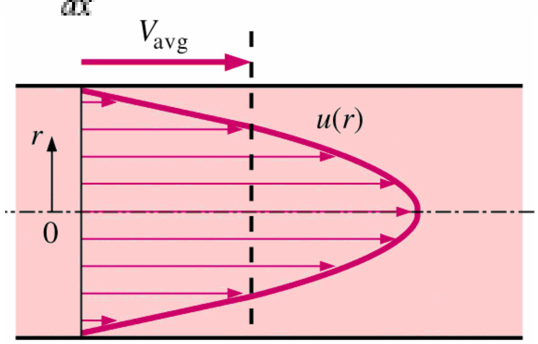
laminar turbulent spark photo

Laminar Flow in Pipes

Velocity profile for laminar pipe flow

$$u(r) = \frac{r^2 - r_0^2}{4\mu}\frac{d}{dx}(p + \gamma z),$$

$$u(r) > 0 \text{ for } \frac{d}{dx}(p + \gamma z) < 0, \text{ i.e., favorable pressure gradient}$$



Laminar flow

Head loss and friction factor for laminar pipe flow (These are exact solutions!!):

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

$$\Delta h=\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)$$

$$h_L=\frac{p_1-p_2}{\gamma}+(z_1-z_2)=-\Delta h$$

$$h_L=\frac{L}{\gamma}\left[-\frac{d}{ds}(p+\gamma z)\right] \qquad L = \text{length of pipe} = ds$$

$$=\frac{L}{\gamma}\left[\frac{8\mu\overline{V}}{r_o^2}\right]=-\Delta h\alpha\overline{V} \qquad h_L=L\left[-\frac{d}{ds}\left(\frac{p}{\gamma}+z\right)\right]=L\left(-\frac{dh}{ds}\right)$$

or $$h_f=h_L=\frac{32\mu L\overline{V}}{\gamma D^2} \qquad h_f = \text{head loss due to friction}$$

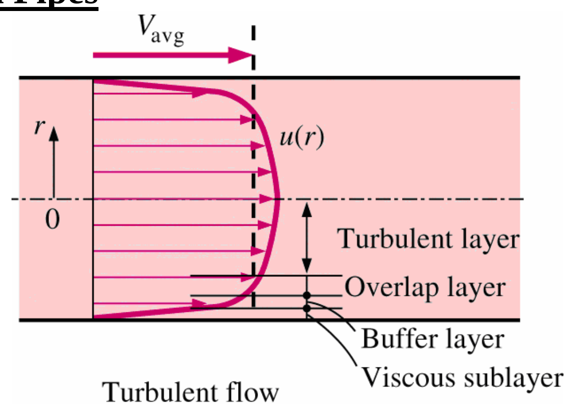
exact solution

$$h_L=\frac{32\mu LV}{\gamma D^2}=f\frac{L}{D}\frac{V^2}{2g}$$

$$\tau_0=\frac{D}{4}\frac{\gamma h_L}{L}\Rightarrow h_L=\frac{4L\tau_0}{D\gamma}=4L\frac{1}{8}\rho V^2 f/D\gamma=f\frac{L}{D}\frac{V^2}{2g}$$

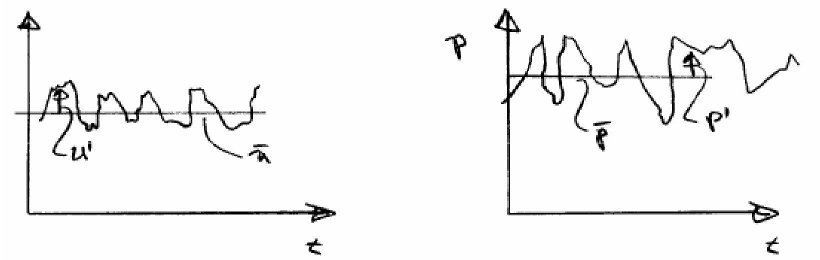
$$\therefore f=\frac{64}{Re} \qquad Re=\frac{VD}{\nu}$$

Turbulent Flow in Pipes



$$h_f = f \cdot \frac{L}{D} \cdot \frac{\overline{V}^2}{2g} \qquad \text{Darcy – Weisbach Equation}$$

Description of turbulent flow



Velocity and pressure are random functions of time!!
They can be separated into two parts such as mean and fluctuation components:

$$u = \overline{u} + u' \qquad\qquad p = \overline{p} + p'$$
$$v = \overline{v} + v' \qquad\qquad \text{and for compressible flow}$$
$$w = \overline{w} + w' \qquad\qquad \rho = \overline{\rho} + \rho' \text{ and } T = \overline{T} + T'$$

Most important influence of turbulence on the mean motion:

→An increase in the fluid stress by "Reynolds stresses"

$$\tau'_{ij} = -\rho\overline{u'_i u'_j}$$
$$= \begin{bmatrix} -\rho\overline{u'^2} & -\rho\overline{u'v'} & -\rho\overline{u'w'} \\ -\rho\overline{u'v'} & -\rho\overline{v'^2} & -\rho\overline{v'w'} \\ -\rho\overline{u'w'} & -\rho\overline{v'w'} & -\rho\overline{w'^2} \end{bmatrix}$$

Mean flow equations for turbulent flow

→Reynolds Averaged Navier Stokes (RANS) equations

Continuity $\quad \nabla \cdot \underline{V} = 0 \quad \text{i.e. } \nabla \cdot \overline{\underline{V}} = 0 \text{ and } \nabla \cdot \underline{V}' = 0$

Momentum $\quad \rho\frac{D\overline{V}}{Dt} = -\rho g\hat{k} - \nabla\overline{p} + \nabla \cdot \tau_{ij}$

$$\tau_{ij} = \mu\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \underbrace{-\rho\overline{u'_i u'_j}}_{\tau'_{ij}}$$

$u_1 = u \quad x_1 = x$
$u_2 = v \quad x_2 = y$
$u_3 = w \quad x_3 = z$

‘Modeling’ required!!

Turbulence Modeling:

a) Eddy viscosity, Mixing-length theory, One-equation model, Two- equation model
→(k-ε model, k-ω model: **Recall CFD-PreLab2, Lab2!!**)

b) Mean-flow velocity profile correlations

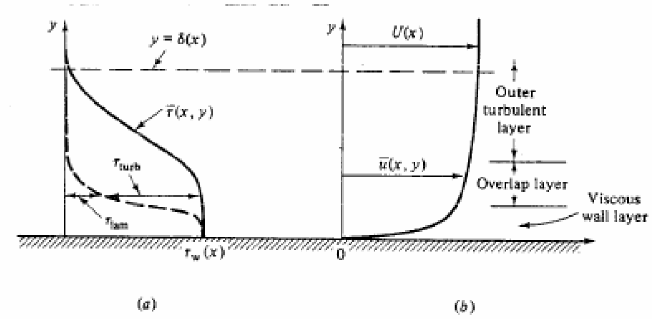


Fig. 6.8 Typical velocity and shear distributions in turbulent flow near a wall: (a) shear; (b) velocity. (measurements)

Inner layer: viscous stress dominates

Outer layer: turbulent stress dominates

Overlap layer: both types of stress important

1) Inner layer

$$u = f(\mu, \tau_w, \rho, y) \qquad \text{note: not } f(\delta)$$

$$u^+ = f(y^+) \qquad \text{law-of-the-wall}$$

$$u^+ = y^+$$

where: $u^+ = \dfrac{u}{u^*}$

u^* = friction velocity = $\sqrt{\tau_w/\rho}$

$$y^+ = \frac{yu^*}{\nu}$$

very near the wall:

$$\tau \sim \tau_w \sim \text{constant} = \mu\frac{du}{dy} \quad \Rightarrow \quad u = cy \quad \text{or} \quad u^+ = y^+$$

2) Outer layer

$$\frac{U_e - u}{u^*} = f\left(\frac{y}{\delta}\right) \qquad \text{velocity defect law}$$

3) Overlap layer

$$\frac{u}{u^*} = \frac{1}{\kappa}\ln y^+ + B \qquad \text{log-law}$$

$$\kappa \approx .41 \qquad B \approx 5$$

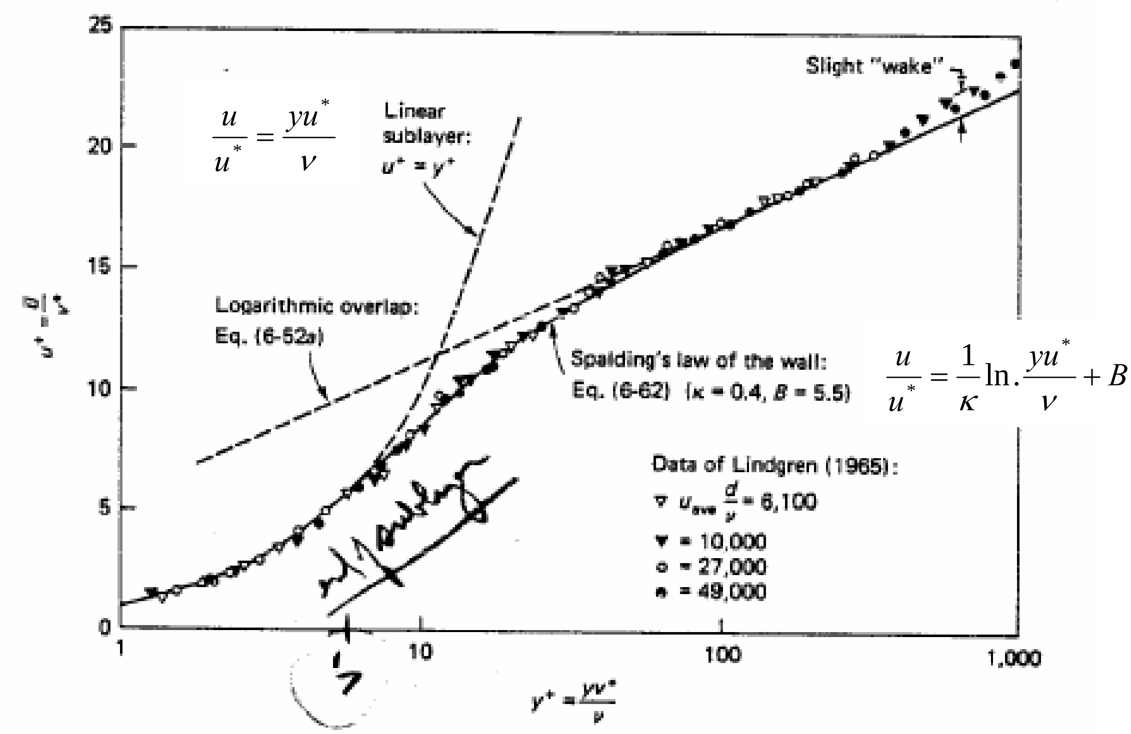


FIGURE 6-6
Comparison of Spalding's inner-law expression with the pipe-flow data of Lindgren (1965).

Velocity distribution and resistance in smooth pipes

$$\frac{V}{u^*} = 2.44\ln\underbrace{\frac{r_o u^*}{\nu}}_{\frac{1}{2}\text{Re}\left(\frac{f}{8}\right)^{1/2}} + 1.34 = \left(\frac{\rho V^2}{\tau_o}\right)^{1/2} = \left(\frac{8}{f}\right)^{1/2}$$

$$\frac{1}{\sqrt{f}} = 2\log\left(\text{Re}\, f^{1/2}\right) - .8 \qquad \text{Re} > 3000$$

$$h_f = .316\left(\frac{\mu}{\rho VD}\right)^{1/4}\frac{L}{D}\frac{V^2}{2g}$$

Velocity distribution and resistance in rough pipes:

Inner layer:

$$u^+ = u^+(y/k)$$

Overlap layer:

$$u_R^+ = \frac{1}{\kappa}\ln\frac{y}{k} + \text{constant} \qquad \text{rough}$$

$$k^+ = \frac{ku^*}{\nu}$$

1. $k^+ < 5$ hydraulically smooth (no effect of roughness)
2. $5 < k^+ < 70$ transitional roughness (Re dependence)
3. $k^+ > 70$ fully rough (independent Re)

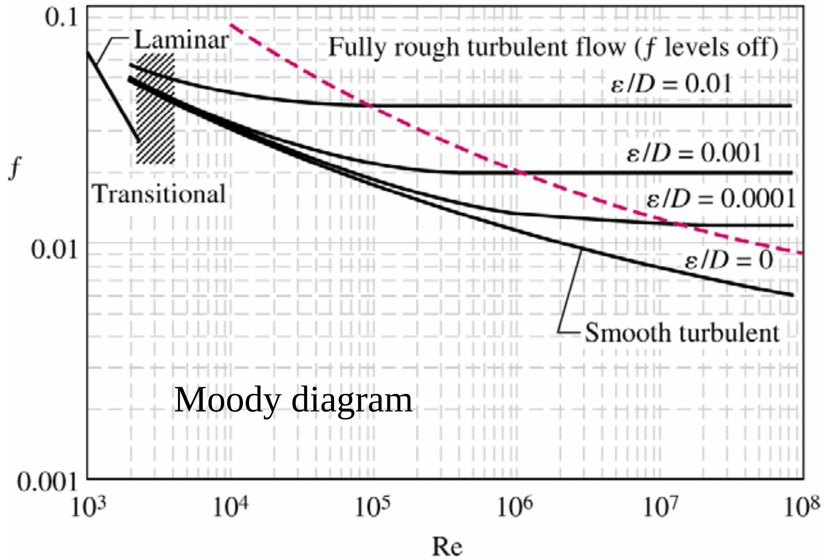
Head losses and friction factor for turbulent pipe flow:

Friction factor:

$1/\sqrt{f} = 1.14 - 2\log\left(k_s/D + 9.35/\mathrm{Re}\sqrt{f}\right)$, as shown in Moody diagram

$k_s/D =$ roughness parameter

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = h_1 - h_2,\ h_1 = \frac{p_1}{\gamma} + z_1,\ h_2 = \frac{p_2}{\gamma} + z_2$$



0.1
0.01
0.001
f
Laminar
Transitional
Fully rough turbulent flow (f levels off)
ε/D = 0.01
ε/D = 0.001
ε/D = 0.0001
ε/D = 0
Smooth turbulent
Moody diagram
10^3
10^4
10^5
10^6
10^7
10^8
Re

Types of problems for turbulent pipe flow

Three Cononical Types of Problems

1. Determine The Head Loss

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = -\Delta h = \left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_2\right)$$

$$f = f\left(\mathrm{Re}, k/D\right)$$

2. Determine The Flow Rate

$$V = \underbrace{\left[\frac{2gh_f}{L/D}\right]^{1/2}}_{\text{known from problem statement}} f^{-1/2}$$

Given $f \rightarrow \underbrace{V \rightarrow \mathrm{Re} \rightarrow f}$, repeat to convergence

2. Determine The Pipe Diameter Rate

$$D = \underbrace{\left[\frac{8LQ^2}{\pi^2 g h_f}\right]^{1/5}}_{\text{known from problem statement}} f^{-1/5}$$

Given $f \rightarrow \underbrace{D \rightarrow \mathrm{Re}, k/D \rightarrow f}$, repeat to convergence

<u>Flows at pipe inlets and losses from fittings</u>

1. entrance and exit effects
2. expansions and contractions
3. bends, elbows, tees, and other fittings
4. valves (open or partially closed)

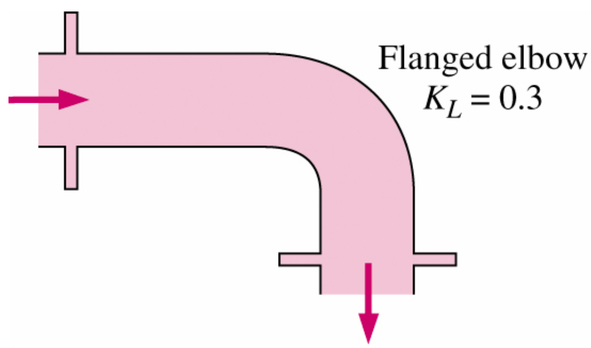
} can be large effect

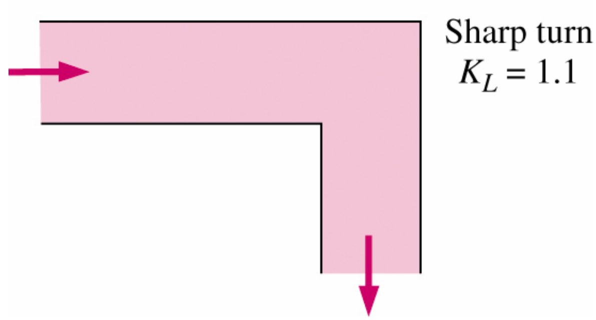
Modified Energy Equation to Include Minor Losses:

$$\frac{p_1}{\gamma} + z_1 + \frac{1}{2g}\alpha_1 V_1^2 + h_p = \frac{p_2}{\gamma} + z_2 + \frac{1}{2g}\alpha_2 V_2^2 + h_t + h_f + \sum h_m$$

$$h_m = K\frac{V^2}{2g}$$

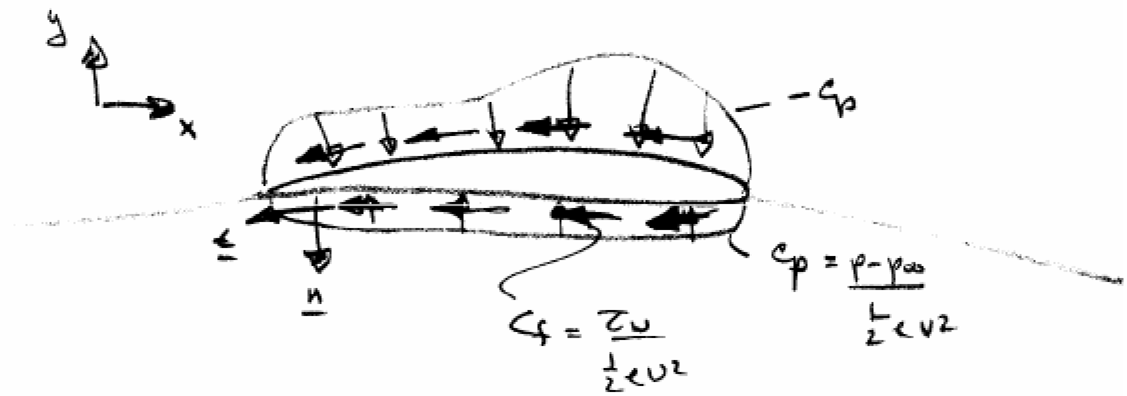
(K: minor loss coefficient→Depending on the shape of the pipe inlet/exit/curvature)





Chapter 9: Flow over Immersed Bodies

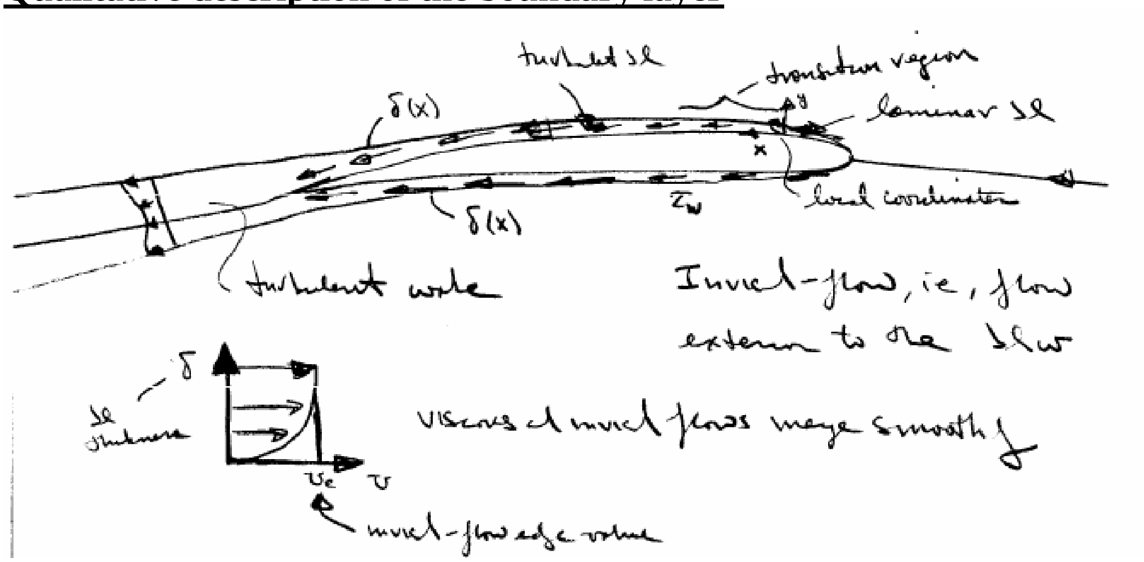
Separation of drag: Form and skin-friction



$$C_D = \frac{1}{\frac{1}{2}\rho V^2 A}\Big\{\underbrace{\int_S (p-p_\infty)\underline{n}\cdot\hat{i}\,dA}_{C_{Dp}} + \underbrace{\int_S \tau_w\,\underline{t}\cdot\hat{i}\,dA}_{C_f}\Big\} \quad : \text{Drag}$$

$$C_L = \frac{1}{\frac{1}{2}\rho V^2 A}\Big\{\int_S (p-p_\infty)\underline{n}\cdot\hat{j}\,dA\Big\} \quad : \text{Lift}$$

Qualitative description of the boundary layer



Boundary layer equation (2D case presented)

→Obtained from NS equation order of magnitude for each terms

Variable	order of magnitude		
u	U	O(1)	
v	δ<<L	O(ε)	ε = δ/L
$\frac{\partial}{\partial x}$	L	O(1)	
$\frac{\partial}{\partial y}$	1/δ	$O(\varepsilon^{-1})$	
ν	δ^2	ε^2	

Boundary layer equation in 2D:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Quantitative relations for the laminar boundary layer

Laminar boundary layer over a flat plate→Blasius solution

$$\delta = \frac{5x}{\sqrt{Re}} \qquad \text{value of y where } u/U_\infty = .99$$

$$Re_x = \frac{U_\infty x}{\nu}$$

$$c_f = \frac{2\tau_w}{\rho U_\infty^2} = \frac{0.664}{\sqrt{Re_x}} = \frac{\theta}{x}$$

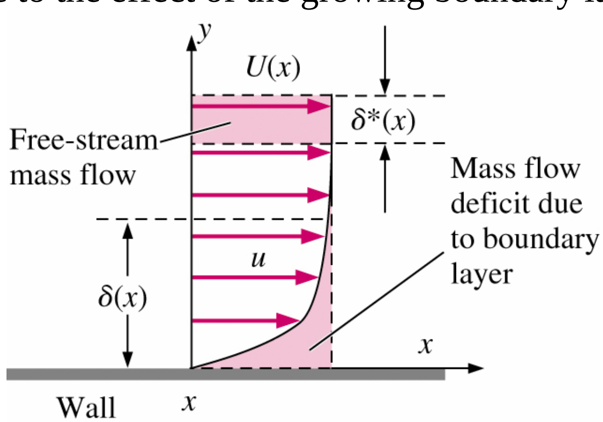
$$C_f = \frac{1}{L}\int_0^L c_f\,dx = 2c_f(L)$$

$$= \frac{1.328}{\sqrt{Re_L}}$$

$$\frac{U_\infty L}{\nu}$$

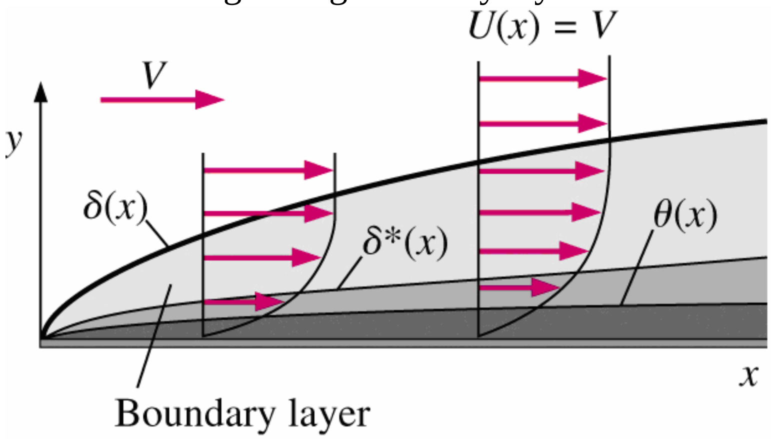
$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy = 1.7208 \frac{x}{\sqrt{Re_x}} \qquad \text{displacement thickness}$$

Displacement thickness: imaginary increase in thickness of the wall, as seen by the outer flow, due to the effect of the growing boundary layer.



$$\theta = \int_0^{\delta} \left(1 - \frac{u}{U_\infty}\right) \frac{u}{U_\infty} dy = 0.664 \frac{x}{\sqrt{Re_x}} \quad \text{momentum thickness}$$

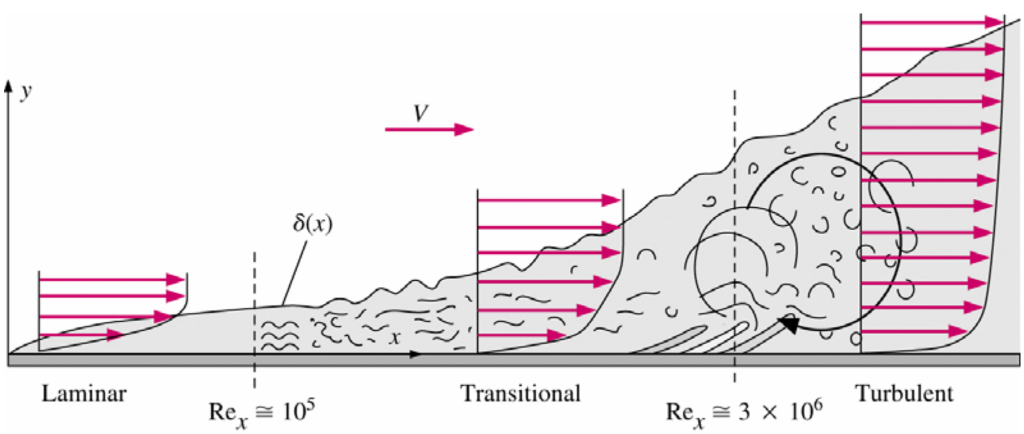
Momentum thickness: the loss of momentum flux per unit width divided by ρU^2 due to the presence of the growing boundary layer.



$$H = \text{shape parameter} = \frac{\delta^*}{\theta} = 2.5916$$

Quantitative relations for the turbulent boundary layer

Transition from laminar boundary-layer to turbulent boundary-layer:

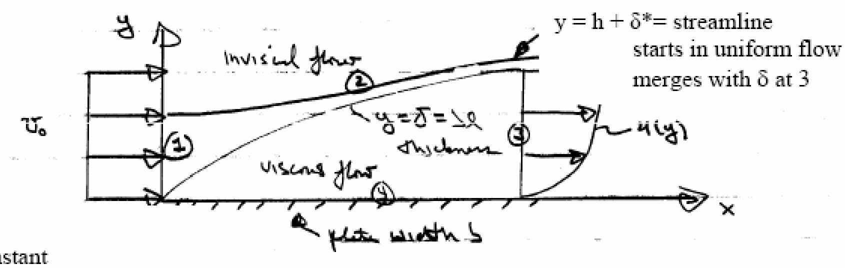


Engineering critical Reynolds number: $Re_{x,\,cr} = 5 \times 10^5$

→ $Re_x < Re_{x,\,cr}$: boundary layer is most likely laminar.

→ $Re_x > Re_{x,\,cr}$: boundary layer is most likely turbulent.

Momentum Integral Analysis: obtain general momentum integral relation
→Valid for both laminar and turbulent flow



Steady
ρ = constant
neglect g
$v << u = u_o \Rightarrow p$ = constant
i.e., $-\nabla p = 0$

CV = 1, 2, 3, 4

$$-D = \text{drag} = b\int_0^x \tau_w dx$$

pressure force = 0 for $v << U_o$

force on CV (−D); wall shear stress (τ_w)

$u \sim U_o$

$$\sum F_x = -D = \rho\int_1 u(\underline{V}\cdot\underline{dA}) + \rho\int_3 u(\underline{V}\cdot\underline{dA})$$

$$= \rho(-U_o^2 bh) + \rho b\int_3 u^2 dy$$

$$D(x) = \rho b U_o \int_0^\delta u dy - \rho b\int_0^\delta u^2 dy$$

$$= \rho b\int_0^\delta u(U_o - u)dy$$

$$C_D = \frac{D}{\frac{1}{2}\rho U_o^2 bL} = \frac{2}{L}\underbrace{\int_0^\delta \frac{u}{U_o}\left(1-\frac{u}{U_o}\right)dy}_{\theta\,=\,\text{momentum thickness}}$$

On the other hand, C_D can be expressed as

$$C_D = \frac{D}{\frac{1}{2}\rho U_o^2 A} = \frac{b\int_0^x \tau_w dx}{\frac{1}{2}\rho U_o^2 bL} = \frac{2\theta}{L}$$

Then,

$$\int_0^x \frac{\tau_w}{\frac{1}{2}\rho U_o^2}(x)dx = 2\theta(x)$$

Therefore,

$$\frac{1}{2}\left(\frac{\tau_w}{\frac{1}{2}\rho U_o^2}\right) = \frac{d\theta}{dx}$$

Finally, momentum integral relation is obtained as

$$\frac{c_f}{2} = \frac{d\theta}{dx}$$

c_f = local skin friction coefficient

momentum integral relation for flat plate boundary layer

Approximate solutions for a **laminar** boundary layer obtained from momentum integral analysis:

	Exact Blassius	
$\delta = \frac{4.65x}{\sqrt{Re_x}}$	$\frac{5x}{\sqrt{Re_x}}$	7% ↓
$\tau_w = \frac{.323\rho V^2}{\sqrt{Re_x}}$	$\frac{.332\rho U^2}{\sqrt{Re_x}}$	3%↓
$c_f = \frac{.646}{\sqrt{Re_x}}$	$\frac{.664}{\sqrt{Re_x}}$	
$C_f = \frac{1.29}{\sqrt{Re_L}}$	$\frac{1.33}{\sqrt{Re_L}}$	

Approximate solutions for a turbulent boundary layer obtained from momentum integral analysis:

$$\frac{\delta}{x} = .16 Re_x^{-1/7}$$

$$c_f = \frac{.027}{Re_x^{1/7}}$$

$$C_f = \frac{.031}{Re_L^{1/7}} = \frac{7}{6} C_f(L)$$

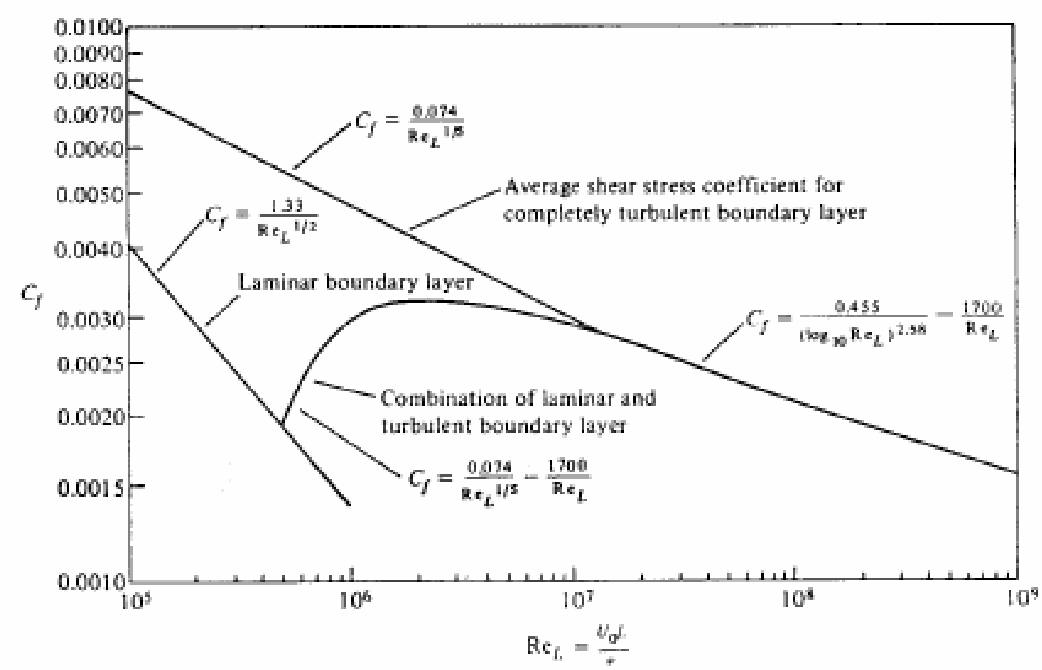
Total shear-stress coefficient

$$C_f = \frac{.455}{(\log_{10} Re_L)^{2.58}} \frac{-1700}{Re_L} \qquad Re > 10^7$$

$$\frac{\delta}{L} = c_f(.98 \log Re_L - .732)$$

Local shear-stress coefficient

$$c_f = (2\log Re_x - .65)^{-2.3}$$



C_f vs Re_L relationship in laminar and turbulent boundary layer

Drag of 2D bodies

Flat-plate parallel to the flow:

$$C_{Dp} = \frac{1}{\frac{1}{2}\rho V^2 A}\int_S (p - p_\infty)\underline{n}\cdot \mathbf{\imath} = 0$$

$$C_f = \frac{1}{\frac{1}{2}\rho V^2 A}\int_S \tau_w \underline{t}\cdot \hat{i}\, dA$$

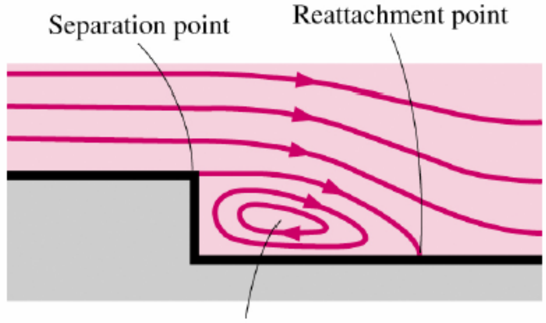
$$= \frac{1.33}{Re_L^{1/2}} \qquad \text{laminar flow}$$

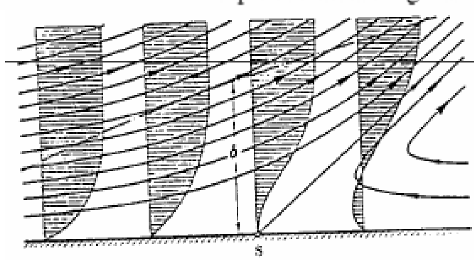
$$= \frac{.074}{Re_L^{1/5}} \qquad \text{turbulent flow}$$

Flow separation:

→The fluid stream detaches itself from the surface of the body at sufficiently high velocities. Only appeared in viscous flow!!

Flow separation forms the region called 'separated region'





S = point of separation

Fig. 2.12. Diagrammatic representation of flow in the boundary layer near a point of separation

Drag coefficients in common geometries:

Table 7.2
DRAG OF TWO-DIMENSIONAL BODIES AT Re = 10^5

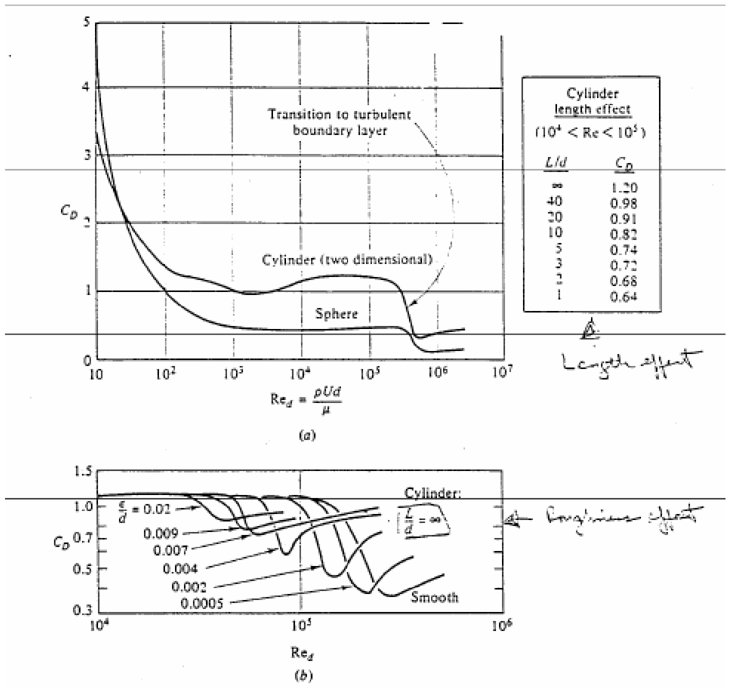
Shape	*C_D based on frontal area*	*Shape*	*C_D based on frontal area*
Plate:		Half-cylinder:	
	2.0		1.2
Square cylinder:			
	2.1		1.7
		Equilateral triangle:	
	1.6		1.6
Half tube:			
	1.2		2.0
	2.3		

Table 7.3
DRAG OF THREE-DIMENSIONAL BODIES AT Re ≈ 10^5

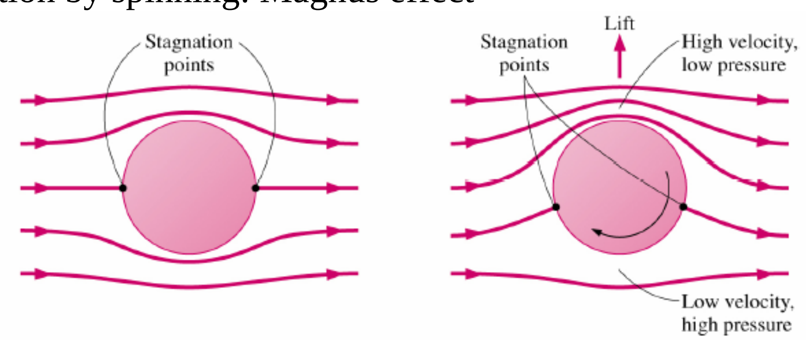
Body	*Ratio*	*C_D based on frontal area*
Cube:		
		1.07
		0.81
60° cone:		
		0.5
Disk:		
		1.17
Cup:		
		1.4
		0.4
Parachute (low porosity):		
		1.2

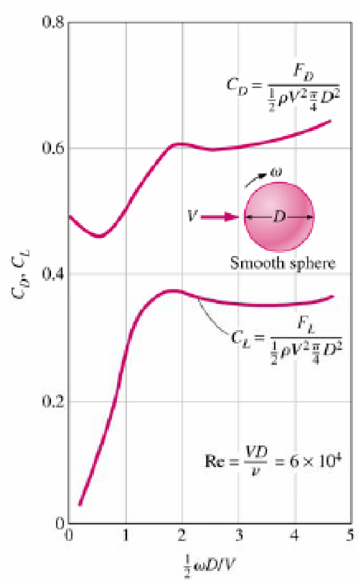
Flow over cylinder and spheres:

- Re<1: Creeping flow, C_D=24/Re, No-flow separation regime
- 10^5<Re<10^6: boundary layer become turbulent, large reduction in C_D

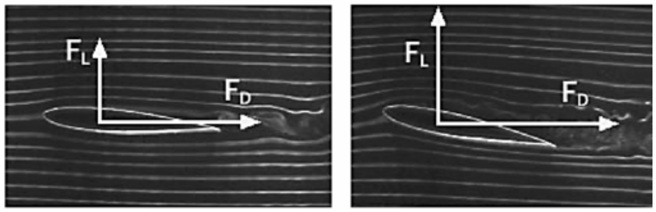


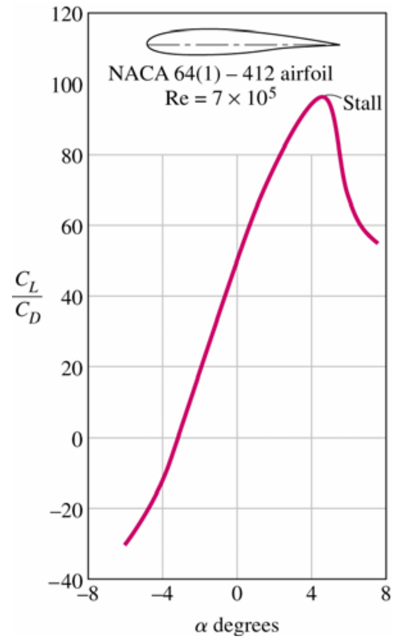
Lift generation by spinning: Magnus effect





Lift acting on the airfoil;





Minimum flight velocity :

$$V_{\min} = \sqrt{\frac{2W}{\rho C_{L,\max} A}}$$