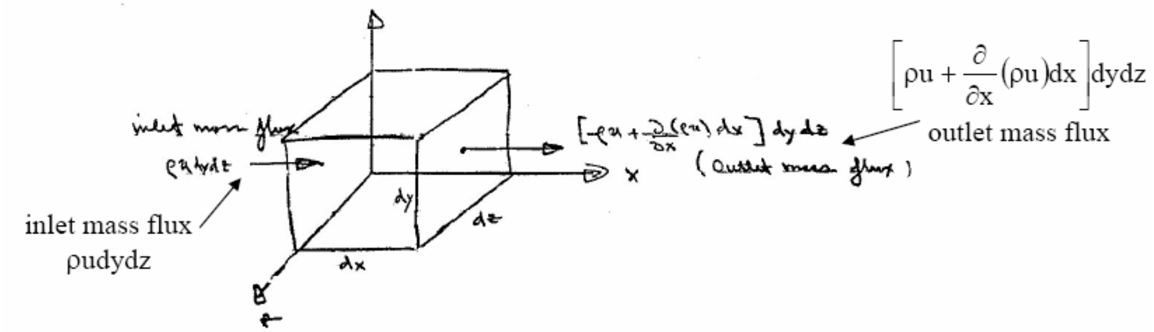
Review for Exam 3

Chapter 9: Differential analysis of Fluid Flow

Governing equation of:

Integral form→Useful for large scale CV analysis
Differential form→Useful for relatively small-scale point analysis

9.1Continuity equation in differential form



$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

per unit ~~V~~
differential form of
continuity equations

$$\frac{\partial\rho}{\partial t}+\underbrace{\nabla\cdot(\rho\underline{V})}_{\rho\nabla\cdot\underline{V}+\underline{V}\cdot\nabla\rho}=0$$

$$\frac{D\rho}{Dt}+\rho\nabla\cdot\underline{V}=0 \qquad \frac{D}{Dt}=\frac{\partial}{\partial t}+\underline{V}\cdot\nabla$$

Simplifications:

1. Steady flow: $\nabla\cdot(\rho\underline{V})=0$

2. $\rho=$ constant: $\nabla\cdot\underline{V}=0$

i.e., $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \text{3D}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{2D}$$

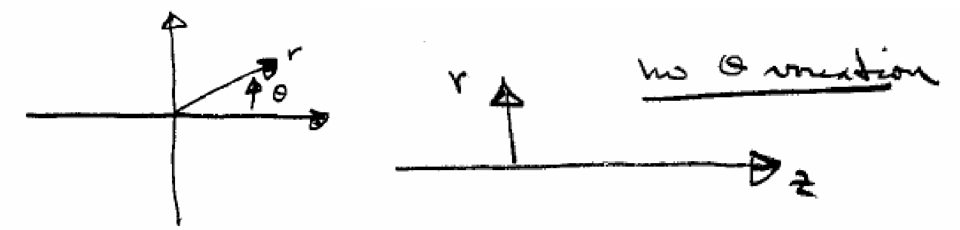
Simplification of 2. denotes the **incompressibility of the fluid!!

9.2The stream function ψ

Cartrsian coordinates: $u = \dfrac{\partial \psi}{\partial y}, v = -\dfrac{\partial \psi}{\partial x}$

Polar coordinates: $u_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta}, u_\theta = -\dfrac{\partial \psi}{\partial r}$

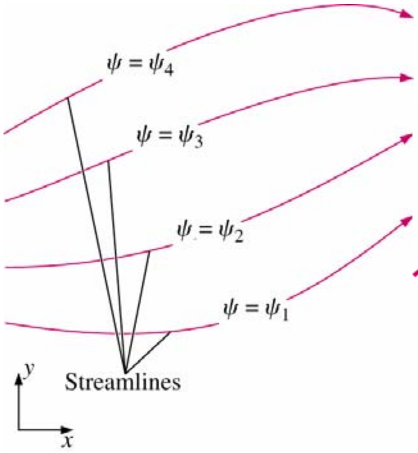
Axisymmetric cylindrical coordinates: $u_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial z}, u_z = -\dfrac{\partial \psi}{\partial r}$

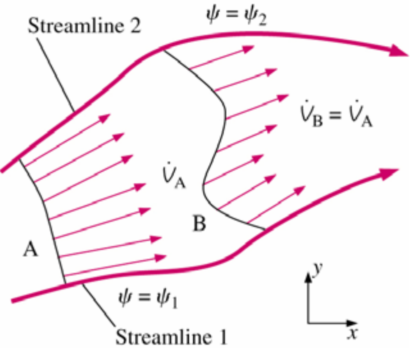


Polar coordinates (left), Axisymmetric cylindrical coordinates (right)

Important features of ψ

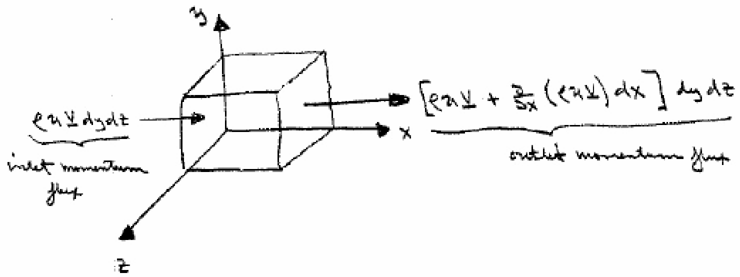
- Curves of constant ψ are streamlines of the flow
- The difference in the value of ψ from one streamline to another is equal to the volume flow rate per unit width between the two streamlines





9.3Navier-Stokes (NS) Equations

NS equation is a conservation of momentum!!



Start from 1-D flow approximation:

$$\sum \underline{F} = \underbrace{\frac{d}{dt}\int_{CV} \rho \underline{V} dV}_{\frown} + \underbrace{\int_{CS} \underline{V} \rho \underline{V} \cdot \underline{dA}}_{} \qquad \text{1-D flow approximation}$$

$$\frown = \sum(\dot{m}_i \underline{V}_i)_{out} - \sum(\dot{m}_i \underline{V}_i)_{in}$$

$$\text{where } \underbrace{\dot{m} = \rho A V = \rho dydzu}_{\text{mass flux}} \text{ x-face}$$

$$\frown \sim \frac{d}{dt}(\rho \underline{V}) dxdydz$$

$$\frown = \left[\underset{\text{x-face}}{\frac{\partial}{\partial x}(\rho u \underline{V})} + \underset{\text{y-face}}{\frac{\partial}{\partial y}(\rho v \underline{V})} + \underset{\text{z-face}}{\frac{\partial}{\partial z}(\rho w \underline{V})}\right] dxdydz$$

$$\sum \underline{F} = \rho \frac{D\underline{V}}{Dt} dxdydz$$

$$\text{where } \sum \underline{F} = \sum \underline{F}_{body} + \sum \underline{F}_{surface}$$

Notice that:

Body force→due to external fields such as gravity or magnetics

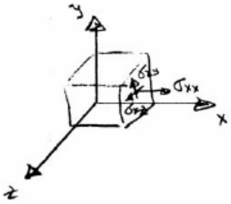
$$\sum \underline{F}_{body} = d\underline{F}_{grav} = \rho \underline{g} dxdydz$$

$$\text{and } \underline{g} = -g\hat{k} \quad \text{for } g\downarrow \; z\uparrow$$

$$\text{i.e., } \underline{f}_{body} = -\rho g \hat{k}$$

Surface force→due to the stresses acting on the sides of CS

$$\sigma_{ij} = \underset{\text{normal pressure}}{-p\delta_{ij}} + \underset{\text{viscous stress}}{\tau_{ij}}$$



$$= \begin{bmatrix} -p+\tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & -p+\tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & -p+\tau_{zz} \end{bmatrix}$$

$\delta_{ij} = 1 \quad i = j$
$\delta_{ij} = 0 \quad i \neq j$

$$\underline{f}_{surf} = -\nabla p + \nabla \cdot \tau_{ij} = \nabla \cdot \sigma_{ij} \qquad \sigma_{ij} = -p\delta_{ij} + \tau_{ij}$$

Putting together the above results

$$\sum \underline{f} = \underline{f}_{body} + \underline{f}_{surf} = \rho \frac{D\underline{V}}{Dt}$$

$$\underline{f}_{body} = -\rho g \hat{k}$$

$$\underline{f}_{surface} = -\nabla p + \nabla \cdot \tau_{ij}$$

$$\underline{a} = \frac{D\underline{V}}{Dt} = \frac{\partial \underline{V}}{\partial t} + \underline{V} \cdot \nabla \underline{V}$$

The physical meanings of each term in NS equation are:

$$\underset{\substack{\text{inertia}\\ \text{force}}}{\rho \underline{a}} = \underset{\substack{\text{body}\\ \text{force}\\ \text{due to}\\ \text{gravity}}}{-\rho g \hat{k}} \underset{\substack{\text{surface}\\ \text{force due}\\ \text{to p}}}{-\nabla p} + \underset{\substack{\text{surface force}\\ \text{due to viscous}\\ \text{shear and normal}\\ \text{stresses}}}{\nabla \cdot \tau_{ij}}$$

Write viscous shear and normal stresses in the form as:

$$\tau_{ij} = \mu \varepsilon_{ij} \qquad \mu = \text{coefficient of viscosity}$$

$$\varepsilon_{ij} = \text{rate of strain tensor}$$

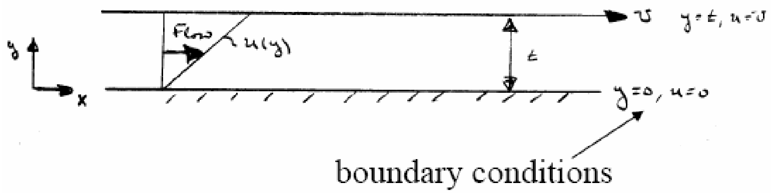
$$= \begin{bmatrix} \dfrac{\partial u}{\partial x} & \left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) & \left(\dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z}\right) \\ \left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) & \dfrac{\partial v}{\partial y} & \left(\dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z}\right) \\ \left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) & \left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) & \dfrac{\partial w}{\partial z} \end{bmatrix}$$

After some mathematical manipulation, NS and continuity equations are obtained as:

$$\boxed{\begin{aligned} &\text{x:} \quad \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \\ &\text{y:} \quad \rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \\ &\text{z:} \quad \rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] \\ &\qquad\qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \end{aligned}}$$

9.4 Differential Analysis of Fluid Flow

- Couette Flow



Governing equations and boundary conditions for flow field:

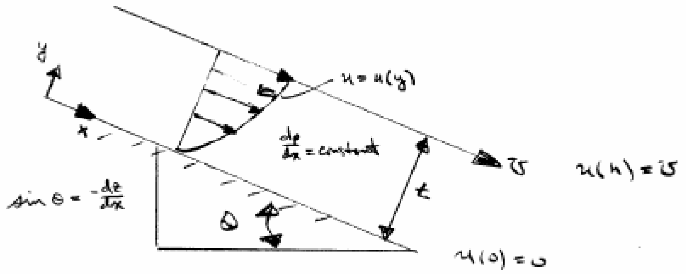
$$\left.\begin{array}{ll} \text{Continuity} & \dfrac{\partial u}{\partial x}=0 \\ \text{Momentum} & 0=\mu\dfrac{d^2u}{dy^2} \end{array}\right\} \begin{array}{l} u=u(y) \\ v=o \\ \dfrac{\partial p}{\partial x}=\dfrac{\partial p}{\partial y}=0 \end{array}$$

Solution (velocity profile and shear stress):

$$u=\frac{U}{t}y$$

$$\tau=\mu\frac{du}{dy}=\frac{\mu U}{t}=\text{constant}$$

- Generalization of inclined flow with a constant pressure gradient



Governing equations and boundary conditions for flow field:

$$\left.\begin{array}{ll} \text{Continutity} & \dfrac{\partial u}{\partial x}=0 \\ \text{Momentum} & 0=-\dfrac{\partial}{\partial x}(p+\gamma z)+\mu\dfrac{d^2u}{dy^2} \end{array}\right\} \begin{array}{l} u=u(y) \\ v=o \\ \dfrac{\partial p}{\partial y}=0 \end{array}$$

Solution with non-dimensional form (velocity profile):

$$\frac{u}{U}=-\frac{\gamma t^2}{2\mu U}\frac{dh}{dx}\left(1-\frac{y}{t}\right)\frac{y}{t}+\frac{y}{t}$$

Further non-dimensionalization of the obtained velocity profile yields:

define: P = non-dimensional pressure gradient

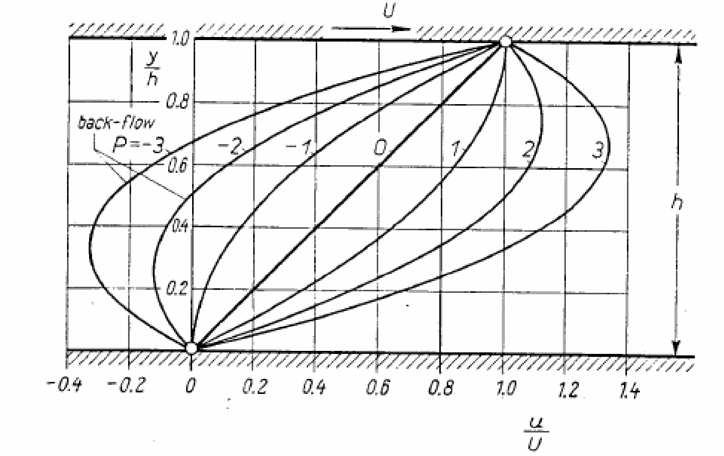
$$= -\frac{\gamma t^2}{2\mu U}\frac{dh}{dx} \qquad h = \frac{p}{\gamma} + z$$

$$Y = y/t \qquad = -\frac{\gamma z^2}{2\mu U}\left[\frac{1}{\gamma}\frac{dp}{dx} + \frac{dz}{dx}\right]$$

$$\boxed{\Rightarrow \quad \frac{u}{U} = P \cdot Y(1-Y) + Y}$$

parabolic velocity profile

Effect of pressure gradient to the velocity profile of Couette flow:



$P > 0$: Favorable pressure gradient → Velocity is positive over the entire width

$P < 0$: Adverse pressure gradient → Backflow near the stationary wall

$P = 0$: Zero pressure gradient → The velocity profile is linear

Chapter 10: Approximate Solutions of the NS equations

10.1 Creeping flow approximation

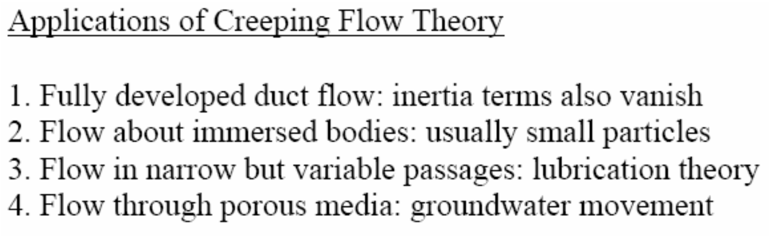
Basic assumption for creeping flow: the inertia terms are negligible in the momentum equation if Re<<1.

In non-dimensional form of NS equation,

$$\mathrm{Re}\frac{D\mathbf{V}^*}{Dt^*} = -\nabla^* p^* + \nabla^{*2}\mathbf{V}^*$$

Since Re<<1, we have:

$$\nabla^* p^* \approx \nabla^{*2}\mathbf{V}^*$$



Applications of Creeping Flow Theory

1. Fully developed duct flow: inertia terms also vanish
2. Flow about immersed bodies: usually small particles
3. Flow in narrow but variable passages: lubrication theory
4. Flow through porous media: groundwater movement

10.2 Approximation for Inviscid Regions of Flow→See Text pp.481-pp.485

10.3 The Irrotational Flow Approximation→ See Text pp.485-pp.510

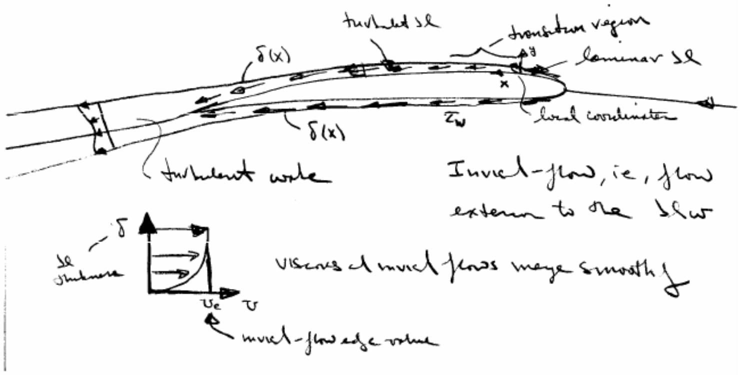
10.4 Qualitative Description of the Boundary layer

Boundary layer:

A very thin region of flow near a solid wall where **viscous forces and rotationality cannot be ignored**.

Boundary-layer theory:

The asymptotic form of the NS equations for high-Re flow about the slender bodies.



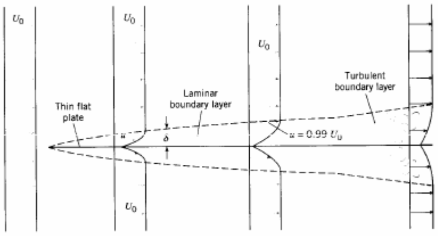
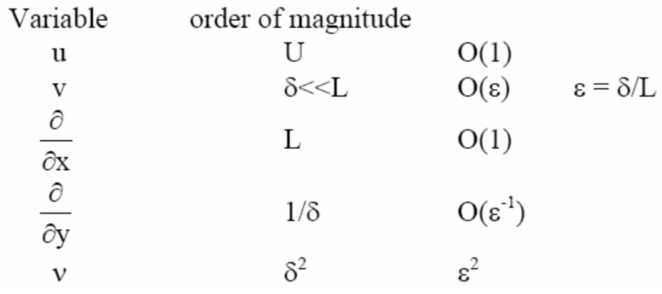


FIGURE 9.4 *Development of boundary layer and distribution of shear stress along a thin, flat plate. (a) Flow pattern in boundary layers above and below the plate. (b) Shear-stress distribution on either side of the plate.*

The order assumptions of boundary-layer theory:



Variable	order of magnitude		
u	U	O(1)	
v	δ<<L	O(ε)	ε = δ/L
$\frac{\partial}{\partial x}$	L	O(1)	
$\frac{\partial}{\partial y}$	1/δ	$O(\varepsilon^{-1})$	
ν	δ^2	ε^2	

Use the order assumptions above to obtain boundary-layer equations from NS equations:

NS equations:

$$\left.\begin{array}{l}
u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=-\dfrac{\partial p}{\partial x}+\mu\left(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}\right) \\
1 \quad 1 \qquad \varepsilon \quad \varepsilon^{-1} \qquad\qquad \varepsilon^{2} \quad 1 \qquad \varepsilon^{-2} \\[2ex]
u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}=-\dfrac{\partial p}{\partial y}+\mu\left(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial x^2}\right) \\
1 \quad \varepsilon \qquad \varepsilon \quad 1 \qquad\qquad \varepsilon^{2} \quad 1 \qquad \varepsilon^{-1} \\[2ex]
\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \\
1 \qquad 1
\end{array}\right\}\ \text{elliptic}$$

Boundary-layer equations:

Retaining terms of O(1) only results in the celebrated boundary-layer equations

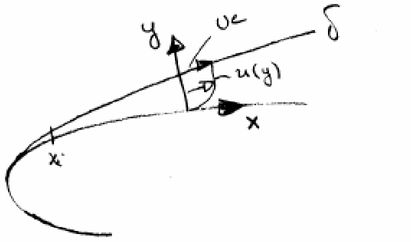
$$\left.\begin{array}{l}
u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=-\dfrac{\partial p}{\partial x}+\mu\dfrac{\partial^2 u}{\partial y^2} \\[1ex]
\dfrac{\partial p}{\partial y}=0 \\[1ex]
\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0
\end{array}\right\}\ \text{parabolic}$$

Important aspects of boundary-layer equations:

1. $\dfrac{\partial P}{\partial y}=0$ i.e. $P=P_e$=const. across the boundary layer
2. Continuity equation holds.
3. Boundary conditions to solve the boundary-layer equations are:

$$u = v = 0 \qquad y = 0$$

$$u = U_e \qquad y = \delta$$

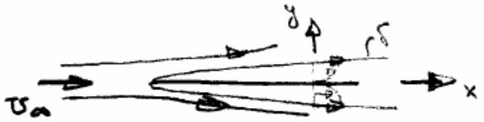


10.5 Quantitative relations for the Laminar Boundary Layer

Laminar boundary layer over a flat plate→Blasius solution

Governing equations and boundary conditions to obtain Blasius solution:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$



Note: $\frac{\partial p}{\partial x}=0$ for a flat plate

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

$$u=v=0 \ @ \ y=0 \qquad u=U_\infty \quad @ \ y=\delta$$

Results:

$$\delta=\frac{5x}{\sqrt{\text{Re}}}$$ value of y where $u/U_\infty = .99$

$$\text{Re}_x=\frac{U_\infty x}{\nu}$$

$$c_f=\frac{2\tau_w}{\rho U_\infty^2}=\frac{0.664}{\sqrt{\text{Re}_x}}=\frac{\theta}{x}$$

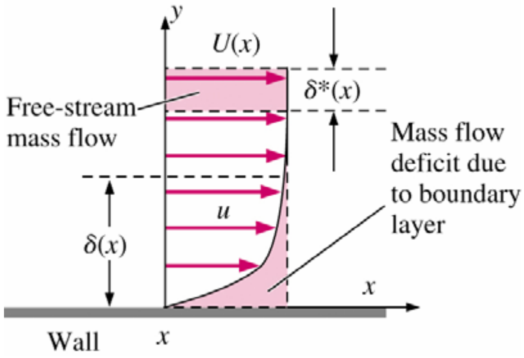
$$C_f=\frac{1}{L}\int_0^L c_f dx=2c_f(L)$$

$$=\frac{1.328}{\sqrt{\text{Re}_L}}$$

$$\frac{U_\infty L}{\nu}$$

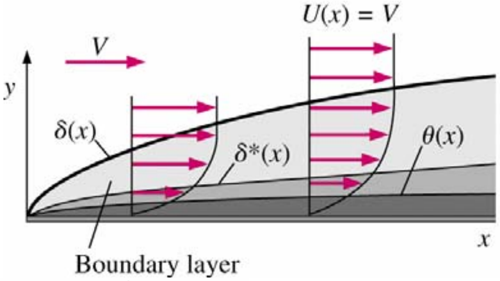
$$\delta^*=\int_0^\delta\left(1-\frac{u}{U_\infty}\right)dy=1.7208\frac{x}{\sqrt{\text{Re}_x}}$$ displacement thickness

Displacement thickness: imaginary increase in thickness of the wall, as seen by the outer flow, due to the effect of the growing boundary layer.



$$\theta = \int_0^{\delta} \left(1 - \frac{u}{U_\infty}\right) \frac{u}{U_\infty} dy = 0.664 \frac{x}{\sqrt{\text{Re}_x}} \quad \text{momentum thickness}$$

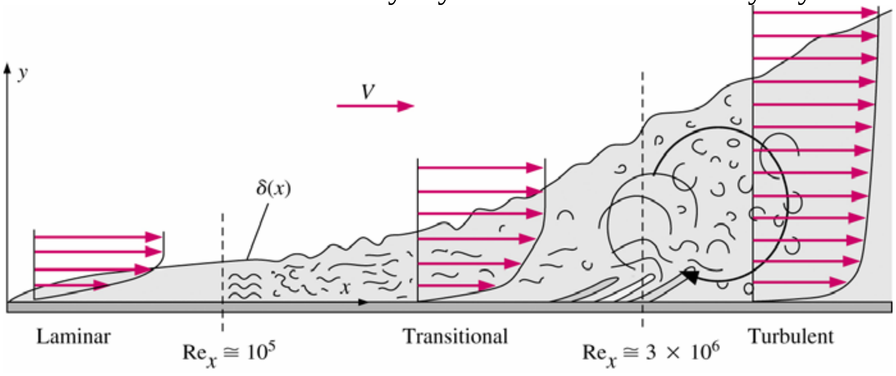
Momentum thickness: the loss of momentum flux per unit width divided by ρU^2 due to the presence of the growing boundary layer.



$$H = \text{shape parameter} = \frac{\delta^*}{\theta} = 2.5916$$

10.6 Qualitative relations for the Turbulent Boundary layer

Transition from laminar boundary-layer to turbulent boundary-layer:

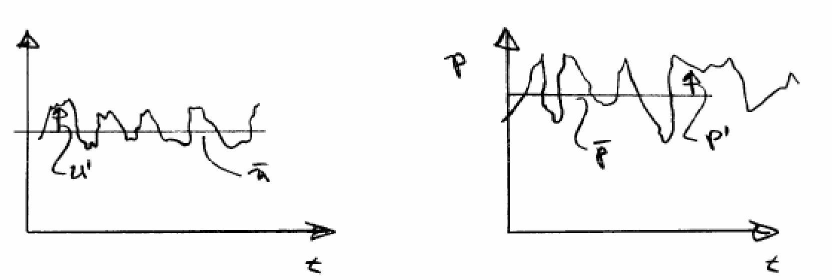


Engineering critical Reynolds number: $Re_{x,\,cr} = 5\times10^5$

→ $Re_x < Re_{x,\,cr}$: boundary layer is most likely laminar.

→ $Re_x > Re_{x,\,cr}$: boundary layer is most likely turbulent.

Description of turbulent flow:



Velocity and pressure are random functions of time!!

They can be separated into two parts such as mean and fluctuation components:

$$u = \overline{u} + u' \qquad p = \overline{p} + p'$$

$$v = \overline{v} + v' \qquad \text{and for compressible flow}$$

$$w = \overline{w} + w' \qquad \rho = \overline{\rho} + \rho' \text{ and } T = \overline{T} + T'$$

Most important influence of turbulence on the mean motion:

→An increase in the fluid stress by "Reynolds stresses"

$$\tau'_{ij} = -\rho\overline{u'_i u'_j}$$

$$= \begin{bmatrix} -\rho\overline{u'^2} & -\rho\overline{u'v'} & -\rho\overline{u'w'} \\ -\rho\overline{u'v'} & -\rho\overline{v'^2} & -\rho\overline{v'w'} \\ -\rho\overline{u'w'} & -\rho\overline{v'w'} & -\rho\overline{w'^2} \end{bmatrix}$$

'Modeling' required!!

Mean flow equations for turbulent flow

→Reynolds Averaged Navier Stokes (RANS) equations

Continuity $\nabla \cdot \underline{V} = 0$ i.e. $\nabla \cdot \underline{\overline{V}} = 0$ and $\nabla \cdot \underline{V}' = 0$

Momentum $$\rho \frac{D\overline{V}}{Dt} = -\rho g\hat{k} - \nabla\overline{p} + \nabla \cdot \tau_{ij}$$

$$\tau_{ij} = \mu\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \underbrace{-\rho\overline{u_i'u_j'}}_{\tau_{ij}'}$$

$u_1 = u$	$x_1 = x$
$u_2 = v$	$x_2 = y$
$u_3 = w$	$x_3 = z$

Turbulence Modeling:

a) Eddy viscosity, Mixing-length theory, One-equation model,
Two- equation model (*k-ε* model, *k-ω* model: **Recall CFD-PreLab2, Lab2!!**)

b) Mean-flow velocity profile correlations

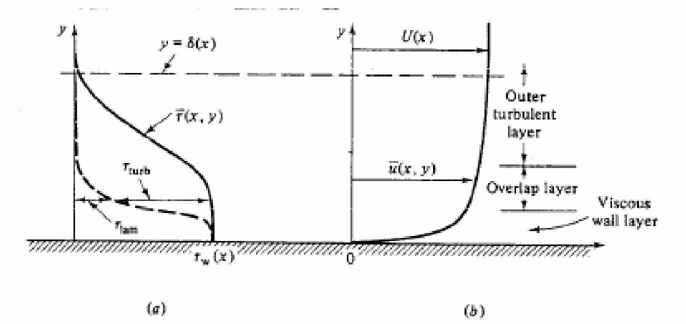


Fig. 6.8 Typical velocity and shear distributions in turbulent flow near a wall: (a) shear; (b) velocity. (measurements)

Inner layer: viscous stress dominates

Outer layer: turbulent stress dominates

Overlap layer: both types of stress important

1) Inner layer

$u = f(\mu, \tau_w, \rho, y)$ note: not f(δ)

$u^+ = f(y^+)$ law-of-the-wall

$u^+ = y^+$

where: $u^+ = \frac{u}{u^*}$

$u^* = \text{friction velocity} = \sqrt{\tau_w / \rho}$

$y^+ = \frac{yu^*}{\nu}$

very near the wall:

$$\tau \sim \tau_w \sim \text{constant} = \mu\frac{du}{dy} \quad \Rightarrow \quad u = cy \quad \text{or} \quad u^+ = y^+$$

2) Outer layer

$$\frac{U_e - u}{u^*} = f\left(\frac{y}{\delta}\right) \qquad \text{velocity defect law}$$

3) Overlap layer

$$\frac{u}{u^*} = \frac{1}{\underset{\underset{.41}{\uparrow}}{\kappa}} \ln y^+ + \underset{\underset{5}{\uparrow}}{B} \qquad \text{log-law}$$

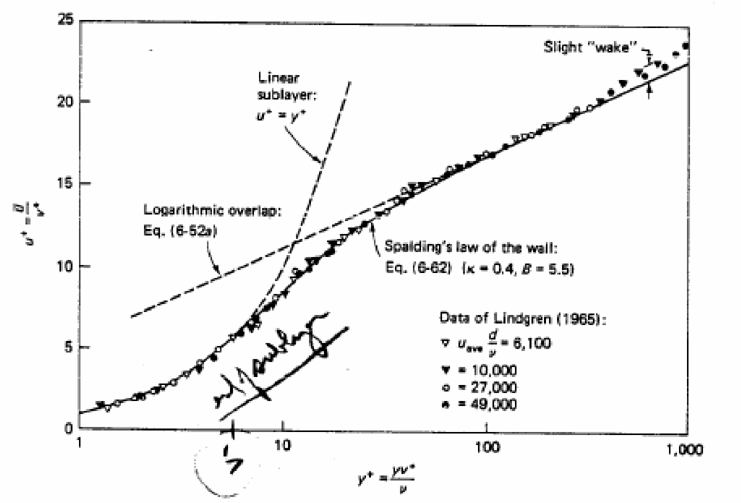
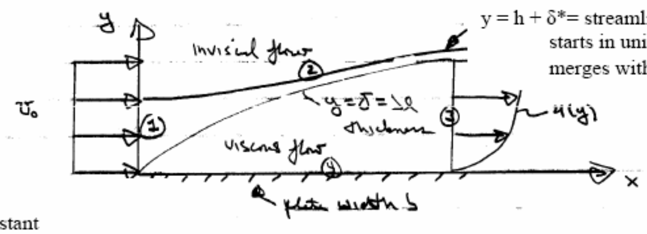


FIGURE 6-6
Comparison of Spalding's inner-law expression with the pipe-flow data of Lindgren (1965).

Momentum Integral Analysis: obtain general momentum integral relation

→Valid for both laminar and turbulent flow



$y = h + \delta^* =$ streamline
starts in uniform flow
merges with δ at 3

Steady
ρ = constant
neglect g
$v << u = u_o \Rightarrow p = \text{constant}$
i.e., $-\nabla p = 0$

CV = 1, 2, 3, 4

$$\underset{\underset{\text{force on CV}}{\uparrow}}{-D} = \text{drag} = b\int_0^x \underset{\underset{\text{wall shear stress}}{\uparrow}}{\tau_w} dx \qquad \text{pressure force} = 0 \text{ for } v << U_o$$

$u \sim U_o$

$$\sum F_x = -D = \rho\int_1 u(\underline{V}\cdot\underline{dA}) + \rho\int_3 u(\underline{V}\cdot\underline{dA})$$

$$= \rho\left(-U_o^2 bh\right) + \rho b\int_3 u^2 dy$$

$$D(x) = \rho b U_o \int_0^{\delta} u\,dy - \rho b \int_0^{\delta} u^2\,dy$$

$$= \rho b \int_0^{\delta} u(U_o - u)\,dy$$

$$C_D = \frac{D}{\frac{1}{2}\rho U_o^2 bL} = \frac{2}{L}\underbrace{\int_0^{\delta} \frac{u}{U_o}\left(1 - \frac{u}{U_o}\right)dy}_{\theta\ =\ \text{momentum thickness}}$$

On the other hand, C_D can be expressed as;

$$C_D = \frac{D}{\frac{1}{2}\rho U_o^2 A} = \frac{b\int_0^{x} \tau_w\,dx}{\frac{1}{2}\rho U_o^2 bL} = \frac{2\theta}{L}$$

Then,

$$\int_0^{x} \frac{\tau_w}{\frac{1}{2}\rho U_o^2}(x)\,dx = 2\theta(x)$$

Therefore,

$$\frac{1}{2}\left(\frac{\tau_w}{\frac{1}{2}\rho U_o^2}\right) = \frac{d\theta}{dx}$$

Finally, momentum integral relation is obtained as:;

$$\frac{c_f}{2} = \frac{d\theta}{dx} \qquad c_f = \text{local skin friction coefficient}$$

momentum integral relation for
flat plate boundary layer

Approximate solutions for a laminar boundary layer obtained from momentum integral analysis:

	Exact Blassius	
$\delta = \frac{4.65x}{\sqrt{Re_x}}$	$\frac{5x}{\sqrt{Re_x}}$	7% ↓
$\tau_w = \frac{.323\rho V^2}{\sqrt{Re_x}}$	$\frac{.332\rho U^2}{\sqrt{Re_x}}$	3%↓
$c_f = \frac{.646}{\sqrt{Re_x}}$	$\frac{.664}{\sqrt{Re_x}}$	
$C_f = \frac{1.29}{\sqrt{Re_L}}$	$\frac{1.33}{\sqrt{Re_L}}$	

Approximate solutions for a turbulent boundary layer obtained from momentum integral analysis:

Velocity profile inside the turbulent boundary layer to obtain the solutions:

1) log-law

$$\frac{u}{u^*} = \frac{1}{\kappa} \ln \frac{yu^*}{\nu} + B$$

neglect laminar sub layer and velocity defect region

2) 1/7 power law

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

Obtained approximate solutions:

$$\frac{\delta}{x} = .16\,\mathrm{Re}_x^{-1/7}$$

$$c_f = \frac{.027}{\mathrm{Re}_x^{1/7}}$$

$$C_f = \frac{.031}{\mathrm{Re}_L^{1/7}} = \frac{7}{6} C_f(L)$$

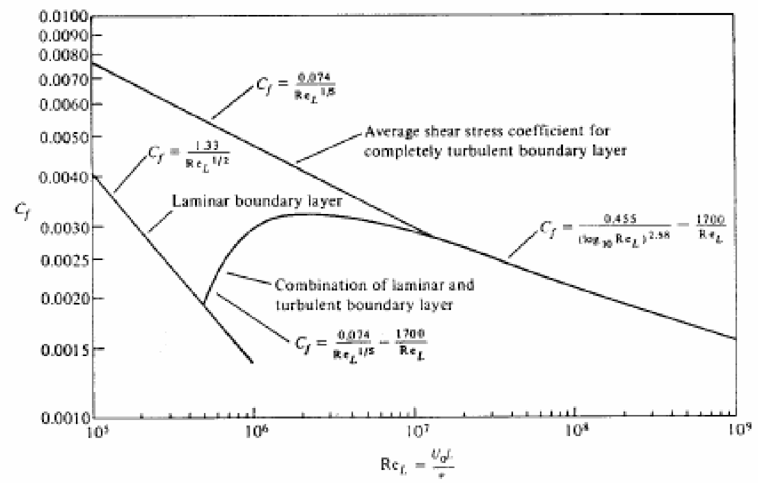
Total shear-stress coefficient

$$C_f = \frac{.455}{(\log_{10} \mathrm{Re}_L)^{2.58}} \frac{-1700}{\mathrm{Re}_L} \qquad \mathrm{Re} > 10^7$$

$$\frac{\delta}{L} = c_f(.98 \log \mathrm{Re}_L - .732)$$

Local shear-stress coefficient

$$c_f = (2 \log \mathrm{Re}_x - .65)^{-2.3}$$

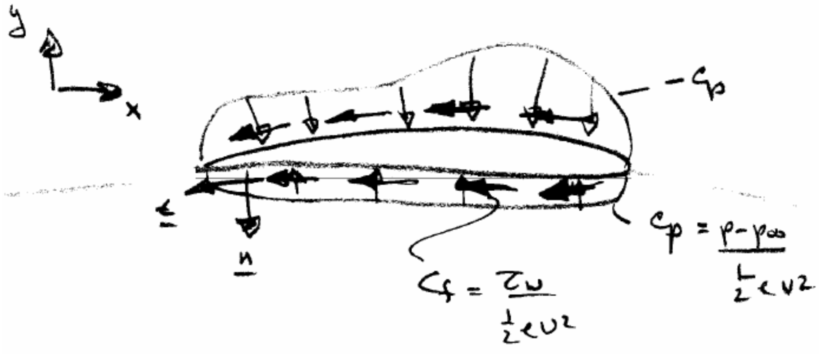


C_f vs Re_L relationship in laminar and turbulent boundary layer

Chapter 11: Drag and Lift

11.1 Basic consideration

[Drag]=[form drag]+[skin-friction drag]



Drag

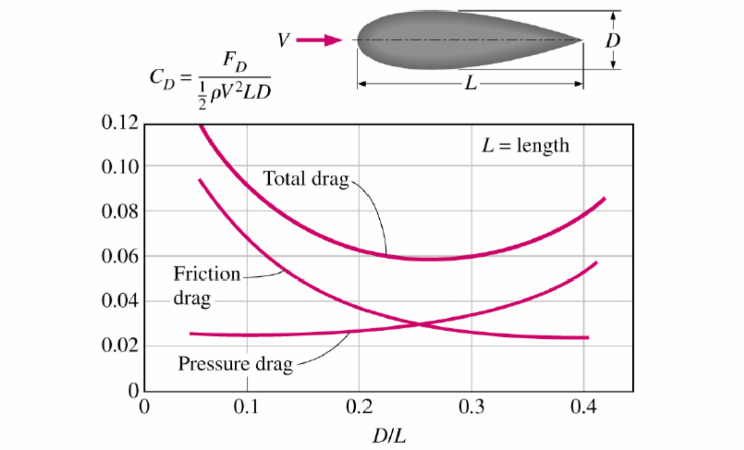
$$C_D = \frac{1}{\frac{1}{2}\rho V^2 A}\left\{\underbrace{\int_S (p - p_\infty)\underline{n}\cdot\hat{i}\,dA}_{C_{Dp}} + \underbrace{\int_S \tau_w \underline{t}\cdot\hat{i}\,dA}_{C_f}\right\}$$

Lift

$$C_L = \frac{1}{\frac{1}{2}\rho V^2 A}\left\{\int_S (p - p_\infty)\underline{n}\cdot\hat{j}\,dA\right\}$$

Drag reduction by streamlining:

Trade-off relationship between pressure drag and friction drag



Trade-off relationship between pressure drag and friction drag

11.2 Drag of 2-D and 3-D bodies

Flat-plate parallel to the flow:

$$C_{Dp} = \frac{1}{\frac{1}{2}\rho V^2 A}\int_S (p - p_\infty)\underline{n}\cdot\imath = 0$$

$$C_f = \frac{1}{\frac{1}{2}\rho V^2 A}\int_S \tau_w \underline{t}\cdot\hat{\imath}dA$$

$$= \frac{1.33}{Re_L^{1/2}} \qquad \text{laminar flow}$$

$$= \frac{.074}{Re_L^{1/5}} \qquad \text{turbulent flow}$$

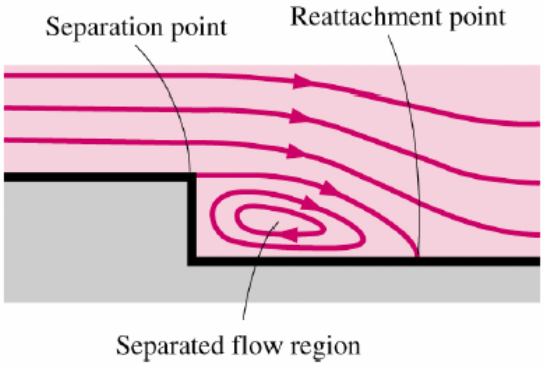
In general,

$$C_D = \underbrace{\frac{Drag}{\frac{1}{2}\rho V^2 A}}_{\text{scale factor}} = f\left(Re, \overset{c/L}{Ar}, \frac{t}{L}, \frac{\varepsilon}{L}, T, \text{etc.}\right)$$

Flow separation:

→The fluid stream detaches itself from the surface of the body at sufficiently high velocities. Only appeared in viscous flow!!

Flow separation forms the region called ‘separated region’



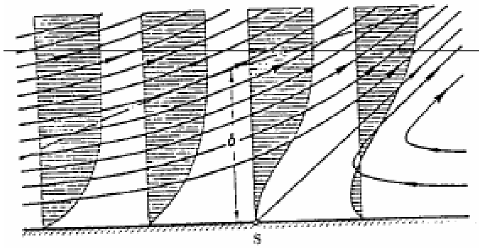


Fig. 2.12. Diagrammatic representation of flow in the boundary layer near a point of separation

Drag coefficients if common geometries (2D and 3D):

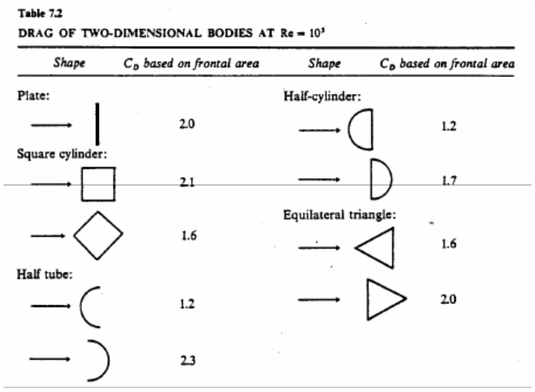


Table 7.2

DRAG OF TWO-DIMENSIONAL BODIES AT Re = 10^5

Shape	*C_D based on frontal area*	*Shape*	*C_D based on frontal area*
Plate:		Half-cylinder:	
	2.0		1.2
Square cylinder:			
	2.1		1.7
		Equilateral triangle:	
	1.6		1.6
Half tube:			
	1.2		2.0
	2.3		

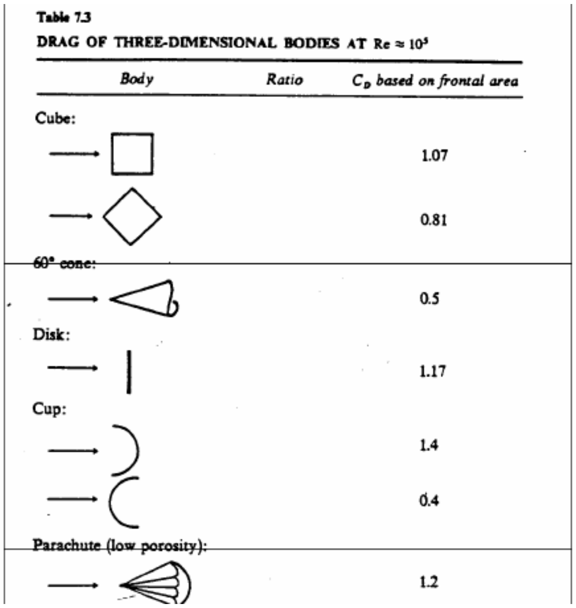


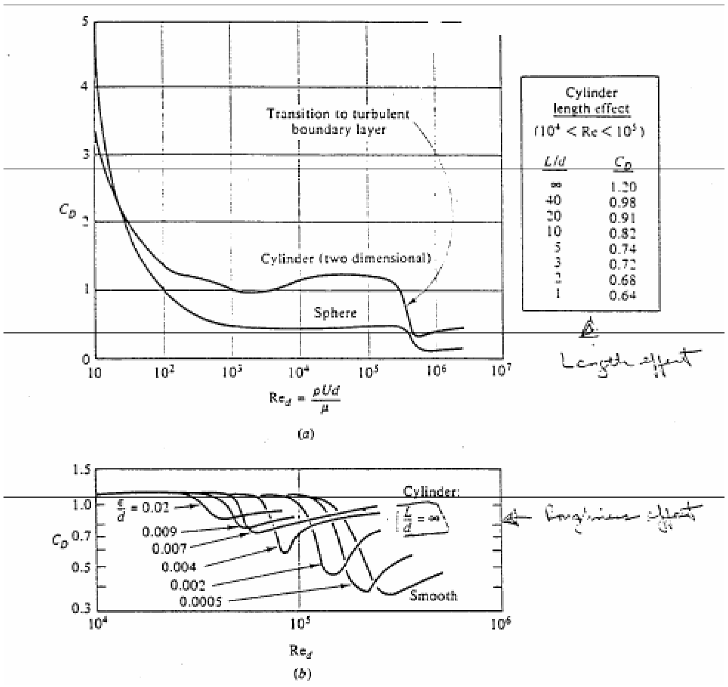
Table 7.3

DRAG OF THREE-DIMENSIONAL BODIES AT Re ≈ 10^5

Body	*Ratio*	*C_D based on frontal area*
Cube:		
		1.07
		0.81
60° cone:		
		0.5
Disk:		
		1.17
Cup:		
		1.4
		0.4
Parachute (low porosity):		
		1.2

Flow over cylinder and spheres:

- Re<1: Creeping flow, C_D=24/Re, No-flow separation regime
- 10^5<Re<10^6: boundary layer become turbulent, large reduction in CD



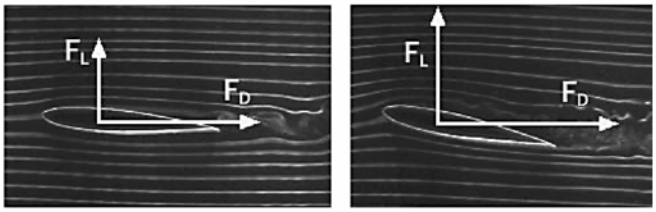
Cylinder length effect (10^4 < Re < 10^5)

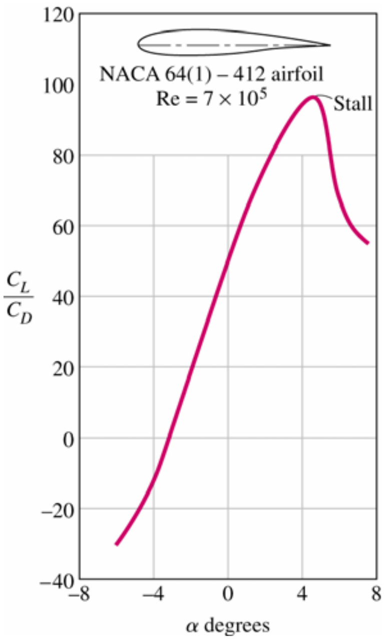
L/d	C_D
∞	1.20
40	0.98
20	0.91
10	0.82
5	0.74
3	0.72
2	0.68
1	0.64

Length effect

Roughness effect

Lift:





Minimum flight velocity :

$$V_{\min} = \sqrt{\frac{2W}{\rho C_{L,\max} A}}$$

Lift generation by spinning: Magnus effect