Solution for review problems for Exam 3, 057:020-Fall 2007

Friction factor and head loss

SOLUTION The average flow velocity in a pipe is given. The head loss, the pressure drop, and the pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors.

Properties The density and dynamic viscosity of water are given to be ρ = 62.42 lbm/ft^3 and μ = 1.038 × 10^{-3} lbm/ft · s, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number is

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{(62.42\ \text{lbm/ft}^3)(3\ \text{ft/s})(0.01\ \text{ft})}{1.038 \times 10^{-3}\ \text{lbm/ft} \cdot \text{s}} = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the head loss become

$$f = \frac{64}{\text{Re}} = \frac{64}{1803} = 0.0355$$

$$h_L = f\frac{L}{D}\frac{V_{\text{avg}}^2}{2g} = 0.0355\,\frac{30\ \text{ft}}{0.01\ \text{ft}}\,\frac{(3\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = \mathbf{14.9\ ft}$$

(*b*) Noting that the pipe is horizontal and its diameter is constant, the pressure drop in the pipe is due entirely to the frictional losses and is equivalent to the pressure loss,

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.0355\,\frac{30\ \text{ft}}{0.01\ \text{ft}}\,\frac{(62.42\ \text{lbm/ft}^3)(3\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm} \cdot \text{ft/s}^2}\right)$$

$$= \mathbf{929\ lbf/ft^2 = 6.45\ psi}$$

(*c*) The volume flow rate and the pumping power requirements are

$$\dot{V} = V_{\text{avg}} A_c = V_{\text{avg}}(\pi D^2/4) = (3\ \text{ft/s})[\pi(0.01\ \text{ft})^2/4] = 0.000236\ \text{ft}^3/\text{s}$$

$$\dot{W}_{\text{pump}} = \dot{V}\,\Delta P = (0.000236\ \text{ft}^3/\text{s})(929\ \text{lbf/ft}^2)\left(\frac{1\ \text{W}}{0.737\ \text{lbf} \cdot \text{ft/s}}\right) = \mathbf{0.30\ W}$$

Therefore, power input in the amount of 0.30 W is needed to overcome the frictional losses in the flow due to viscosity.

Minor loss

10.91 Information and Assumptions

from Table 10.2 $k_s = 0.046$ mm
assume from Table A.5 $\nu = 1.31 \times 10^{-6}$ mm
provided in problem statement

Find

pump power

Energy equation

$$
\begin{aligned}
p_1/\gamma - V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 100 + h_p &= 0 + V_2^2/2g + 150 + V_2^2/2g(0.03 + fL/D) \\
V_2 &= Q/A_p = 20/((\pi/4) \times 1.5^2) = 11.32 \text{ m/s} \\
\text{Re} &= VD/\nu = 11.32 \times 1.5/(1.31 \times 10^{-6}) = 1.3 \times 10^7 \\
k_s/D &= 0.046/1500 = 0.00003
\end{aligned}
$$

From Fig. 10.8 $f = 0.010$. Then

$$
\begin{aligned}
h_p &= 140 - 100 + V_2^2/2g(1.03 + 0.010 \times 300/1.5) \\
h_p &= 40 + 19.8 = 59.8 \text{ m}
\end{aligned}
$$

Pump power

$$
P = Q\gamma h_p = 15 \times 9,810 \times 59.8 = \underline{\underline{8.80 \text{ MW}}}
$$

Boundary layer

Assumptions **1** The flow is steady and incompressible. **2** The walls are smooth, and disturbances and vibrations are kept to a minimum. **3** The boundary layer is laminar.

Properties The kinematic viscosity of air at 19°C is $\nu = 1.507 \times 10^{-5}$ m²/s.

Analysis (*a*) The Reynolds number at the end of the test section is approximately

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(4.0\ \mathrm{m/s})(0.30\ \mathrm{m})}{1.507 \times 10^{-5}\ \mathrm{m^2/s}} = 7.96 \times 10^4$$

Since Re_x is lower than the engineering critical Reynolds number, $\mathrm{Re}_{x,\,\mathrm{cr}} = 5 \times 10^5$, and is even lower than $\mathrm{Re}_{x,\,\mathrm{critical}} = 1 \times 10^5$, and since the walls are smooth and the flow is clean, we may assume that the boundary layer on the wall remains laminar throughout the length of the test section. As the boundary layer grows along the wall of the wind tunnel test section, air in the region of irrotational flow in the central portion of the test section accelerates as in Fig. 10–105 in order to satisfy conservation of mass. We use Eq. 10–73 to estimate the displacement thickness at the end of the test section,

$$\delta^* \cong \frac{1.72x}{\sqrt{\mathrm{Re}_x}} = \frac{1.72(0.30\ \mathrm{m})}{\sqrt{7.96 \times 10^4}} = 1.83 \times 10^{-3}\ \mathrm{m} = 1.83\ \mathrm{mm} \qquad (1)$$

Two cross-sectional views of the test section are sketched in Fig. 10–107, one at the beginning and one at the end of the test section. The effective radius at the end of the test section is reduced by δ^* as calculated by Eq. 1. We apply conservation of mass to calculate the average air speed at the end of the test section,

$$V_{\mathrm{end}} A_{\mathrm{end}} = V_{\mathrm{beginning}} A_{\mathrm{beginning}} \quad \rightarrow \quad V_{\mathrm{end}} = V_{\mathrm{beginning}} \frac{\pi R^2}{\pi (R - \delta^*)^2} \qquad (2)$$

which yields

$$V_{\mathrm{end}} = (4.0\ \mathrm{m/s}) \frac{(0.15\ \mathrm{m})^2}{(0.15\ \mathrm{m} - 1.83 \times 10^{-3}\ \mathrm{m})^2} = \mathbf{4.10\ m/s} \qquad (3)$$

Drag force

SOLUTION A pipe is submerged in a river. The drag force that acts on the pipe is to be determined.

Assumptions **1** The outer surface of the pipe is smooth so that Fig. 11–34 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The direction of water flow is normal to the pipe. **4** Turbulence in river flow is not considered.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1$ kg/m³ and $\mu = 1.138 \times 10^{-3}$ kg/m · s.

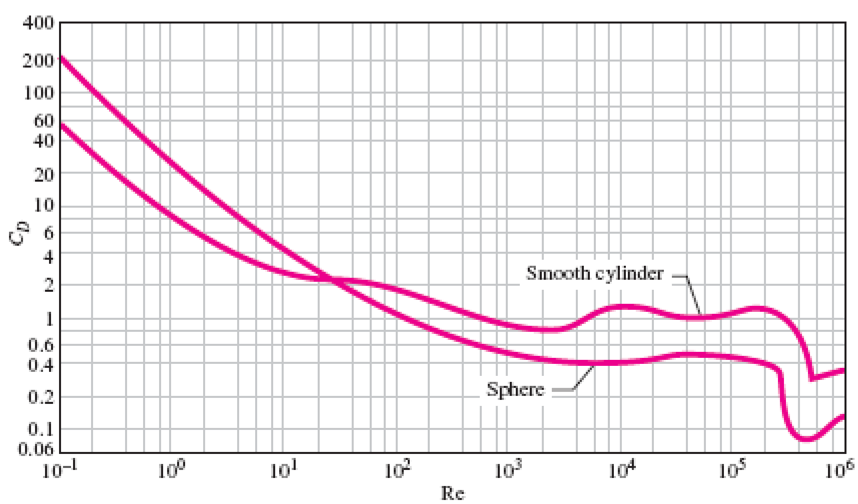
Analysis Noting that $D = 0.022$ m, the Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})(0.022\ \text{m})}{1.138 \times 10^{-3}\ \text{kg/m}\cdot\text{s}} = 7.73 \times 10^4$$

The drag coefficient corresponding to this value is, from Fig. 11–34, $C_D = 1.0$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the pipe becomes

$$F_D = C_D A \frac{\rho V^2}{2} = 1.0(30 \times 0.022\ \text{m}^2)\frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 5275\ \text{N} \cong \mathbf{5300\ N}$$



Dimensional Analysis

SOLUTION We are to calculate and plot C_D as a function of Re for a given set of wind tunnel measurements and determine if dynamic similarity and/or Reynolds number independence have been achieved. Finally, we are to estimate the aerodynamic drag force acting on the prototype truck.

Assumptions **1** The model truck is geometrically similar to the prototype truck. **2** The aerodynamic drag on the strut(s) holding the model truck is negligible.

Properties For air at atmospheric pressure and at $T = 25°C$, $\rho = 1.184$ kg/m³ and $\mu = 1.849 \times 10^{-5}$ kg/m · s.

Analysis We calculate C_D and Re for the last data point listed in Table 7–7 (at the fastest wind tunnel speed),

$$C_{D,m} = \frac{F_{D,m}}{\frac{1}{2}\rho_m V_m^2 A_m} = \frac{89.9\text{ N}}{\frac{1}{2}(1.184\text{ kg/m}^3)(70\text{ m/s})^2(0.159\text{ m})(0.257\text{ m})}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)$$
$$= 0.758$$

and

$$\text{Re}_m = \frac{\rho_m V_m W_m}{\mu_m} = \frac{(1.184\text{ kg/m}^3)(70\text{ m/s})(0.159\text{ m})}{1.849 \times 10^{-5}\text{ kg/m}\cdot\text{s}} = 7.13 \times 10^5 \qquad (1)$$

We repeat these calculations for all the data points in Table 7–7, and we plot C_D versus Re in Fig. 7–41.

Have we achieved dynamic similarity? Well, we have *geometric* similarity between model and prototype, but the Reynolds number of the prototype truck is

$$\text{Re}_p = \frac{\rho_p V_p W_p}{\mu_p} = \frac{(1.184\text{ kg/m}^3)(26.8\text{ m/s})[16(0.159\text{ m})]}{1.849 \times 10^{-5}\text{ kg/m}\cdot\text{s}} = 4.37 \times 10^6 \qquad (2)$$

where the width of the prototype is specified as 16 times that of the model. Comparison of Eqs. 1 and 2 reveals that the prototype Reynolds number is more than six times larger than that of the model. Since we cannot match the independent Π's in the problem, **dynamic similarity has not been achieved.**

Have we achieved Reynolds number independence? From Fig. 7–41 we see that **Reynolds number independence has indeed been achieved**—at Re greater than about 5×10^5, C_D has leveled off to a value of about 0.76 (to two significant digits).

Since we have achieved Reynolds number independence, we can extrapolate to the full-scale prototype, assuming that C_D remains constant as Re is increased to that of the full-scale prototype.

Predicted aerodynamic drag on the prototype:

$$F_{D,p} = \tfrac{1}{2}\rho_p V_p^2 A_p C_{D,p}$$
$$= \tfrac{1}{2}(1.184\text{ kg/m}^3)(26.8\text{ m/s})^2[16^2(0.159\text{ m})(0.257\text{ m})](0.76)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$
$$= \mathbf{3400\text{ N}}$$