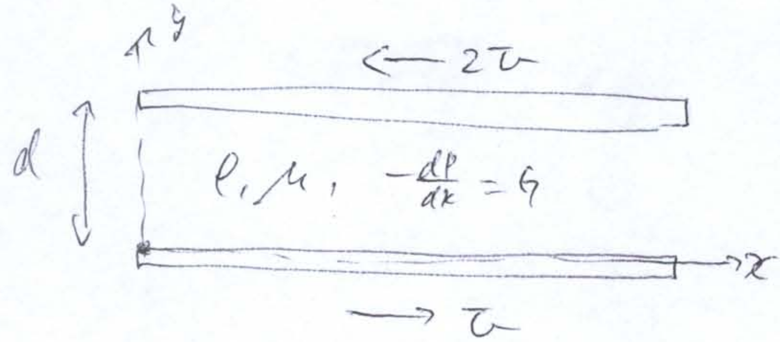
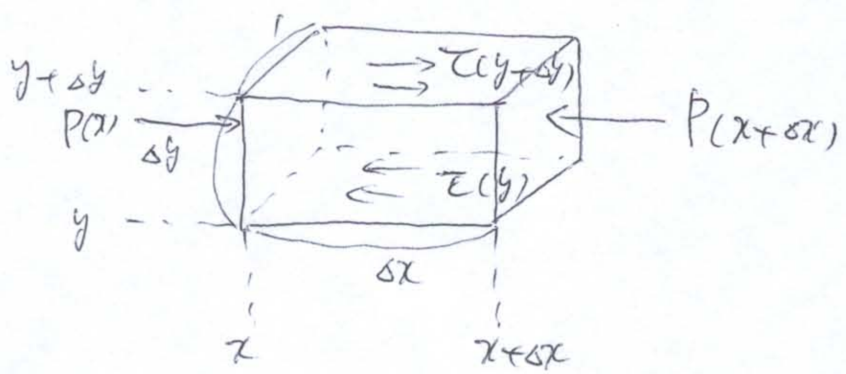
[1] (Couttee flow between two parallel plates)



(a) Assume the small fluid element. with unit width



Balance of the force on fluid element :

$$\underbrace{P(x)\times\Delta y\times 1 + \tau(y+\Delta y)\times\Delta x\times 1}_{\rightarrow} = \underbrace{P(x+\Delta x)\times\Delta y\times 1 + \tau(y)\times\Delta x\times 1}_{\leftarrow}$$

$\rightarrow$ $$\{P(x+\Delta x) - P(x)\}\Delta y - \{\tau(y+\Delta y) - \tau(y)\}\Delta x = 0$$

$\rightarrow$ devide both sides by $\Delta x \Delta y$

$$\frac{P(x+\Delta x) - P(x)}{\Delta x} - \frac{\tau(y+\Delta y) - \tau(y)}{\Delta y} = 0$$

$\rightarrow$ take $\lim_{\Delta x, \Delta y \to 0}$

$\rightarrow$ $$\frac{\partial P}{\partial x} - \frac{\partial \tau}{\partial y} = 0$$

By definition, $\tau = \mu \frac{\partial u}{\partial y}$

$\therefore \frac{\partial P}{\partial x} - \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = 0 \rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$

Given that $\frac{\partial P}{\partial x} = -G \quad \therefore \frac{\partial^2 u}{\partial y^2} = -\frac{G}{\mu}$

$\rightarrow \frac{\partial u}{\partial y} = -\frac{G}{\mu} y + C_1$

$\rightarrow u = -\frac{G}{2\mu} y^2 + C_1 y + C_2$

Apply Boundary conditions to get C_1 and C_2.

At $y = 0$: $u = U \rightarrow C_2 = U$

At $y = d$: $u = -2U \rightarrow -2U = -\frac{G}{2\mu} d^2 + C_1 d + U$

$\therefore C_1 = \frac{Gd}{2\mu} - \frac{3U}{d}$

$\therefore u(y) = -\frac{G}{2\mu} y^2 + \left(\frac{Gd}{2\mu} - \frac{3U}{d}\right) y + U$ //

$\tau = \mu \frac{\partial u}{\partial y}$

$= \mu \left(-\frac{G}{\mu} y + \left(\frac{Gd}{2\mu} - \frac{3U}{d}\right)\right)$ //

(b)

$$Q = \int_0^d u(y)\,dy \times 1$$ unit width.

$$= \int_0^d \left\{ -\frac{G}{2\mu} y^2 + \left(\frac{Gd}{2\mu} - \frac{3U}{d} \right) y + U \right\} dy$$

$$= \left[-\frac{G}{6\mu} y^3 + \frac{1}{2} \left(\frac{Gd}{2\mu} - \frac{3U}{d} \right) y^2 + Uy \right]_0^d$$

$$= \frac{Gd^3}{12\mu} - \frac{1}{2} U d$$

//.

$\boxed{2}$ (From 9-29)

The condition of incompressibility

$$\rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$ (i.e. continuity holds)

Given that

$$\vec{V} = (axy^2 - b)\vec{i} + cy^3\vec{j} + dxy\vec{k}$$

$$= \begin{bmatrix} axy^2 - b \\ cy^3 \\ dxy \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Then,

$$\frac{\partial u}{\partial x} = ay^2, \quad \frac{\partial v}{\partial y} = 3cy^2, \quad \frac{\partial w}{\partial z} = 0$$

$$\therefore \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = ay^2 + 3cy^2 + 0$$

$$= (a+3c)y^2$$

Use the condition of incompressibility, then

$$a + 3c = 0 \rightarrow \underline{a = -3c}$$ //

General form of continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \qquad \text{(Compressible / incompressible)}$$

$$\downarrow$$

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} + \boxed{u\frac{\partial \rho}{\partial x}} + \rho\frac{\partial v}{\partial y} + \boxed{v\frac{\partial \rho}{\partial y}} + \rho\frac{\partial w}{\partial z} + \boxed{w\frac{\partial \rho}{\partial z}} = 0$$

$$\downarrow$$

$$\underbrace{\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}}_{\bigcirc} + \underbrace{\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)}_{\square} = 0$$

$$\downarrow$$

$$\frac{D\rho}{Dt} + \rho\,\nabla\cdot\vec{u} = 0 \qquad (\vec{u} = u\,i + v\,j + w\,k)$$

$$\downarrow$$

$$\frac{1}{\rho}\frac{\partial \rho}{Dt} + \nabla\cdot\vec{u} = 0$$

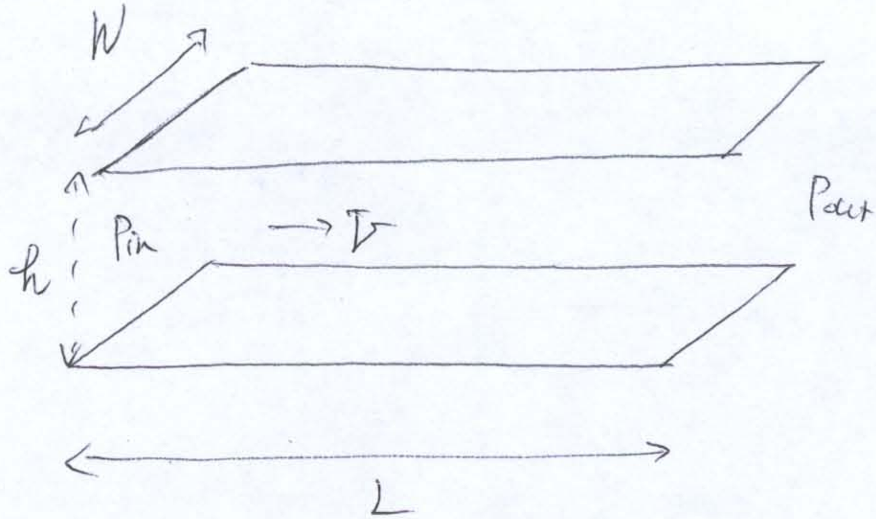
Incompressible flow : Changing in the density of the material element (the element moving with fluid) are negligibly small

$$\frac{1}{\rho}\frac{D\rho}{Dt} \simeq 0$$

$$\longrightarrow \boxed{\nabla\cdot\vec{u} = 0 \quad : \quad \text{Incompressibility !}}$$

[3] (From 9-82)

Flow is assumed to be 2D. & Steady.



Start from NS & Continuity eqn.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad : x\text{-momentum} \quad — ①$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad : y\text{-momentum} \quad — ②$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad : \text{Continuity} \quad — ③$$

Make assumptions as:

① Flow is parallel flow. $\Rightarrow v = 0$

② Flow is fully developed. $\Rightarrow u = u(y)$

Then, simplify ① ~ ③ based on the assumptions above.

$$①: \; u\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{0} + \underbrace{\cancel{v}}_{0}\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{0} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\longrightarrow \; -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} = 0 \quad — ①'$$

② : $\underbrace{u\frac{\partial v}{\partial x}}_{0} + \underbrace{\cancel{v\frac{\partial v}{\partial y}}}_{0} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\underbrace{\frac{\partial^2 v}{\partial x^2}}_{0} + \underbrace{\frac{\partial^2 v}{\partial y^2}}_{0}\right)$

$$\rightarrow \quad -\frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \qquad \text{— ②'}$$

③ : $\underbrace{\frac{\partial u}{\partial x}}_{0} + \underbrace{\frac{\partial v}{\partial y}}_{0} = 0 \quad \rightarrow \quad 0 = 0 \qquad \text{— ③'}$

Solve ①' to get velocity profile.

$$\frac{\partial^2 u}{\partial y^2} = -\frac{1}{\rho\nu}\frac{\partial p}{\partial x} = -\frac{1}{\mu}\frac{\partial p}{\partial x} \quad \left(\because \nu = \frac{\mu}{\rho}\right)$$

integrate

$$\rightarrow \quad \frac{\partial u}{\partial y} = -\frac{1}{\mu}\frac{\partial p}{\partial x}y + C_1$$

$$\rightarrow \quad u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 + C_1 y + C_2$$

integrate again.

Apply Boundary conditions to determine C_1 and C_2.

At $y = 0$: $u = 0$ $\rightarrow$ $0 = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\times 0^2 + C_1\times 0 + C_2$

$\rightarrow$ $C_2 = 0$

At $y = h$: $u = 0$ $\rightarrow$ $0 = -\frac{1}{2\mu}\frac{\partial p}{\partial x}h^2 + C_1 h$

$\rightarrow$ $C_1 = \frac{1}{2\mu}\frac{\partial p}{\partial x}h$

$$\therefore \quad u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 + \frac{1}{2\mu}\frac{\partial p}{\partial x}hy = -\frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - hy)$$

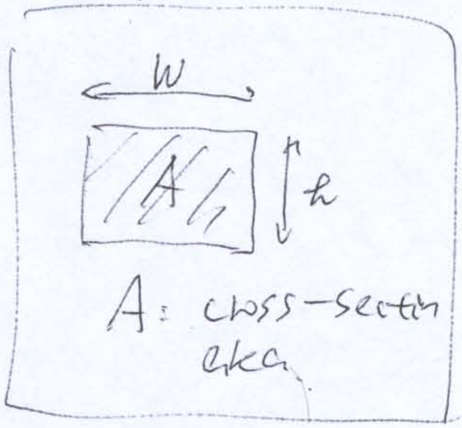
As long as the pressure at inlet and outlet are given,
we need to think $\frac{\partial p}{\partial x}$..

$\frac{\partial p}{\partial x}$: rate of change of pressure in x-direction

$$\therefore \frac{\partial p}{\partial x} = \frac{P_{out} - P_{in}}{L} = \frac{0 - 101.325\times10^{3}}{1.5} = -67550$$

μ is obtained from Table of fluid property. //

<u>Flow rate : Q</u>



$$Q = W \times \int_0^h u\,dy$$

$$= W \times \int_0^h -\frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - hy)\,dy$$

$$= -\frac{W}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{3}y^3 - \frac{1}{2}hy^2\right]_0^h$$

$$= -\frac{W}{12\mu}\frac{\partial p}{\partial x}$$

OK to remain!!

$$\rightarrow Q = -\frac{W}{12\mu}\times(-67550) = 9.10\times10^{-4}\ (m^3/s)$$ //

$$V_{Ave} = \frac{Q}{A} = \frac{9.10\times10^{-4}}{0.0025\times0.75} = 0.485\ (m/s)$$

$$\rightarrow Re = \frac{\rho V_{ave} h}{\mu} = 14.5$$: laminar //

[4] (from 10-92)

Given the velocity profile as $u = \frac{Uy}{\delta}$

Displacement thickness δ^* is

$$\boxed{\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy}$$

$$\rightarrow \delta^* = \int_0^{\delta} \left(1 - \frac{1}{\cancel{U}} \cdot \frac{\cancel{U}y}{\delta}\right) dy = \int_0^{\delta} \left(1 - \frac{y}{\delta}\right) dy$$

$$= \left[y - \frac{y^2}{2\delta} \right]_0^{\delta} = \frac{1}{2}\delta$$

Momentum thickness θ is

$$\boxed{\theta = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy}$$

$$= \int_0^{\delta} \frac{1}{\cancel{U}} \cdot \frac{\cancel{U}y}{\delta} \left(1 - \frac{1}{\cancel{U}} \cdot \frac{\cancel{U}y}{\delta}\right) dy$$

$$= \int_0^{\delta} \frac{y}{\delta} \left(1 - \frac{y}{\delta}\right) dy$$

$$= \frac{1}{\delta} \left[\frac{1}{2} y^2 - \frac{y^3}{3\delta} \right]_0^{\delta}$$

$$= \frac{1}{\delta} \left(\frac{1}{2}\delta^2 - \frac{1}{3}\delta^2 \right) = \frac{1}{6}\delta$$

$\boxed{5}$ (from 10-100)

By definition, Karman's integral equation is

$$\frac{d}{dx}(U^2\theta) + U\frac{dU}{dx}\delta^* = \frac{\tau_w}{\rho}$$

$$\rightarrow 2U\theta\frac{dU}{dx} + U^2\frac{d\theta}{dx} + U\frac{dU}{dx}\delta^* = \frac{\tau_w}{\rho} \quad \text{— ①}$$

For flat-plate problem, U is defined as free-stream.

$\rightarrow U = \text{const.} \rightarrow \frac{dU}{dx} = 0$

Then, simplify ①

$$2U\theta\underbrace{\cancel{\frac{dU}{dx}}}_{0} + U^2\frac{d\theta}{dx} + U\underbrace{\cancel{\frac{dU}{dx}}}_{0}\delta^* = \frac{\tau_w}{\rho}$$

$$\rightarrow \frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2}$$

Recall $\frac{\tau_w}{\frac{1}{2}\rho U^2} = C_{f,x}$, then

$$\frac{d\theta}{dx} = \frac{1}{2}C_{f,x}$$

Given that $\theta = 0.097\delta$, $C_{f,x} = 0.059\left(\frac{Ux}{\nu}\right)^{-\frac{1}{5}}$, then

$$0.097\frac{d\delta}{dx} = \frac{1}{2}\times 0.059\times\left(\frac{Ux}{\nu}\right)^{-\frac{1}{5}}$$

$$\rightarrow \frac{d\delta}{dx} = \frac{0.059}{2\times 0.097}\times\left(\frac{Ux}{\nu}\right)^{-\frac{1}{5}}$$

Let $A = \dfrac{0.059}{2 \times 0.097}$, $B = \dfrac{U}{\nu}$, then

$$\frac{d\delta}{dx} = A (Bx)^{-\frac{1}{5}}$$
$$= AB^{-\frac{1}{5}} x^{-\frac{1}{5}}$$

$(x=0 \to \delta=0)$
Constants disappear

$$\to \delta = \int AB^{-\frac{1}{5}} x^{-\frac{1}{5}} dx$$
$$= AB^{-\frac{1}{5}} \left[\frac{4}{5} x^{\frac{4}{5}} \right] = \frac{4}{5} AB^{-\frac{1}{5}} x^{\frac{4}{5}} + C_1 \quad = 0$$

$$\to \delta = \frac{4}{5} AB^{-\frac{1}{5}} x^{\frac{4}{5}}$$

Devide both sides by x to get $\frac{\delta}{x}$

$$\frac{\delta}{x} = \frac{4}{5} A \underbrace{B^{-\frac{1}{5}} x^{-\frac{1}{5}}}$$
$$= \frac{4}{5} A \left(\frac{Ux}{\nu}\right)^{-\frac{1}{5}} //$$

$\parallel$ Re_x : local Reynolds number.

Text. PP 581

Distinguish!! C_f vs $C_{f,x}$

Average

local

$$C_f = \frac{1}{L} \int_0^L \boxed{C_{f,x}} \, dx$$

$$C_{f,x} = \begin{cases} \dfrac{0.664}{\sqrt{Re_x}} & (Re_x < 5 \times 10^5) \\ \dfrac{0.059}{Re_x^{\frac{1}{5}}} & (5 \times 10^5 \leq Re_x \leq 10^7) \end{cases}$$

$\boxed{6}$

Let the drag coefficient when the window closed : $C_{DC} = 0.32$

" open : $C_{DO} = 0.41$

Then,

$$C_{DC} = \frac{F_{DC}}{\frac{1}{2}\rho v^2 A} \quad (A : \text{frontal area})$$

$$C_{DO} = \frac{F_{DO}}{\frac{1}{2}\rho v^2 A}$$

$$\rightarrow F_{DC} = \frac{1}{2}\rho v^2 A\, C_{DC}, \quad F_{DO} = \frac{1}{2}\rho v^2 A\, C_{DO}$$

$$\therefore \Delta F = F_{DO} - F_{DC}$$

$$= \frac{1}{2}\rho v^2 A\,(C_{DO} - C_{DC})$$

$$= \frac{1}{2}\rho v^2 A\,(0.41 - 0.32) = 0.045\,\rho v^2 A$$

(a) $V = 35$ mph $= 51.33$ (ft/s)

$$\therefore \Delta F = \frac{0.045 \times 0.075 \times (51.33)^2 \times 18}{32.174}$$

$$= 4.974 \text{ (lbf)}$$

$$\therefore P = \frac{4.974 \times 51.33}{737.56} = \underline{0.345 \text{ (kW)}} \;//$$

$\boxed{1\,(\text{kW}) = 737.56\ (\frac{\text{lbf}\cdot\text{ft}}{\text{s}})}$

(h) $V = 70\ (mi/h) = 102.66\ (ft/s)$

Use the same approach as (C), then

$$\Delta F = \frac{0.045 \times 0.075 \times (102.66)^2 \times 18}{32.174}$$

$$= 19.89\ (lbf)$$

$$\therefore P = \frac{19.89 \times 102.66}{737.56} = \underline{2.77\ (kW)}$$