Review problems for final exam – Fall2006 –

11–96E The passenger compartment of a minivan traveling at 60 mi/h in ambient air at 1 atm and 80°F can be modeled as a 3.2-ft-high, 6-ft-wide, and 11-ft-long rectangular box. The airflow over the exterior surfaces can be assumed to be turbulent because of the intense vibrations involved. Determine the drag force acting on the top and the two side surfaces of the van and the power required to overcome it.

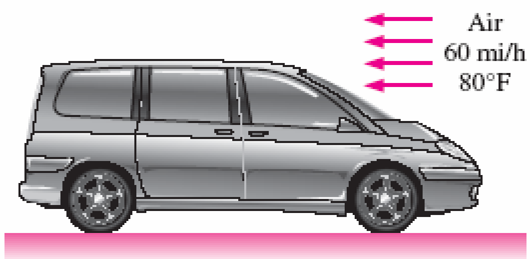


FIGURE P11–96E

10–84 In order to avoid boundary layer interference, engineers design a "boundary layer scoop" to skim off the boundary layer in a large wind tunnel (Fig. P10–84). The scoop is constructed of thin sheet metal. The air is at 20°C, and flows at $V = 65.0$ m/s. How high (dimension h) should the scoop be at downstream distance $x = 1.45$ m?

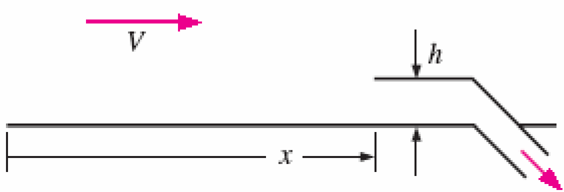


FIGURE P10–84

8–59 Consider flow from a water reservoir through a circular hole of diameter D at the side wall at a vertical distance H from the free surface. The flow rate through an actual hole with a sharp-edged entrance ($K_L = 0.5$) will be considerably less than the flow rate calculated assuming "frictionless" flow and thus zero loss for the hole. Disregarding the effect of the kinetic energy correction factor, obtain a relation for the "equivalent diameter" of the sharp-edged hole for use in frictionless flow relations.

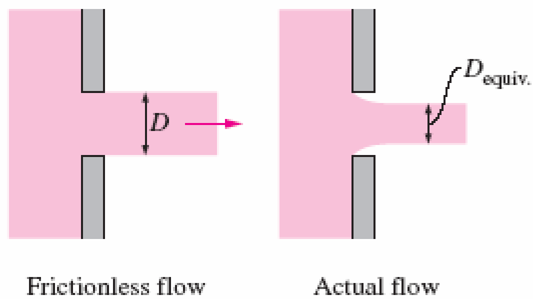


FIGURE P8–59

7–73 Use dimensional analysis to show that in a problem involving shallow water waves (Fig. P7–73), both the Froude number and the Reynolds number are relevant dimensionless parameters. The wave speed c of waves on the surface of a liquid is a function of depth h, gravitational acceleration g, fluid density ρ, and fluid viscosity μ. Manipulate your Π's to get the parameters into the following form:

$$\mathrm{Fr} = \frac{c}{\sqrt{gh}} = f(\mathrm{Re}) \qquad \text{where } \mathrm{Re} = \frac{\rho c h}{\mu}$$

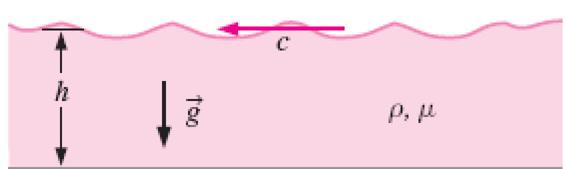


FIGURE P7–73

5–100 A wind tunnel draws atmospheric air at 20°C and 101.3 kPa by a large fan located near the exit of the tunnel. If the air velocity in the tunnel is 80 m/s, determine the pressure in the tunnel.

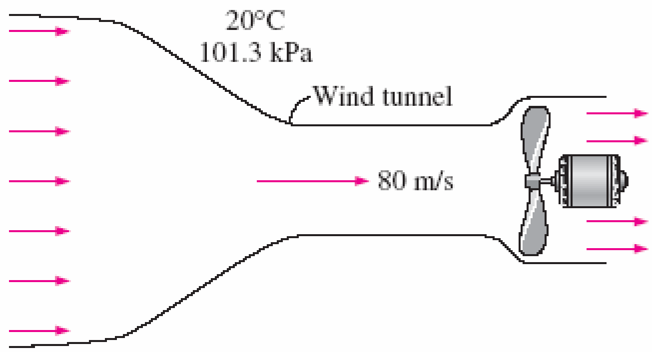


FIGURE P5–100

5–49E Water flows through a horizontal pipe at a rate of 1 gal/s. The pipe consists of two sections of diameters 4 in and 2 in with a smooth reducing section. The pressure difference between the two pipe sections is measured by a mercury manometer. Neglecting frictional effects, determine the differential height of mercury between the two pipe sections. *Answer:* 0.52 in

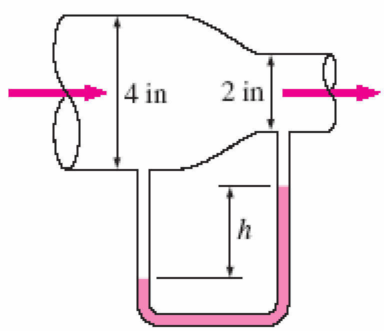


FIGURE P5–49E

Solutions for the review problems – Fall 2006–

10-84

Solution The height of a boundary layer scoop in a wind tunnel test section is to be calculated.

Assumptions **1** The flow is steady and incompressible. **2** The walls are smooth. **3** The boundary layers starts growing at $x = 0$.

Properties The kinematic viscosity of air at 20°C is $\nu = 1.516 \times 10^{-5}$ m²/s.

Analysis (a) As the boundary layer grows along the wall of the wind tunnel test section, the Reynolds number increases. The Reynolds number at location x is

Re$_x$:
$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(65.0 \text{ m/s})(1.45 \text{ m})}{1.516\times10^{-5} \text{m}^2/\text{s}} = 6.22\times10^6$$

Since Re_x is greater than the transition Reynolds number, $\mathrm{Re}_{x,\text{transition}} \approx 3 \times 10^6$, we assume that the boundary layer is turbulent throughout the length of the test section. We estimate the boundary layer thickness at the location of the scoop,

Table 10-4a:
$$\delta \approx \frac{0.16x}{(\mathrm{Re}_x)^{1/7}} = \frac{0.16(1.45 \text{ m})}{(6.22\times10^6)^{1/7}} = 2.48\times10^{-2} \text{ m} = 24.8 \text{ mm} \qquad (1)$$

or,

Table 10-4b:
$$\delta \approx \frac{0.38x}{(\mathrm{Re}_x)^{1/5}} = \frac{0.38(1.45 \text{ m})}{(6.22\times10^6)^{1/5}} = 2.41\times10^{-2} \text{ m} = 24.1 \text{ mm} \qquad (1)$$

11-96E The passenger compartment of a minivan is modeled as a rectangular box. The drag force acting on the top and the two side surfaces and the power needed to overcome it are to be determined. √

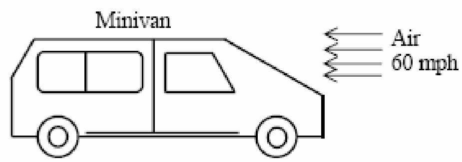
Assumptions **1** The air flow is steady and incompressible. **2** The air flow over the exterior surfaces is turbulent because of constant agitation. **3** Air is an ideal gas. **4** The top and side surfaces of the minivan are flat and smooth (in reality they can be rough). **5** The atmospheric air is calm (no significant winds).

Properties The density and kinematic viscosity of air at 1 atm and 80°F are ρ = 0.07350 lbm/ft^3 and ν = 0.6110 ft^2/h = 1.697×10^{-4} ft^2/s.

Analysis The Reynolds number at the end of the top and side surfaces is

$$\text{Re}_L = \frac{VL}{\upsilon} = \frac{[60\times1.4667\text{ ft/s}](11\text{ ft})}{1.697\times10^{-4}\text{ ft}^2\text{/s}} = 5.704\times10^6$$

The air flow over the entire outer surface is assumed to be turbulent. Then the friction coefficient becomes



$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} = \frac{0.074}{(5.704\times10^6)^{1/5}} = 0.00330$$

The area of the top and side surfaces of the minivan is

$$A = A_{\text{top}} + 2A_{\text{side}} = 6\times11+2\times3.2\times11 = 136.4\text{ ft}^2$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a plane surface, the drag force acting on these surfaces becomes

$$F_D = C_f A\frac{\rho V^2}{2} = 0.00330\times(136.4\text{ ft}^2)\frac{(0.074\text{ lbm/ft}^3)(60\times1.4667\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{4.0\ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

$$\dot{W}_{\text{drag}} = F_D V = (4.0\text{ lbf})(60\times1.4667\text{ ft/s})\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.48\ kW}$$

8-59 Water is discharged from a water reservoir through a circular hole of diameter D at the side wall at a vertical distance H from the free surface. A relation for the "equivalent diameter" of the sharp-edged hole for use in frictionless flow relations is to be obtained.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **3** The effect of the kinetic energy correction factor is disregarded, and thus $\alpha = 1$.

Analysis The loss coefficient is $K_L = 0.5$ for the sharp-edged entrance, and $K_L = 0$ for the "frictionless" flow. We take point 1 at the free surface of the reservoir and point 2 at the exit of the hole, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is zero ($V_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine,e}}+h_L \quad\rightarrow\quad H=\alpha_2\frac{V_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L\dfrac{V^2}{2g}$. Substituting and solving for V_2 gives

$$H=\alpha_2\frac{V_2^2}{2g}+K_L\frac{V_2^2}{2g} \quad\rightarrow\quad 2gH=V_2^2(\alpha_2+K_L) \quad\rightarrow\quad V_2=\sqrt{\frac{2gH}{\alpha_2+K_L}}=\sqrt{\frac{2gH}{1+K_L}}$$

since $\alpha_2 = 1$. Then the volume flow rate becomes

$$\dot{V}=A_cV_2=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}} \qquad (1)$$

Note that in the special case of $K_L = 0$ (frictionless flow), the velocity relation reduces to the Toricelli equation, $V_{2,\text{frictionless}}=\sqrt{2gH}$. The flow rate in this case through a hole of D_e (equivalent diameter) is

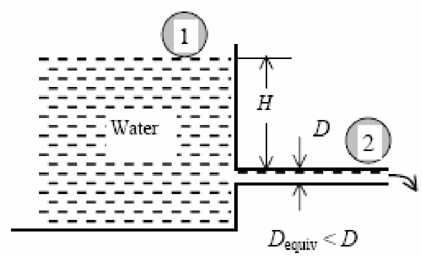
$$\dot{V}=A_{c,\text{equiv}}V_{2,\text{frictionless}}=\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH} \qquad (2)$$

Setting Eqs. (1) and (2) equal to each other gives the desired relation for the equivalent diameter,

$$\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH}=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}}$$

which gives

$$D_{\text{equiv}}=\frac{D}{(1+K_L)^{1/4}}=\frac{D}{(1+0.5)^{1/4}}=0.904D$$



Discussion Note that the effect of frictional losses of a sharp-edged entrance is to reduce the diameter by about 10%. Also, noting that the flow rate is proportional to the square of the diameter, we have $\dot{V}\propto D_{\text{equiv}}^2=(0.904D)^2=0.82D^2$. Therefore, the flow rate through a sharp-edged entrance is about 18% less compared to the frictionless entrance case.

Analysis We perform a dimensional analysis using the method of repeating variables.

Step 1 There are five parameters in this problem; $n = 5$,

List of relevant parameters: $c = f(h, \rho, \mu, g) \qquad n = 5$

Step 2 The primary dimensions of each parameter are listed,

c	h	ρ	μ	g
$\{L^1t^{-1}\}$	$\{L^1\}$	$\{m^1L^{-3}\}$	$\{m^1L^{-1}t^{-1}\}$	$\{L^1t^{-2}\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $k = n - j = 5 - 3 = 2$

Step 4 We need to choose three repeating parameters since $j = 3$. We pick length scale h, density difference ρ, and gravitational constant g.

Repeating parameters: h, ρ, and g

Step 5 The Πs are generated. Note that for the first Π we do the algebra in our heads since the relationship is very simple. The dependent Π is

Π_1 = *Froude number:* $$\Pi_1 = \mathrm{Fr} = \frac{c}{\sqrt{gh}} \qquad (1)$$

This Π is the *Froude number*. Similarly, the Π formed with viscosity is generated,

$$\Pi_2 = \mu h^a \rho^b g^c \qquad \{\Pi_2\} = \left\{(m^1L^{-1}t^{-1})(L^1)^a(m^1L^{-3})^b(L^1t^{-2})^c\right\}$$

mass: $\{m^0\} = \{m^1m^b\} \qquad 0 = 1 + b \qquad b = -1$

time: $\{t^0\} = \{t^{-1}t^{-2c}\} \qquad 0 = -1 - 2c \qquad c = -\frac{1}{2}$

length: $\{L^0\} = \{L^{-1}L^aL^{-3b}L^c\} \qquad 0 = -1 + a - 3b + c$, $0 = -1 + a + 3 - \frac{1}{2} \qquad a = -\frac{3}{2}$

which yields

Π_2:
$$\Pi_2 = \frac{\mu}{\rho h^{\frac{3}{2}}\sqrt{g}}$$

We can manipulate this Π into the Reynolds number if we invert it and then multiply by Fr (Eq. 1) The final form is

Modified Π_2 = *Reynolds number*:
$$\Pi_2 = \text{Re} = \frac{\rho c h}{\mu}$$

Step 6 We write the final functional relationship as

Relationship between Πs:
$$\text{Fr} = \frac{c}{\sqrt{gh}} = f(\text{Re}) \quad \text{where} \quad \text{Re} = \frac{\rho c h}{\mu}$$

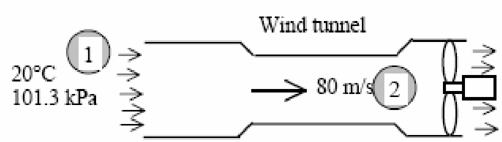
5-100 A wind tunnel draws atmospheric air by a large fan. For a given air velocity, the pressure in the tunnel is to be determined. √

Assumptions **1**The flow through the pipe is steady, incompressible, and irrotational with negligible friction (so that the Bernoulli equation is applicable). **2** Air is and ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa·m³/kg·K.

Analysis We take point 1 in atmospheric air before it enters the wind tunnel (and thus $P_1 = P_{atm}$ and $V_1 \cong 0$), and point 2 in the wind tunnel. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases), the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad P_2 = P_1 - \frac{\rho V_2^2}{2} \qquad (1)$$



where

$$\rho = \frac{P}{RT} = \frac{101.3 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293 \text{ K})} = 1.205 \text{ kg/m}^3$$

Substituting, the pressure in the wind tunnel is determined to be

$$P_2 = (101.3 \text{ kPa}) - (1.205 \text{ kg/m}^3)\frac{(80 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) = \mathbf{97.4 \text{ kPa}}$$

Discussion Note that the velocity in a wind tunnel increases at the expense of pressure. In reality, the pressure will be even lower because of losses.

5-49E Water flows through a horizontal pipe that consists of two sections at a specified rate. The differential height of a mercury manometer placed between the two pipe sections is to be determined. √

Assumptions **1**The flow through the pipe is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The losses in the reducing section are negligible.

Properties The densities of mercury and water are ρ_{Hg} = 847 lbm/ft^3 and ρ_w = 62.4 lbm/ft^3.

Analysis We take points 1 and 2 along the centerline of the pipe over the two tubes of the manometer. Noting that $z_1 = z_2$, the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\frac{\rho_w(V_2^2-V_1^2)}{2} \qquad (1)$$

We let the differential height of the mercury manometer be h and the distance between the centerline and the mercury level in the tube where mercury is raised be s. Then the pressure difference $P_2 - P_1$ can also be expressed as

$$P_1+\rho_w g(s+h)=P_2+\rho_w gs+\rho_{Hg}gh \quad\rightarrow\quad P_1-P_2=(\rho_{Hg}-\rho_w)gh \qquad (2)$$

Combining Eqs. (1) and (2) and solving for h,

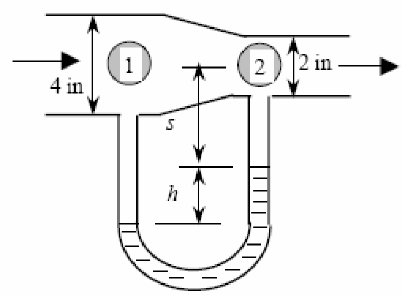
$$\frac{\rho_w(V_2^2-V_1^2)}{2}=(\rho_{Hg}-\rho_w)gh \quad\rightarrow\quad h=\frac{\rho_w(V_2^2-V_1^2)}{2g(\rho_{Hg}-\rho_w)}=\frac{V_2^2-V_1^2}{2g(\rho_{Hg}/\rho_w-1)}$$

Calculating the velocities and substituting,

$$V_1=\frac{\dot V}{A_1}=\frac{\dot V}{\pi D_1^2/4}=\frac{1\text{ gal/s}}{\pi(4/12\text{ ft})^2/4}\left(\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right)=1.53\text{ ft/s}$$

$$V_2=\frac{\dot V}{A_2}=\frac{\dot V}{\pi D_2^2/4}=\frac{1\text{ gal/s}}{\pi(2/12\text{ ft})^2/4}\left(\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right)=6.13\text{ ft/s}$$

$$h=\frac{(6.13\text{ ft/s})^2-(1.53\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)(847/62.4-1)}=0.0435\text{ ft}=\mathbf{0.52\text{ in}}$$



Therefore, the differential height of the mercury column will be 0.52 in.