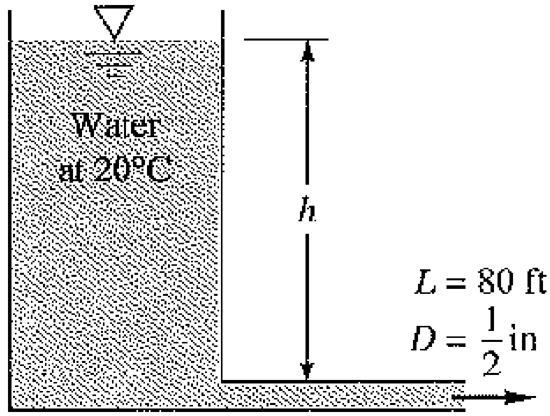
1. Head loss

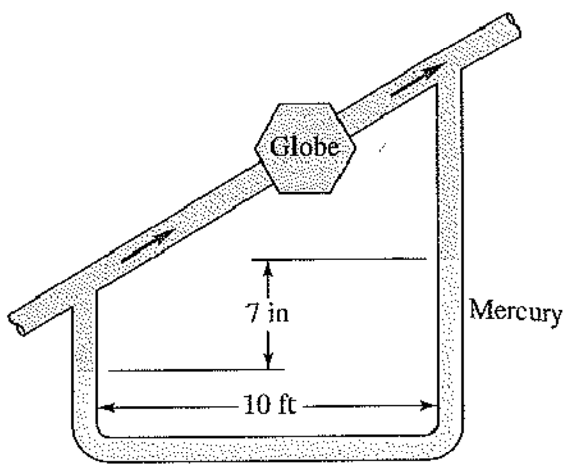
What level h must be maintained in Fig. P6.61 to deliver a flow rate of 0.015 ft^3/s through the ½ -in commercial steel pipe?



P6.61

2. Minor loss

The water pipe in Fig. P6.106 slopes upward at 30°. The pipe has a 1-in diameter and is smooth. The flanged globe valve is fully open. If the mercury manometer shows a 7-in defection, what is the flow rate in ft^3/s?

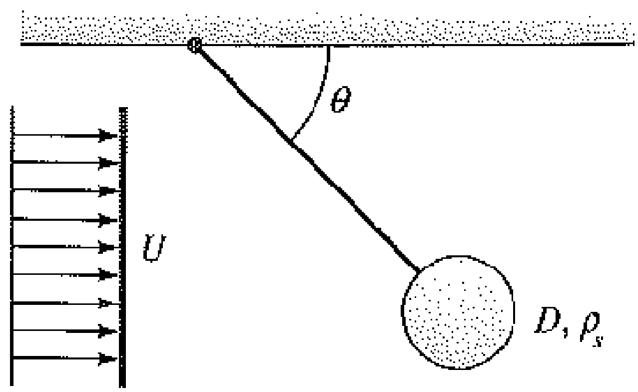


P6.106

3. Lift and drag coefficient

A heavy sphere attached to a strung should hang at an angel θ when immersed in a stream of velocity U, as in Fig. P7.80. Derive an expression for θ as a function of the sphere and flow properties. What is θ if the sphere is steel (SG = 7.86) of diameter 3 cm and the flow is sea-level standard air at U =40 m/s? Neglect the string drag.

P7.80



4. Boundary layer

A sharp flat plate with L = 50 cm and b = 3 m is parallel to a stream of velocity 2.5 m/s. Find the drag on *one side* of the plate, and the boundary thickness δ at the trailing edge for (a) air and (b) water at 20 °C and 1 atm.

5. Fluid kinematics

Following the fluid particle, calculate the *y* component of acceleration for a particle whose velocity vector is given by $\mathbf{V} = (3z - x^2)\mathbf{i} + yt^2\mathbf{j} + xz^2\mathbf{k}$ in ft/sec at the point x = 1ft, y =1 ft, and t = 2sec.

6. Mass and momentum conservation

A 45° reducing elbow can be found in domestic water piping system. As illustrated in Figure 3.15, water flows into the elbow in the positive direction and is deflected through an angle of 45°. The inlet diameter is 2.5 cm and the outlet diameter is 1.2 cm. The volume flow rate of water is 0.0008 m^3/s. The inlet and outlet pressures are, respectively, 160 kPa and 150 kPa. If the elbow is located in a horizontal plane, determine the forces exerted on it by moving water.

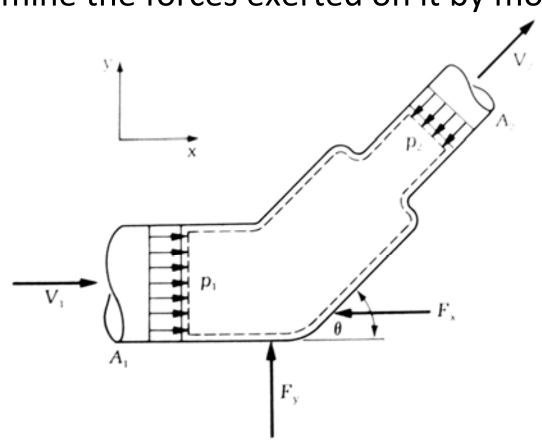


Figure 3.15. The reducing elbow of Example 3.7.

Solutions

1.

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. For commercial steel, take $\varepsilon \approx 0.00015$ ft, or $\varepsilon/d = 0.00015/(0.5/12) \approx 0.0036$. Compute

$$V = \frac{Q}{A} = \frac{0.015}{(\pi/4)(0.5/12)^2} = 11.0\ \frac{ft}{s};$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.94(11.0)(0.5/12)}{2.09E-5} \approx 42500 \quad \varepsilon/d = 0.0036, \quad f_{Moody} \approx 0.0301$$

The energy equation, with p1 = p2 and V1 ≈ 0, yields an expression for surface elevation:

$$h = h_f + \frac{V^2}{2g} = \frac{V^2}{2g}\left(1 + f\frac{L}{d}\right) = \frac{(11.0)^2}{2(32.2)}\left[1 + 0.0301\left(\frac{80}{0.5/12}\right)\right] \approx \mathbf{111\ ft} \quad \textit{Ans.}$$

2.

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. The pipe length and elevation change are

Fig. P6.106

$$L = \frac{10\ ft}{\cos 30°} = 11.55\ ft; \quad z_2 - z_1 = 10\tan 30° = 5.77\ ft, \quad \text{Open 1'' globe valve: } K \approx 13$$

The manometer indicates the total pressure change between (1) and (2):

$$p_1 - p_2 = (\rho_{Merc} - \rho_w)gh + \rho_w g\Delta z = (13.6 - 1)(62.4)\left(\frac{7}{12}\right) + 62.4(5.77) \approx 819\ psf$$

The energy equation yields

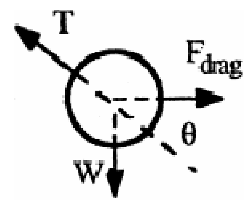
$$\frac{p_1 - p_2}{\rho g} = \Delta z + h_f + h_m = 5.77 + \frac{V^2}{2(32.2)}\left[f\frac{11.55}{1/12} + 13\right] \approx \frac{819\ lbf/ft^2}{62.4\ lbf/ft^3}$$

$$\text{or:} \quad V^2 \approx \frac{2(32.2)(7.35)}{(139f + 13)}. \quad \text{Guess } f \approx 0.02,\ V \approx 5.48\ \frac{ft}{s},\ Re \approx 42400,\ f_{new} \approx 0.0217$$

Rapid convergence to $f \approx 0.0217$, $V \approx 5.44$ ft/s, $Q = V(\pi/4)(1/12)^2 \approx$ **0.0296 ft^3/s.** *Ans.*

3.

Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E−5 kg/m·s. The sphere should hang so that string tension balances the resultant of drag and net weight:



$$\tan\theta = \frac{W_{net}}{Drag}, \quad \text{or} \quad \theta = \tan^{-1}\left[\frac{(\rho_s - \rho)g(\pi/6)D^3}{(\pi/8)C_D\rho U^2 D^2}\right] \quad \textit{Symbolic answer.}$$

For the given numerical data, first check Re and the drag coefficient, then find the angle:

$$Re_D = \frac{1.225(40)(0.03)}{1.78E{-}5} \approx 83000, \quad \text{Fig. 7.16b: } C_D \approx \mathbf{0.5}$$

$$\mathbf{F} = \frac{\pi}{8}(0.5)(1.225)(40)^2(0.03)^2 \approx 0.346 \text{ N};$$

$$W = [7.86(998) - 1.225](9.81)\frac{\pi}{6}(0.03)^3 \approx 1.09 \text{ N} \quad \therefore \quad \theta = \tan^{-1}(1.09/0.346) \approx \mathbf{72°} \quad \textit{Ans.}$$

4.

- *Assumptions:* Laminar flat-plate flow, but we should check the Reynolds numbers.
- *Approach:* Find the Reynolds number and use the appropriate boundary layer formulas.
- *Property values:* From Table A.2 for air at 20°C, ρ = 1.2 kg/m^3, ν = 1.5E-5 m^2/s. From Table A.1 for water at 20°C, ρ = 998 kg/m^3, ν = 1.005E-6 m^2/s.
- *(a) Solution for air:* Calculate the Reynolds number at the trailing edge:

$$Re_L = \frac{UL}{\nu_{air}} = \frac{(2.5 \text{ m/s})(0.5 \text{ m})}{1.5\text{E-5 m}^2/\text{s}} = 83{,}300 < 5\text{E5 therefore assuredly laminar}$$

The appropriate thickness relation is Eq. (7.24):

$$\frac{\delta}{L} = \frac{5}{Re_L^{1/2}} = \frac{5}{(83{,}300)^{1/2}} = 0.0173, \text{ or } \delta_{x=L} = 0.0173(0.5 \text{ m}) \cong 0.0087 \text{ m} \quad \textit{Ans. (a)}$$

The laminar boundary layer is only 8.7 mm thick. The drag coefficient follows from Eq. (7.27):

$$C_D = \frac{1.328}{Re_L^{1/2}} = \frac{1.328}{(83{,}300)^{1/2}} = 0.0046$$

$$\text{or } D_{\text{one side}} = C_D\frac{\rho}{2}U^2bL = (0.0046)\frac{1.2 \text{ kg/m}^3}{2}(2.5 \text{ m/s})^2(3 \text{ m})(0.5 \text{ m}) \approx 0.026 \text{ N} \quad \textit{Ans. (a)}$$

5.

Solution.

$$a_y = \frac{DV_y}{Dt} = V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z} + \frac{\partial V_y}{\partial t}$$

$$= (3z - x^2)\frac{\partial(yt^2)}{\partial x} + yt^2\frac{\partial(yt^2)}{\partial y} + xz^2\frac{\partial(yt^2)}{\partial z} + \frac{\partial(yt^2)}{\partial t}$$

$$= (3z - x^2)(0) + yt^2(t^2) + xz^2(0) + 2yt = yt^4 + 2yt$$

$$a_y(1,1,9,2) = (1)(2)^4 + 2(1)(2) = 20 \text{ ft/sec}^2$$

6.

$$-F_x + p_1A_1 - p_2A_2\cos\theta = \frac{\dot{m}}{g_c}(V_{x\,out} - V_{x\,in}) = \frac{\rho Q}{g_c}(V_2\cos\theta - V_1)$$

Now

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.025)^2}{4} = 0.000\,49 \text{ m}^2$$

Similarly,

$$A_2 = 0.000\,11 \text{ m}^2$$

From the continuity equation, we have

$$V_1 = \frac{Q}{A_1} = \frac{0.000\,8}{0.000\,49} = 1.63 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.000\,8}{0.000\,11} = 7.27 \text{ m/s}$$

After rearranging and substituting into the x-directed momentum equation, we obtain

$$-F_x = p_2A_2\cos\theta - p_1A_1 + \frac{\rho Q}{g_c}(V_2\cos\theta - V_1)$$

$$= (150\,000 \text{ N/m}^2)(0.000\,11 \text{ m}^2)\cos 45° - (160\,000 \text{ N/m}^2)(0.000\,49 \text{ m}^2) + (1\,000 \text{ kg/m}^3)(0.000\,8 \text{ m}^3\text{/s})(7.27\cos 45° - 1.63) \text{ m/s}$$

$$F_x = 59.1 \text{ N}$$

The momentum equation written for the y direction is

$$F_y - p_2A_2\sin\theta = \frac{\dot{m}}{g_c}(V_{y\,out} - V_{y\,in}) = \frac{\rho Q}{g_c}(V_2\sin\theta)$$

After substitution, we find

$$F_y = (150\,000)(0.000\,11)\sin 45° + 1\,000(0.000\,8)(7.27)\sin 45°$$

or

$$F_y = 15.78 \text{ N}$$