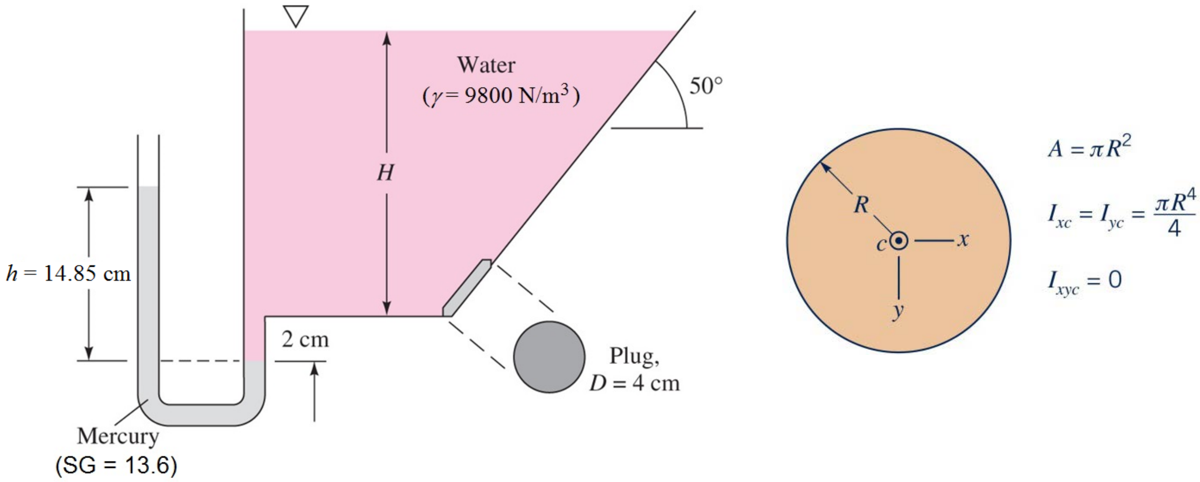
1. [Static pressure & monometer]



(a) (b)

(a) Monometer (**+2 points**)

$$p_{atm} + \gamma \cdot \mathrm{SG} \cdot h - \gamma(H + 0.02) = p_{atm}$$
$$\therefore H = \mathrm{SG} \cdot h - 0.02 = (13.6)(0.1485) - 0.02 = \mathbf{2.0\ m}$$

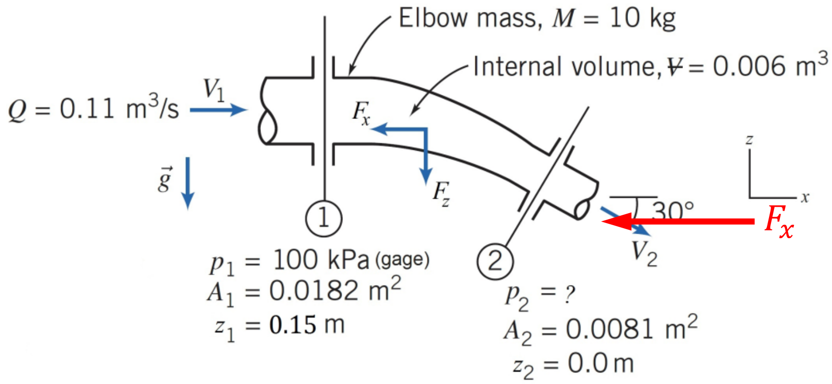
(b1) Hydrostatic pressure force (**+4 points**)

$$h_c = H - \left(\frac{D}{2}\right)\sin 50° = 2 - \left(\frac{0.04}{2}\right)\sin 50° = 1.985\ \mathrm{m}$$
$$A = \frac{\pi D^2}{4} = \frac{\pi \times 0.04^2}{4} = 1.257 \times 10^{-3}\ \mathrm{m}^2$$
$$F_R = \gamma h_c A = (9800)(1.987)(1.257 \times 10^{-3}) = \mathbf{24.5\ N}$$

(b2) Pressure center (**+4 points**)

$$\bar{y} = \frac{h_c}{\sin 50°} = \frac{1.985}{\sin 50°} = 2.591\ \mathrm{m}$$
$$I_x = \frac{\pi R^4}{4} = \frac{(\pi)(0.04/2)^4}{4} = 1.257 \times 10^{-7}\ \mathrm{m}^4$$
$$\therefore y_R = \bar{y} + \frac{I_x}{\bar{y}A} = 2.591 + \frac{1.257 \times 10^{-7}}{(2.591)(1.257 \times 10^{-3})} = \mathbf{2.591\ m}$$

2. [Continuity+Momentum+Energy]



(a) Continuity (**+1 point**)

$$V_1 = \frac{Q}{A_1} = \frac{0.11}{0.0182} = \mathbf{6.0\ m/s}$$
$$V_2 = \frac{Q}{A_2} = \frac{0.11}{0.0081} = \mathbf{13.6\ m/s}$$

(b) Energy equation (**+3 points**)

$$p_2 = p_1 + \gamma\left[\frac{V_1^2 - V_2^2}{2g} + (z_1 - z_2)\right]$$
$$\therefore p_2 = 100{,}000 + (9800)\left[\frac{6^2 - 13.6^2}{2 \times 9.81} + 0.15\right] = \mathbf{27{,}066\ Pa}$$

(c1) x-momentum (**+3 points**)

$$-F_x + p_1A_1 - p_2A_2\cos\theta = (\rho Q)V_2\cos\theta - (\rho Q)V_1$$

or

$$F_x = +p_1A_1 - p_2A_2\cos 30^\circ - (\rho Q)(V_2\cos 30^\circ - V_1)$$
$$\therefore F_x = (100{,}000)(0.0182) - (27{,}066)(0.0081)\cos 30^\circ - (999)(0.11)(13.6\cos 30^\circ - 6) = \mathbf{995\ N}$$

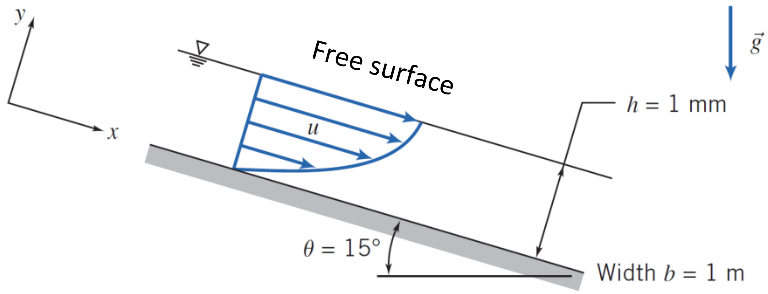
(c2) y-momentum (**+3 points**)

$$-F_z + p_2A_2\sin\theta - Mg - \gamma V\!\!\!\!- = (\rho Q)(-V_2\sin\theta) - (\rho Q)(0)$$

or

$$F_z = p_2A_2\sin\theta - Mg - \gamma V\!\!\!\!- + \rho Q V_2\sin\theta$$
$$\therefore F_z = (27{,}066)(0.0081)\sin 30^\circ - (10)(9.81) - (9800)(0.006) + (999)(0.11)(13.6)\sin 30^\circ$$
$$= \mathbf{700\ N}$$

3. [Navier-Stokes]



(a) B.C. at x = h (**+4 points**)

$$\tau(h) = \mu \left.\frac{du}{dy}\right)_{y=h} = \mu \cdot \frac{\rho g \sin\theta}{\mu}(C - h) = 0$$

$$\therefore \boldsymbol{C = h}$$

(b) Acceleration (**+2 points**)

$$\because \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = \frac{\partial u}{\partial x} = \frac{\partial u}{\partial z} = v = w = 0,$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0 + (u)(0) + (0)\left(\frac{\partial u}{\partial y}\right) + (0)(0) = 0$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = 0 + (u)(0) + (0)(0) + (0)(0) = 0$$

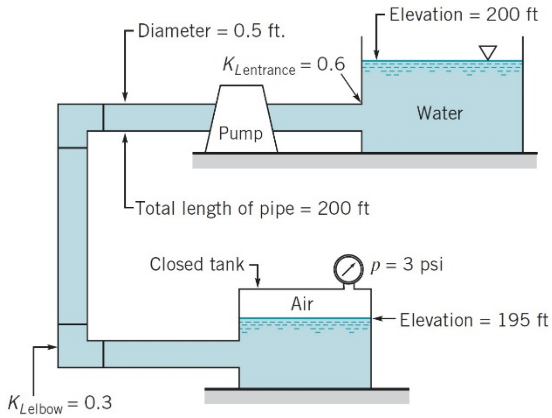
$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = 0 + (u)(0) + (0)(0) + (0)(0) = 0$$

(c) Flow rate (**+4 points**)

$$Q = \int_0^h \frac{\rho g \sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right) b\,dy = \frac{\rho g \sin\theta\, b}{\mu}\left[\frac{hy^2}{2} - \frac{y^3}{6}\right]_0^h$$

$$\therefore Q = \frac{\rho g \sin\theta\, b h^3}{3\mu} = \frac{(912)(9.81)(\sin 15°)(1)(0.001)^3}{(3)(0.38)} = \boldsymbol{2.03 \times 10^{-6}\ \mathrm{m^3/s}}$$

4. [Major+Minor - *Q*]



Energy equation (**+5 points**)

Energy equation with $p_1 = 0$ and $V_1 = V_2 = 0$,

$$0 + 0 + z_1 + h_p = \frac{p_2}{\gamma} + 0 + z_2 + f\frac{L}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$$

or

$$200 + 15 = \frac{3 \times 144}{62.4} + 195 + f\frac{200}{0.5}\frac{V^2}{2 \times 32.2} + (0.6 + 2 \times 0.3)\frac{V^2}{2 \times 32.2}$$

Solve for V,

$$V = \sqrt{\frac{842.2}{400f + 1.2}} \quad \left(\text{or} \quad \sqrt{\frac{13.0769}{6.2112f + 0.0186}}\right) \qquad (1)$$

Iteration process (**+4 points**)

$$Re = \frac{\rho V D}{\mu} = \frac{(1.94)(V)(0.5)}{2.34 \times 10^{-5}} = 41{,}453 \cdot V \qquad (2)$$

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.1} + \frac{6.9}{Re}\right] = -1.8\log\left[\left(\frac{0.003/0.5}{3.7}\right)^{1.1} + \frac{6.9}{Re}\right] \qquad (3)$$

Assume f = 0.03 and using (1), (2), and (3) in sequence,

$$V = 7.988 \rightarrow Re = 3.31 \times 10^5 \rightarrow f = 0.033$$

Assume f = 0.033,

$$V = 7.648 \rightarrow Re = 3.17 \times 10^5 \rightarrow f = 0.033 \;\; \text{(Converged)}$$

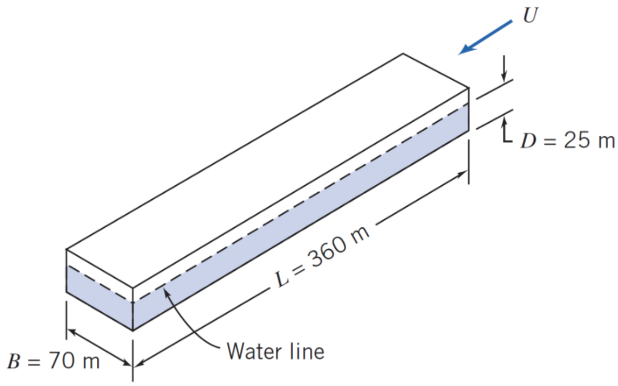
1)

Flow rate (**+1 point**)

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 0.5^2}{4} = 0.196 \text{ m}^2$$

$$Q = VA = (7.648)(0.196) = \mathbf{1.5\ ft^3/s}$$

5. [Boundary layer]



a1) Reynolds number (**+2 points**)

$$U = 5 \times 0.5144 = 2.572 \text{ m/s}$$

$$Re_L = \frac{\rho U L}{\mu} = \frac{(1030)(2.572)(360)}{1.20 \times 10^{-3}} = \mathbf{7.95 \times 10^8}$$

a2) Friction drag coefficient (**+3 points**)

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} = \frac{0.455}{(\log_{10} 7.95 \times 10^8)^{2.58}} = \mathbf{0.00162}$$

or

$$C_f = \frac{0.031}{Re_L^{\frac{1}{7}}} = \frac{0.031}{(7.95 \times 10^8)^{\frac{1}{7}}} = \mathbf{0.00166}$$

b1) Friction Drag (**+3 points**)

$$A = Lb = (360\ )(70 + 2 \times 25) = 43{,}200\ \mathrm{m}^2$$

$$F_D = \frac{1}{2}\rho U^2 A \cdot C_f = \frac{1}{2}(1030)(2.572)^2(43200)(0.00162) = \mathbf{2.38 \times 10^5}\ \boldsymbol{N}$$

Or

$$F_D = \mathbf{2.44 \times 10^5}\ \boldsymbol{N}$$

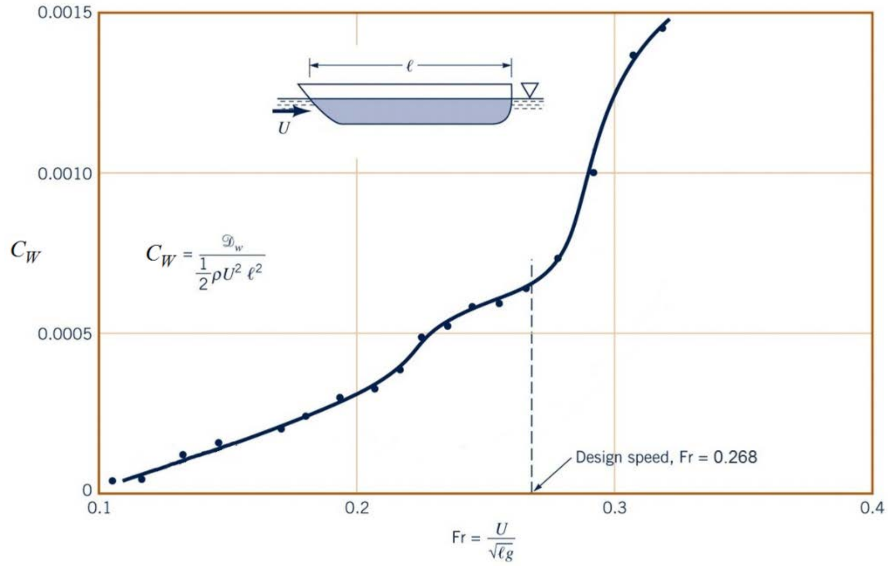
b2) Power (**+2 points**)

$$P = F_D U = (2.38 \times 10^5)(2.572) = \mathbf{6.12 \times 10^5\ W} \approx 0.6\ \mathrm{MW}$$

or

$$P = \mathbf{6.28 \times 10^5}\ \boldsymbol{W}$$

6. [Pi theorem + Drag+Similarity]



(a) Pi theorem (**+4 points**)

D_w	ℓ	U	ρ	g
$\{MLT^{-2}\}$	$\{L\}$	$\{LT^{-1}\}$	$\{ML^{-3}\}$	$\{LT^{-2}\}$
$\{F\}$	$\{L\}$	$\{LT^{-1}\}$	$\{FT^2L^{-4}\}$	$\{LT^{-2}\}$

$$r = n - m = 5 - 3 = 2$$
$$\Pi_1 = \ell^a U^b \rho^c D_w \doteq (L)^a (LT^{-1})^b (ML^{-3})^c (MLT^{-2}) \doteq M^0 L^0 T^0$$
$$\left(\text{or } \doteq (L)^a (LT^{-1})^b (FT^2L^{-4})^c (F) \doteq F^0 L^0 T^0\right)$$
$$\Rightarrow a = -2, \quad b = -2, \quad c = -1$$
$$\therefore \Pi_1 = \ell^{-2} U^{-2} \rho^{-1} D_w = \boldsymbol{\frac{D_w}{\rho U^2 \ell^2}}$$

$$\Pi_2 = \ell^a U^b \rho^c g \doteq (L)^a (LT^{-1})^b (ML^{-3})^c (LT^{-2}) \doteq M^0 L^0 T^0$$
$$\left(\text{or } \doteq (L)^a (LT^{-1})^b (FT^2L^{-4})^c (LT^{-2}) \doteq F^0 L^0 T^0\right)$$
$$\Rightarrow a = 1, \quad b = -2, \quad c = 0$$
$$\therefore \Pi_2 = \ell^1 U^{-2} \rho^0 g = \boldsymbol{\frac{g\ell}{U^2}}$$

(b) Similarity requirement (+2 points)

$$\frac{g\ell_m}{U_m^2} = \frac{g\ell}{U^2}$$
$$U = 16.3 \times 0.5144 = 8.385 \text{ m/s}$$
$$\ell_m = \frac{100}{25} = 4 \text{ m}$$

$$\therefore U_m = U\sqrt{\frac{\ell_m}{\ell}} = (16.3 \times 0.5144)\sqrt{\frac{1}{25}} = \mathbf{1.677\ m/s}$$

(c) Wave-making drag (+4 points)

Model:

$$Fr = \frac{U}{\sqrt{g\ell}} = \frac{1.677}{\sqrt{(9.81)(100/25)}} = 0.268$$

$$C_W = 0.00065 \text{ from the chart}$$

$$\therefore D_w = \frac{1}{2}\rho_m U_m^2 \ell_m^2 \cdot C_W = \left(\frac{1}{2}\right)(999)(1.677)^2(4)^2(0.00065) = \mathbf{14.6\ N}$$

Prototype:

$$Fr = \frac{U}{\sqrt{g\ell}} = \frac{8.385}{\sqrt{(9.81)(100)}} = 0.268$$

$$C_W = 0.00065 \text{ from the chart}$$

$$\therefore D_w = \frac{1}{2}\rho U^2 \ell^2 \cdot C_W = \left(\frac{1}{2}\right)(1{,}030)(8.385)^2(100)^2(0.00065) = \mathbf{235\ kN}$$