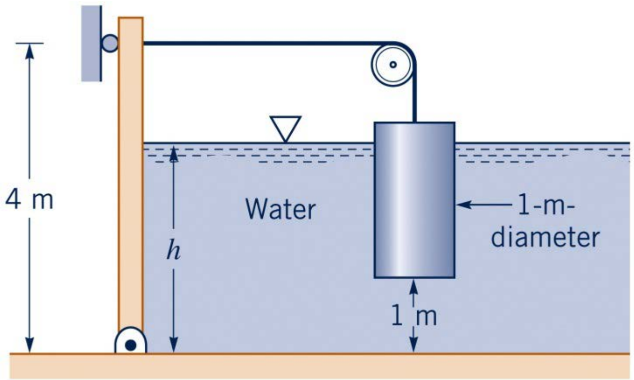
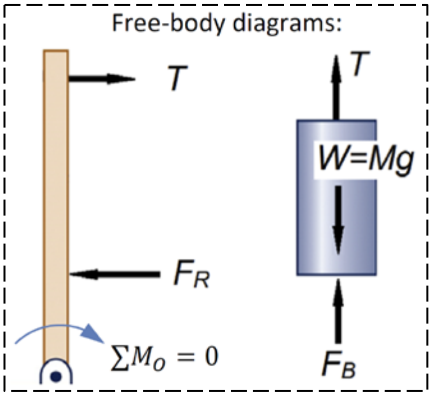
1. [Static pressure & buoyancy]





(a1) Hydrostatic pressure force (**+3 points**)

$$F_R = \gamma h_C A = \gamma \left(\frac{h}{2}\right)(h \times 2) = (9.8 \times 10^3)\left(\frac{2.5}{2}\right)(2.5 \times 2) = \mathbf{61,250\ N}$$

(a2) Pressure center (**+3 points**)

$$y_R = y_C + \frac{I_x}{y_C A} = \frac{2.5}{2} + \frac{\frac{1}{12}(2)(2.5)^3}{\left(\frac{2.5}{2}\right)(2.5 \times 2)} = \mathbf{1.667\ m} \ \ \text{(from free surface)}$$

(b) Tension (**+2 points**)

$$4 \times T = (h - y_R) \times F_R$$

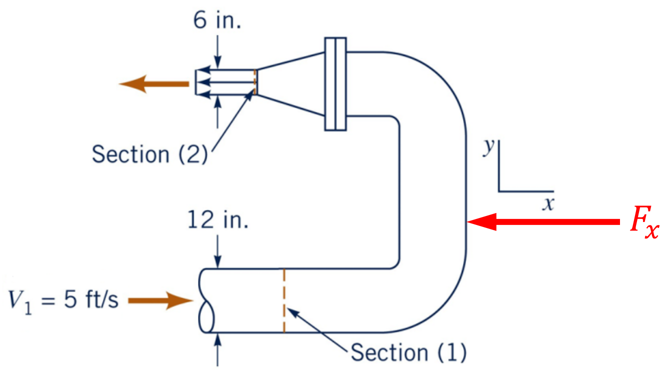
$$\therefore T = \frac{(2.5 - 1.667) \times (61{,}250)}{4} = \mathbf{12,760\ N}$$

(c) Cylinder mass (**+2 points**)

$$W = T + F_B \quad \text{or} \quad Mg = T + \gamma V\!\!\!/$$

$$\therefore M = \frac{T + \gamma V\!\!\!/}{g} = \frac{12{,}760 + (9.8 \times 10^3)\frac{(\pi)(1)^2}{4}(2.5 - 1)}{9.81} = \frac{12{,}760 + 11{,}545}{9.81} = \mathbf{2,480\ kg}$$

2. [Momentum+Energy]



Continuity (**+1 point**)

$$V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1 = \left(\frac{12}{6}\right)^2 (5) = \mathbf{20\ ft/s}$$

Energy equation (**+3 points**)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + 0 = 0 + \frac{V_2^2}{2g} + 0 + h_L$$

$$\therefore p_1 = \gamma \cdot \left(\frac{V_2^2 - V_1^2}{2g} + h_L\right) = (62.4)\left(\frac{20^2 - 5^2}{2 \times 32.2} + 1\right) = 363.4 + 62.4 = \mathbf{425.8\ lb/ft^2}$$

(a) Momentum equation with loss (**+4 points**)

$$-F_x + p_1 A_1 = \dot{m}(-V_2) - \dot{m}V_1$$

where,

$$\dot{m} = \rho A_1 V_1 = \rho A_2 V_2 = 7.62\ \text{kg/m}^3$$

$$\therefore F_x = p_1 A_1 + \dot{m}(V_1 + V_2) = (425.8)\left(\frac{\pi \times 1^2}{4}\right) + (7.62)(20 + 5) = 334 + 191 = \mathbf{525\ lb}$$

Alternatively,

$$-F_x + p_1 A_1 = -\rho A_2 V_2^2 - \rho A_1 V_1^2$$

$$\therefore F_x = p_1 A_1 + \rho A_2 V_2^2 + \rho A_1 V_1^2 = \mathbf{525\ lb}$$

(b) Without loss (**+2 points**)

$$p_1 = \gamma \cdot \left(\frac{V_2^2 - V_1^2}{2g}\right) = \mathbf{363.4\ lb/ft^2}$$

$$\therefore F_x = (363.4)\left(\frac{\pi \times 1^2}{4}\right) + (7.62)(20 + 5) = 285 + 191 = \mathbf{476\ lb}$$

or

$$\therefore F_x = p_1 A_1 + \rho A_2 V_2^2 + \rho A_1 V_1^2 = \mathbf{476\ lb}$$

3. [Exact solution]

(a1) B.C. at $x = 0$ (**+3 points**)

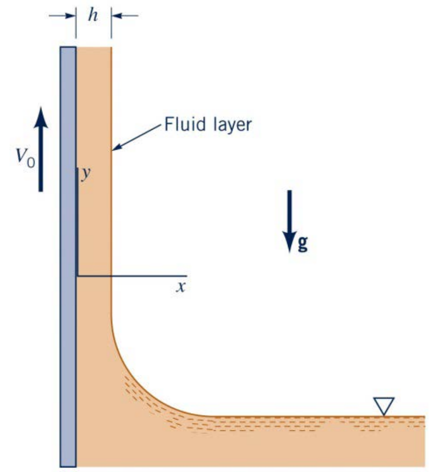
$$v(0) = 0 + 0 + c_2 = V_0$$

$$\therefore \boldsymbol{c_2 = V_0}$$

(a2) B.C. at $x = h$ (**+2 points**)

$$\left.\frac{dv}{dx}\right)_{x=h} = \frac{\gamma}{\mu} \cdot h + c_1 = 0$$

$$\therefore \boldsymbol{c_1 = -\frac{\gamma h}{\mu}}$$



(b1) Shear stress (**+3 points**)

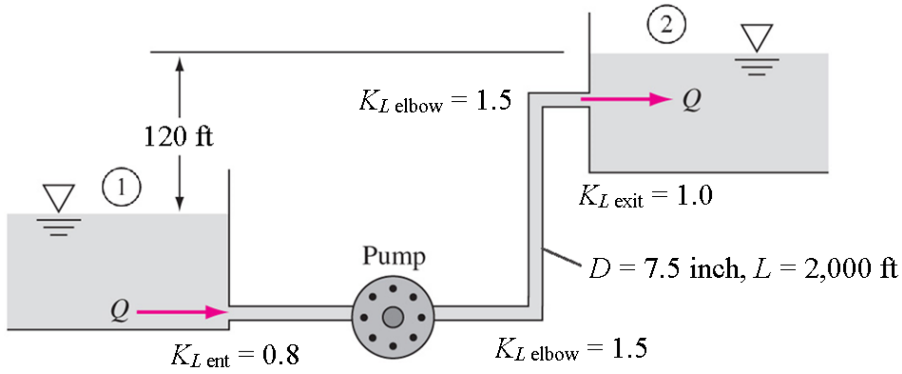
$$\tau_w = \mu \left.\frac{dv}{dx}\right)_{x=0} = \mu \left(\frac{\gamma}{\mu} x - \frac{\gamma h}{\mu}\right)_{x=0} = -\gamma h = (12.4 \times 10^3)(0.002) = \mathbf{-24.8\ N/m^2}$$

(b2) Flow rate (**+2 points**)

$$q = \int_0^h v dx = V_0 h - \frac{\gamma h^3}{3\mu} = (0.015)(0.002) - \frac{(12.4 \times 10^3)(0.002)^3}{3(1.5)} = \mathbf{7.96 \times 10^{-6}\ m^3/s}$$

$$\approx \boldsymbol{8\ cm^3/s}$$

4. [Pipe flow - Q]



Energy equation (**+6 points**)

$$0 + 0 + z_1 + h_p = 0 + 0 + z_2 + \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

with h_p = 250 ft, $z_2 = z_1 + 120$ ft, L = 2,000 ft, D = (7.5/12) = 0.625 ft, and

$$\sum K_L = 0.8 + 2 \times 1.5 + 1.0 = 4.8$$

Thus,

$$0 + 0 + z_1 + 250 = 0 + 0 + (z_1 + 120) + \left(f\frac{2000}{0.625} + 4.8\right)\frac{V^2}{2 \times 32.2}$$

or,

$$V^2 = \frac{(250 - 120)(2)(32.2)}{\frac{2{,}000}{0.625}f + 4.8} = \frac{8{,}372}{3{,}200f + 4.8} \quad (1)$$

Iteration process (**+3 points**)

Guess f = 0.02:

$$V = \left(\frac{8{,}372}{3{,}200\times0.02+4.8}\right)^{\frac{1}{2}} = 11 \text{ ft/s} \rightarrow Re = \frac{\rho VD}{\mu} = \frac{(1.94)(11)(7.5/12)}{2.09\times10^{-5}} = 6.4\times10^5$$

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{0.00085/0.602}{3.7}\right)^{1.1} + \frac{6.9}{6.4\times10^5}\right] \rightarrow f_{new} = 0.022$$

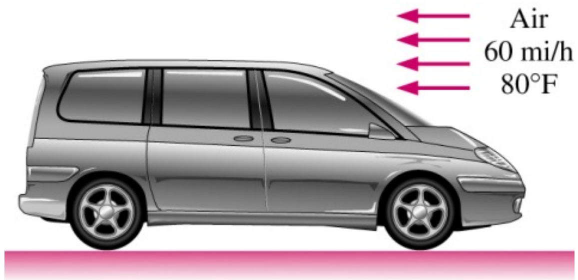
Guess f = 0.022:

$V = 10.55$ ft/s $\rightarrow Re = 6.12\times10^5 \rightarrow f_{new} = 0.022$ (Converged)

Flow rate (**+1 point**)

$$Q = VA = (10.55)\left(\frac{\pi \cdot (7.5/12)^2}{4}\right) = \mathbf{3.24\ ft^3/s}$$

5. [Turbulent boundary layer]



Reynolds number (**+2 points**)

$$Re_L = \frac{VL}{\nu} = \frac{(60 \times 1.4667)(11)}{1.697 \times 10^{-4}} = \frac{(88)(11)}{1.697 \times 10^{-4}} = \mathbf{5.704 \times 10^{6}}$$

Friction drag coefficient (**+3 points**)

$$C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} = \frac{0.074}{(5.704 \times 10^{6})^{\frac{1}{5}}} = \mathbf{0.00330}$$

Alternatively,

$$C_f = \frac{0.031}{Re_L^{\frac{1}{7}}} = 0.00336$$

or

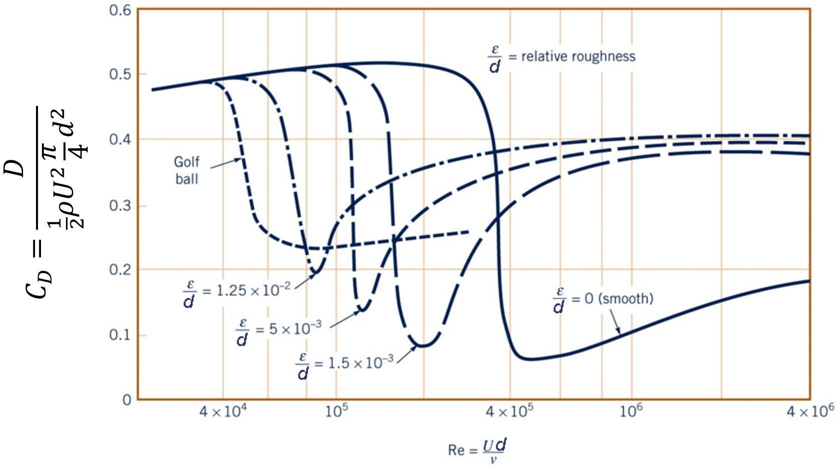
$$C_f = \frac{0.455}{(\log Re_L)^{2.58}} = 0.00329$$

Friction Drag (**+5 points**)

$$D_f = C_f \cdot \frac{1}{2}\rho V^2 A$$

$$\therefore D_f = (0.00330)\left(\frac{1}{2}\right)(2.28 \times 10^{-3})(88)^2\{(6 \times 11) + 2 \times (3.2 \times 11)\}$$
$$= 0.0291 \times (66 + 2 \times 35.2) = 1.9 + 2.1 = \mathbf{4\ lbf}$$

6. [Pi theorem + Drag]



(a) Pi theorem (**+4 points**)

$$D \doteq MLT^{-2}; d \doteq L; U \doteq LT^{-1}; \varepsilon \doteq L; \rho \doteq ML^{-3}; \nu \doteq L^2T^{-1}$$

or

$$D \doteq F; d \doteq L; U \doteq LT^{-1}; \varepsilon \doteq L; \rho \doteq FL^{-4}T^2; \nu \doteq L^2T^{-1}$$

Thus, $k - r = 6 - 3 = 3$. Select ρ, U, d as repeating variables:

$$\Pi_1 = D\rho^a U^b d^c \rightarrow (MLT^{-2})(ML^{-3})^a(LT^{-1})^b(L)^c \doteq M^0L^0T^0 \rightarrow a = -1, b = -2, c = -2$$
$$\Pi_2 = \varepsilon\rho^a U^b d^c \rightarrow (L)(ML^{-3})^a(LT^{-1})^b(L)^c \doteq M^0L^0T^0 \rightarrow a = 0, b = 0, c = -1$$
$$\Pi_3 = \nu\rho^a U^b d^c \rightarrow (L^2T^{-1})(ML^{-3})^a(LT^{-1})^b(L)^c \doteq M^0L^0T^0 \rightarrow a = 0, b = -1, c = -1$$

Alternatively,

$$\Pi_1 = D\rho^a U^b d^c \rightarrow (F)(FL^{-4}T^2)^a(LT^{-1})^b(L)^c \doteq F^0L^0T^0 \rightarrow a = -1, b = -2, c = -2$$
$$\Pi_2 = \varepsilon\rho^a U^b d^c \rightarrow (L)(FL^{-4}T^2)^a(LT^{-1})^b(L)^c \doteq F^0L^0T^0 \rightarrow a = 0, b = 0, c = -1$$
$$\Pi_3 = \nu\rho^a U^b d^c \rightarrow (L^2T^{-1})(FL^{-4}T^2)^a(LT^{-1})^b(L)^c \doteq F^0L^0T^0 \rightarrow a = 0, b = -1, c = -1$$

$$\therefore \Pi_1 = \frac{D}{\rho U^2 d^2}; \quad \Pi_2 = \frac{\varepsilon}{d}; \quad \Pi_1 = \frac{\nu}{Ud}$$

Thus,

$$\boldsymbol{\frac{D}{\rho U^2 d^2} = \phi\left(\frac{\varepsilon}{d}, \frac{Ud}{\nu}\right)}$$

(b) Drag force (+6 points)

$$\frac{\varepsilon}{d} = \frac{3.75 \times 10^{-2}}{3} = 1.25 \times 10^{-2}$$

$$Re = \frac{Ud}{\nu} = \frac{(51)(3/12)}{1.57 \times 10^{-4}} = 8.1 \times 10^4$$

Thus, $C_D \approx 0.2$ from the chart and

$$D = C_D \cdot \tfrac{1}{2}\rho U^2 \frac{\pi}{4} d^2 = (0.2)\left(\frac{1}{2}\right)(2.38 \times 10^{-3})(51)^2 \left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2 = \mathbf{0.03\ lb}$$