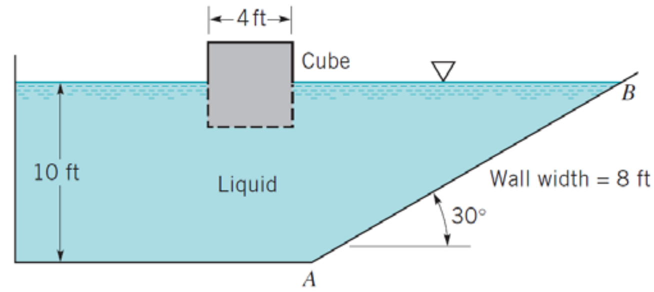
EXAM3 Solutions

Problem 1: Hydrostatic forces on plane surfaces (Chapter 2)

Information and assumptions

- Weight of a cube : 3,000 lb
- Depth of liquid : 10 ft
- Width of the wall : 8 ft



Find

- Specific gravity fo the liquid, γ
- The force acting on the inclined section AB, F_R
- The location of the pressure center from the free surface, y_R, along the inclined wall

Solution

(a) Buoyancy force

$$W = \gamma \forall$$

Or

$$\gamma = \frac{W}{\forall}$$

Since $\forall$ is just for the immersed part only,

$$\forall = (4\text{ ft}) \times (4\text{ ft}) \times (2\text{ ft}) = 32\text{ ft}^3$$

$$\therefore \gamma = \frac{3{,}000\text{ lb}}{32\text{ ft}^3} = \mathbf{93.75\ lb/ft^3}$$

(+3 points)

(b) Hydrostatic force

$$F_R = \bar{p} \cdot A = (\gamma h_c) \cdot A$$

$$h_c = \frac{10\text{ ft}}{2}$$

$$A = (8\text{ ft}) \times \left(\frac{10\text{ ft}}{\sin 30°}\right) = 160\text{ ft}^2$$

$$\therefore F_R = \left(93.75 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{10\text{ ft}}{2}\right)(160\text{ ft}^2) = \mathbf{75{,}000\ lb}$$

(+4 points)

EXAM3 Solutions

(c) Pressure center

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$I_{xc} = \frac{(8\text{ ft})\left(\frac{10\text{ ft}}{\sin 30°}\right)^3}{12} = 5333.33\text{ ft}^4$$

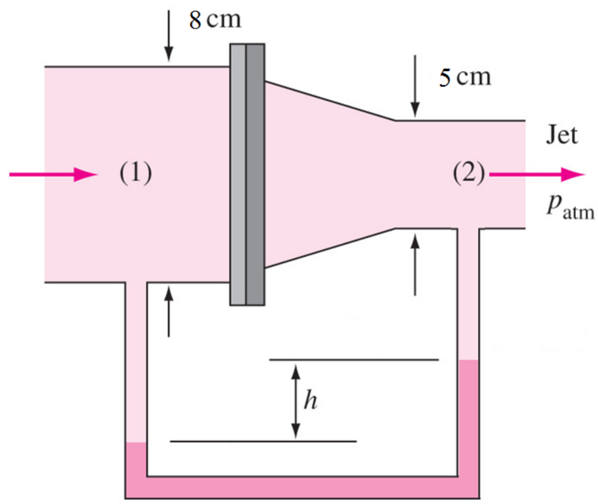
$$y_c = \frac{\left(\frac{10}{\sin 30°}\right)}{2} = 10\text{ ft}$$

$$y_R = \frac{5333.33\text{ ft}^4}{(10\text{ ft})(160\text{ ft}^2)} + (10\text{ ft}) = \mathbf{13.3\ ft}$$

(+3 points)

EXAM3 Solutions

Problem 2. Linear momentum (Chap. 5)



Information and assumptions

- Water, $\gamma = 9{,}790\ \mathrm{N/m^3}$
- SG = 13.56 for the manometer fluid
- Manometer reading, $h = 58\ \mathrm{cm}$
- Friction is neglected.

Find

- The upstream pressure at section 1
- The average velocity at sections 1 and 2
- The axial flange force, F_x, required to keep the nozzle attached to pipe 1

Solution

(a) Manometer equation

$$p_1 - p_2 = (\gamma_m - \gamma)h = (SG - 1)\gamma h$$

where $p_2 = p_{atm} = 0\ (gage)$.

$$\therefore p_1 = (13.56 - 1)(9{,}790\ \mathrm{N/m^3})(0.58\ \mathrm{m}) = \mathbf{71{,}300\ Pa}\ (\boldsymbol{gage})$$

(+2 points)

(b) Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where, p_1 = 71,300 Pa (gage), p_2 = 0 (gage), $V_2 = V_1\, A_1/A_2 = V_1\,(D_1/D_2)^2$, and $z_1 = z_2$. Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{\left(V_1\left(\frac{D_1}{D_2}\right)^2\right)^2}{2g}$$

Solve for V_1,

$$V_1 = \sqrt{\frac{2p_1}{\rho\left[\left(\frac{D_1}{D_2}\right)^4 - 1\right]}}$$

EXAM3 Solutions

$$\therefore V_1 = \sqrt{\frac{(2)(71{,}300\ \mathrm{Pa})}{(999\ \mathrm{kg/m^3})\left[\left(\frac{0.08\ \mathrm{m}}{0.05\ \mathrm{m}}\right)^4 - 1\right]}} = \mathbf{5.1\ m/s}$$

From the Continuity equation,

$$V_2 = V_1 \left(\frac{D_1}{D_2}\right)^2 = (5.1\ \mathrm{m/s})\left(\frac{0.08\ \mathrm{m}}{0.05\ \mathrm{m}}\right)^2 = \mathbf{13.1\ m/s}$$

(+4 points)

(c) Momentum equation

$$p_1 A_1 - p_2 A_2 + F_x = \dot{m}(V_2 - V_1)$$

where $p_2 = 0$ (gage) and $\dot{m} = \rho V_1 A_1 = \rho V_2 A_2$. Thus

$$F_x = \rho V_1 A_1 (V_2 - V_1) - p_1 A_1$$

or

$$F_x = \left(999\frac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(5.1\frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{\pi}{4}\right)(0.08\ \mathrm{m})^2\left(13.1\frac{\mathrm{m}}{\mathrm{s}} - 5.1\frac{\mathrm{m}}{\mathrm{s}}\right) - (71{,}300\ \mathrm{Pa})\left(\frac{\pi}{4}\right)(0.08\ \mathrm{m})^2 \approx \mathbf{-154\ N}$$

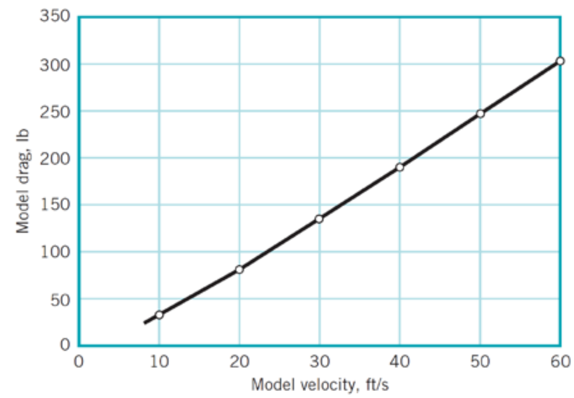
(+4 points)

EXAM3 Solutions

Problem 3. Similarity and model testing (Chap. 7)

Information and assumptions

- Wind velocity, $V = 44$ ft/s
- Prototype length, L = 30 ft
- Prototype diameter, d = 1.25 ft
- Air density, $\rho = 2.38 \times 10^{-3}\,\text{slugs/ft}^3$
- Air viscosity, $\mu = 3.74 \times 10^{-7}\ \text{lb} \cdot \text{s/ft}^2$
- Model length, L_m = 2 ft
- Model diameter, d_m = 1 in.
- Water density, $\rho_m = 1.94\ \text{slugs/ft}^3$
- Water viscosity, $\mu_m = 2.34 \times 10^{-5}\text{lb} \cdot \text{s/ft}^2$



Find

- Re similarity requirement
- Prediction of drag

Solution

Similarity requirements:

$$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho V d}{\mu} \qquad (Re\ similarity)$$

$$\frac{D}{\rho V^2 d^2} = \frac{D_m}{\rho_m V_m^2 d_m^2} \qquad (Prediction\ equation)$$

(+6 points)

From the *Re* similarity requirement

$$V_m = \left(\frac{\rho}{\rho_m}\right)\left(\frac{d}{d_m}\right)\left(\frac{\mu_m}{\mu}\right)V$$

Thus, the model velocity corresponding to the full-size velocity, V = 44 ft/s, is

$$V_m = \left(\frac{2.38 \times 10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}}{1.94\ \frac{\text{slugs}}{\text{ft}^3}}\right)\left(\frac{1.25\ \text{ft}}{1/12\ \text{ft}}\right)\left(\frac{2.34 \times 10^{-5}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}{3.74 \times 10^{-7}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}\right)\left(44\,\frac{\text{ft}}{\text{s}}\right) = \mathbf{50.7\ \frac{ft}{s}}$$

(+2 points)

EXAM3 Solutions

From the prediction equation

$$D = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{d}{d_m}\right)^2 D_m$$

For D_m = 250 lb at V_m = 50.7 ft/s,

$$D = \left(\frac{2.38 \times 10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}}{1.94\ \frac{\text{slugs}}{\text{ft}^3}}\right)\left(\frac{44\,\frac{\text{ft}}{\text{s}}}{50.7\,\frac{\text{ft}}{\text{s}}}\right)^2\left(\frac{1.5\text{ ft}}{1/12\ \text{ft}}\right)^2 (250\text{ lb}) = \mathbf{52\ lb}$$

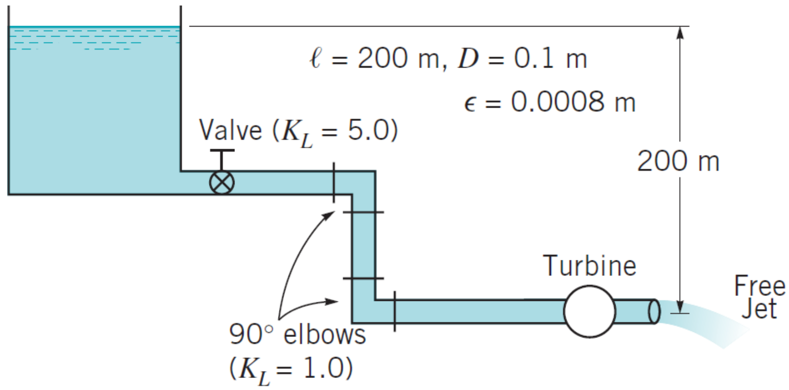
(+2 points)

EXAM3 Solutions

Problem 4. Major and minor loss (Chapt. 8)

Information and assumptions

- Water density, $\rho = 999\ \mathrm{kg/m^3}$
- Viscosity, $\mu = 1.12 \times 10^{-3}\mathrm{N.s/m^2}$
- $h_t = 116\ \mathrm{m}$
- Entrance effects are negligible



Find

- Determine the flow rate, Q

Solution

Energy equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

where, $p_1 = p_2 = 0(gage)$, $V_1 \approx 0$, $V_2 = V$, $z_1 - z_2$ = 200 m, h_t = 116 m, $h_L = h_f + h_m$, and assume a turbulent flow thus $\alpha_2 = 1$

(+3 points)

1) Major loss:

$$h_L = f \frac{\ell}{D} \frac{V^2}{2g}$$

where, ℓ = 200 m and D = 0.1 m.

(+2 points)

2) Minor loss:

$$h_m = \sum K_L \frac{V^2}{2g}$$

where, $\sum K_L$ = 5 + 1 + 1 = 7.

(+2 points)

Thus, the energy equation becomes

$$0 + 0 + (200\ \mathrm{m}) = 0 + \frac{V^2}{2 \times 9.81\ \mathrm{m/s^2}} + (116\ \mathrm{m}) + \left(f \times \frac{200\ \mathrm{m}}{0.1\ \mathrm{m}} + 7\right)\frac{V^2}{2 \times 9.81\ \mathrm{m/s^2}}$$

EXAM3 Solutions

or

$$V = \left(\frac{1648}{2000f + 8}\right)^{\frac{1}{2}}$$

Assume $f = 0.02 \rightarrow V = 5.86 \rightarrow Re = 5.23 \times 10^5 \rightarrow f_{new} = 0.036$ ($\neq$ 0.02)

Assume $f = 0.036 \rightarrow V = 4.54 \rightarrow Re = 4.05 \times 10^5 \rightarrow f_{new} = 0.0360$ (= 0.036; converged)

Thus, f = 0.036 and V = 4.54 m/s.

(+2 points)

$$\therefore Q = VA = \left(4.54\,\frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{\pi}{4}\right)(0.1\ \mathrm{m})^2 = \mathbf{0.0357\ m^3/s}$$

(+1 point)

Note: $Re = \frac{VD}{\nu} = \frac{\left(4.54\frac{\mathrm{m}}{\mathrm{s}}\right)(0.1\ \mathrm{m})}{1.12\times10^{-6}} = 4.05 \times 10^5$; a turbulent flow thus $\alpha_2 = 1$.

EXAM3 Solutions

Problem 5. Boundary layer (Chapt. 9)

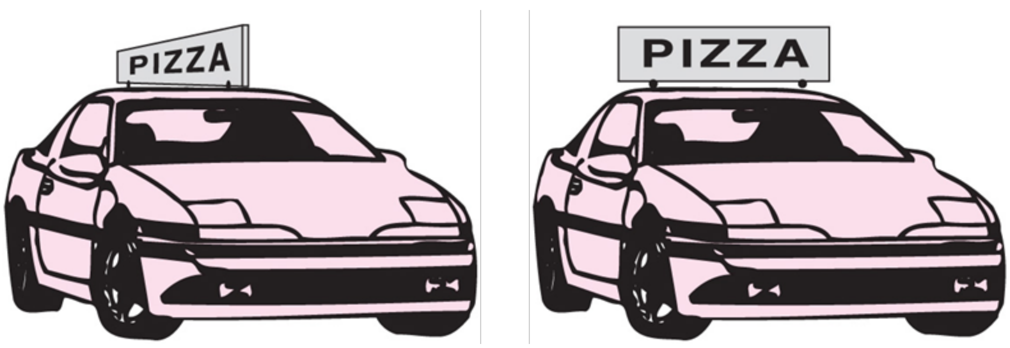
Information and assumptions

- Plate dimension: Height $H = 1.5\text{ ft}$ and length $L = 5\text{ ft}$
- Incoming flow velocity, $V = 40\text{ mph }(58.7\text{ ft/s})$
- Flow is assumed initially laminar for the parallel orientation case
- For the blunt case, drag coefficient $C_D = 1.2$

Find

- Calculate the drag D (in lbf) for both plate orientations

Solution



(a) Parallel orientation:

$$Re_L = \frac{\rho V L}{\mu} = \frac{(2.38 \times 10^{-3}\text{ slugs/ft}^3)(58.7\text{ ft/s})(5\text{ ft})}{3.74 \times 10^{-7}\text{ lb} \cdot \text{s/ft}^2} = 1.87 \times 10^6$$

Thus, the flow is transitional.

(+2 points)

$$C_f = \frac{0.031}{Re_L^{\frac{1}{7}}} - \frac{1440}{RE_L} = \frac{0.031}{(1.87 \times 10^6)^{\frac{1}{7}}} - \frac{1440}{1.87 \times 10^6} = 0.0032$$

(+3 points)

Thus,

EXAM3 Solutions

$$D = C_f \times \frac{1}{2}\rho V^2 A \times (2\,sides)$$

or

$$D = (0.0032)\left(\frac{1}{2}\right)\left(2.38 \times 10^{-3}\frac{\text{slugs}}{\text{ft}^3}\right)\left(58.7\frac{\text{ft}}{\text{s}}\right)^2 (1.5\text{ ft} \times 5\text{ ft} \times 2) = \mathbf{0.2\ lbf}$$

(+3 points)

(b) Normal orientation:

$$D = C_D \times \frac{1}{2}\rho V^2 A$$

or

$$D = (1.2)\left(\frac{1}{2}\right)\left(2.38 \times 10^{-3}\frac{\text{slugs}}{\text{ft}^3}\right)\left(58.7\frac{\text{ft}}{\text{s}}\right)^2 (1.5\text{ ft} \times 5\text{ ft}) = \mathbf{36.9\ lbf}$$

(+2 points)

Equivalent solution 1):

$$C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{1700}{Re_L} = 0.0032$$

$$D = 0.2\text{ lbf}$$

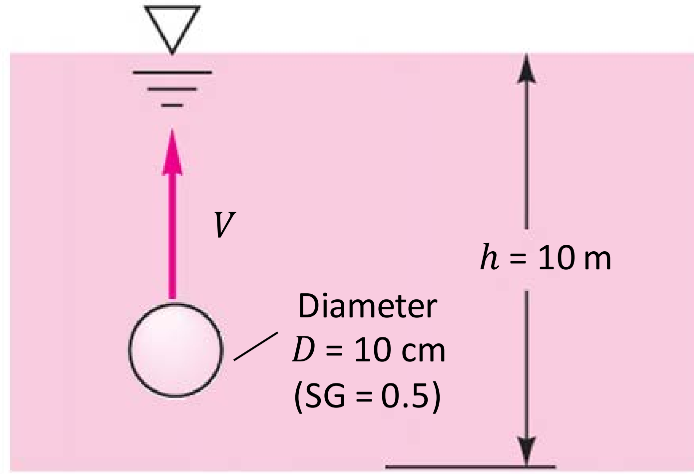
Equivalent solution 2):

$$C_f = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1700}{Re_L} = 0.0031$$

$$D = 0.2\text{ lbf}$$

EXAM3 Solutions

Problem 6. Bluff body drag (Chapt. 9)



Information and assumptions

- SG of buoyant ball, SG=0.5
- Diameter of the ball D=10 cm
- Water depth h=10 m
- Drag coefficient $C_D = 0.5$
- The ball rises up with terminal velocity.
- Density of water, $\rho = 999\ \mathrm{kg/m^3}$
- Viscosity of water, $\mu = 1.12 \times 10^{-3}\mathrm{N.s/m^2}$
- Volume of sphere, $\forall = \pi D^3/6$

Find

- Time the ball takes to reach to the water surface.

Solution

(a) Rising (terminal) velocity

From a vertical force balance about the ball, Buoyancy = Weight + Drag;

$$\gamma\forall = \gamma_{ball} \cdot \forall + C_D \times \frac{1}{2}\rho V^2 A$$

(+5 points)

or

$$\gamma \cdot \frac{\pi D^3}{6} = (SG \cdot \gamma) \cdot \frac{\pi D^3}{6} + C_D \times \frac{1}{2}\rho V^2 \cdot \frac{\pi}{4} D^2$$

Solving for V,

$$V = \sqrt{\frac{4(1 - SG)gD}{3C_D}}$$

or

$$V = \sqrt{\frac{(4)(1 - 0.5)(9.81\ \mathrm{m/s^2})(0.1\mathrm{m})}{(3)(0.5)}} = \mathbf{1.14\ m/s}$$

(+4 points)

(b) Rising time

$$t = \frac{h}{V} = \frac{10\ \mathrm{m}}{1.14\ \mathrm{m/s}} \approx \mathbf{9\ s}$$

(+1 point)