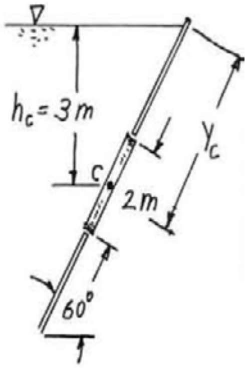
EXAM3 Solutions

Problem 1: Hydrostatic Forces on Plane Surface (Chapter 2)

Information and assumptions

- Circular gate with diameter D = 2 m; R = D/2 = 1 m
- Gate orientation = 60°
- Vertical centroid location h_c = 3 m from the water surface
- γ = 9.8 kN/m^3 for water
- $I_{xc} = \pi R^4/4$ for a circle of radius R



Find

- Water force acting on the gate F_R and the center of pressure y_R.

Solution

(a) Water force:

$$F_R = p_c A = \gamma h_c A \qquad \text{(+3 points)}$$

Thus,

$$F_R = \left(9.8\ \frac{\text{kN}}{\text{m}^3}\right)(3\ \text{m})\left(\frac{\pi}{4}(2\ \text{m})^2\right) = 92.4\ \text{kN} \qquad \text{(+2 points)}$$

(b) Center of pressure:

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

where,

$$I_{xc} = \frac{\pi R^4}{4} = \frac{\pi (1\ \text{m})^4}{4} = \frac{\pi}{4}\ \text{m}^4$$

$$y_c = \frac{h_c}{\cos 30^\circ} = \frac{3\ \text{m}}{\cos 30^\circ} = 3.464\ \text{m} \qquad \text{(+4 points)}$$

Hence,

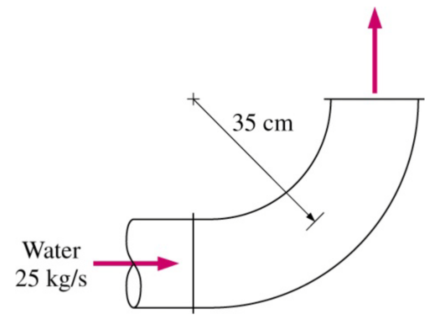
$$y_R = \frac{\frac{\pi}{4}\ \text{m}^4}{(3.464\ \text{m})\left(\frac{\pi}{4}(2\ \text{m})^2\right)} + 3.464\ \text{m} = 3.536\ \text{m} \qquad \text{(+1 point)}$$

EXAM3 Solutions

Problem 2: Linear Momentum (Chapter 5)

Information and assumptions

- Mass flow rate $\dot{m}$ = 25 kg/s
- Elbow diameter D = 0.1 m
- Pressure at the exit $p_2 = p_a = 0$ (gage)
- Elevation change between the in- and out-let Δz = 0.35 m
- ρ = 1000 kg/m^3 for water
- The weight of the elbow and the water in it is negligible
- Steady, frictionless, and irrotational



Find

- Velocities at the inlet and outlet, and the inlet pressure, and the anchoring force

Solution

(a) Continuity equation:

$$V_1 = V_2 = \frac{\dot{m}}{\rho A} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)\left(\frac{\pi}{4}(0.1\ \text{m})^2\right)} = 3.18\ \text{m/s}$$ (+2 points)

(b) Bernoulli equation:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or

$$p_1 = \rho g(z_2 - z_1) = \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.35\ \text{m}) = 3434\ \text{Pa}$$ (+3 points)

(c) Momentum equation along the x (positive to the right) and y (positive upward) axes:

<u>x-direction</u>

$$F_{Rx} + p_1 A_1 = -\dot{m}V_1$$

or

$$F_{Rx} = -\dot{m}V_1 - p_1 A_1$$

$$= -\left(25\ \frac{\text{kg}}{\text{s}}\right)\left(3.18\ \frac{\text{m}}{\text{s}}\right) - \left(3434\ \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4}(0.1\ \text{m})^2\right) = -106.5\ \text{N}$$ (+3 points)

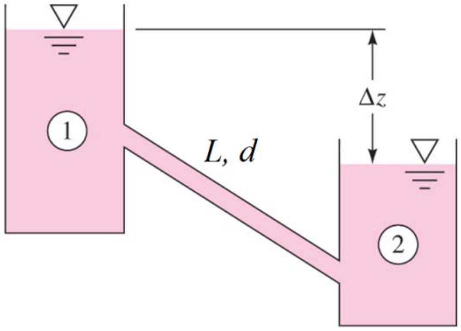
<u>y-direction</u>

$$F_{Ry} = \dot{m}V_2 = \left(25\ \frac{\text{kg}}{\text{s}}\right)\left(3.18\ \frac{\text{m}}{\text{s}}\right) = 79.5\ \text{N}$$ (+2 points)

Note: $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = 133\ \text{N}; \quad \theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = -36.7°$

EXAM3 Solutions

Problem 3: Friction loss (Chapter 8)



Information and assumptions

- Smooth pipe with length L = 4500 m
- Diameter, d = 4 cm = 0.04 m
- Δz = 100 m
- ρ = 998 kg/m^3; μ = 0.001 N·s/m^2
- Initial guess f = 0.0224 for iterative process
- $f = 0.316/Re^{1/4}$

Find

- Flow rate Q

Solution

The energy equation from surface 1 to surface 2 gives

$$\frac{p_1}{\gamma} + \frac{V_1}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2}{2g} + z_2 + f\frac{L}{D}\frac{V^2}{2g}$$

Where, $p_1 = p_2 = p_a$ and $V_1 = V_2 \approx 0$. Then,

$$f\frac{(4500\text{ m})}{(0.04\text{ m})}\frac{V^2}{2(9.81\ \text{m/s}^2)} = 100\text{ m}$$

or

$$fV^2 = 0.01744 \qquad \text{(+5 points)}$$

Iterate with an initial guess of f = 0.0224, calculating V and Re and improving the guess:

$$V = \left(\frac{0.01744}{0.0224}\right)^{\frac{1}{2}} = 0.882\ \frac{\text{m}}{\text{s}};\ Re = \frac{(998)(0.882)(0.04)}{0.001} = 35200;\ f_{new} = \frac{0.316}{(35200)^{0.25}} = 0.0231$$

Let f = 0.0231:

$$V_{new} = \left(\frac{0.01744}{0.0231}\right)^{\frac{1}{2}} = 0.869\ \frac{\text{m}}{\text{s}};\ Re = \frac{(998)(0.869)(0.04)}{0.001} = 34690;\ f_{new} = \frac{0.316}{(34690)^{0.25}} = 0.0232$$

Let f = 0.232:

$$V_{new} = \left(\frac{0.01744}{0.0232}\right)^{\frac{1}{2}} = 0.867\ \frac{\text{m}}{\text{s}};\ Re = \frac{(998)(0.867)(0.04)}{0.001} = 34610;\ f_{new} = \frac{0.316}{(34610)^{0.25}} = 0.0232$$

$$\therefore f_{new} = f = 0.232 \quad \text{(O.K.)} \qquad \text{(+3 points)}$$

Thus, with V = 0.867 m/s the flow rate is

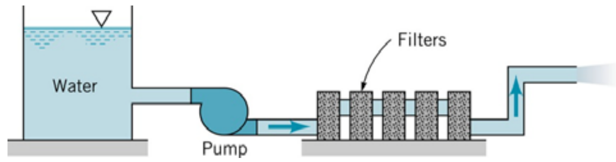
$$Q = AV = \left(\frac{\pi}{4}\right)(0.04\text{ m})^2\left(0.867\ \frac{\text{m}}{\text{s}}\right) = 0.00109\ \text{m}^3/\text{s} = 3.92\ \text{m}^3/\text{h} \qquad \text{(+2 points)}$$

EXAM3 Solutions

Problem 4: Minor loss (Chapter 8)

Information and assumptions

- Q = 0.3 ft^3/s
- L = 80 ft and D = 2 in.
- ε = 0.002 in.
- K_L = 6.0 for each of the five filters
- Pressures at the two points are same
- ρ = 1.94 slug/ft^3; μ = 2.34 × 10^{-5} lb/m·s



Find

- Pump power $\dot{W}$

Solution

From the energy equation between the two locations

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1}{2g}} + \cancel{z_1} + h_p = \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2}{2g}} + \cancel{z_2} + \cancel{h_t} + \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

where $p_1 = p_2$, $V_1 = V_2$, $z_1 \approx z_2$, and $h_t = 0$ as no turbine. Then,

$$h_p = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$ (+4 points)

From continuity

$$V = \frac{Q}{A} = \frac{0.3 \text{ ft}^3/\text{s}}{(\pi/4)(2/12 \text{ ft})^2} = 13.75 \text{ ft/s}$$

Minor losses:

$$\sum K_L = 5(6.0) = 30.0$$ (+3 points)

With $Re = \rho VD/\mu = (1.94)(13.75)(2/12)/2.34 \times 10^{-5} = 1.9 \times 10^5$ and $\varepsilon/D = (0.002 \text{ ft})/(2 \text{ ft}) = 1 \times 10^{-3}$,

$$f = \frac{1.325}{\left\{\ln\left[\left(\frac{\varepsilon/D}{3.7}\right) + \left(\frac{5.74}{Re^{0.9}}\right)\right]\right\}^2} = 0.0213$$

Then,

$$h_p = \left(0.0213\frac{80}{2/12} + 30\right)\frac{(13.75)^2}{2(32.2)} = 118.1 \text{ ft}$$

Thus,

$$\dot{W} = \gamma Q h_p = (62.4)(0.3)(118.1) = 2211\ \frac{\text{ft}\cdot\text{lb}}{\text{s}} = 4.02 \text{ hp}$$ (+3 points)

EXAM3 Solutions

Problem 5: Boundary layer (Chapter 9)

Information and assumptions

- V = 65 mph = 95.34 ft/s
- L = 20 ft; W = 9 ft; H = 8 ft
- ρ = 0.07350 lbm/ft^3; ν = 1.697 × 10^{-4} ft^2/s
- Turbulent smooth-wall flow from the leading edge

Find

- Boundary layer thickness and drag force on the refrigerated compartment

Solution

Reynolds number

$$Re_L = \frac{VL}{\nu} = \frac{(95.34 \text{ ft/s})(20 \text{ ft})}{1.697\times10^{-4} \text{ ft}^2/\text{s}} = 1.124\times10^7$$ (+2 points)

(a) Boundary layer thickness δ

$$\left.\frac{\delta}{x}\right|_{x=L} = \frac{0.16}{Re_L^{1/7}} = \frac{0.16}{(1.124\times10^7)^{1/7}} = 0.0157$$

Thus,

$$\delta = (0.0157)(20 \text{ ft}) = 0.314 \text{ ft} = 3.8 \text{ in}$$ (+2 points)

(b) Drag force

Friction drag coefficient:

$$C_f = \frac{0.031}{Re_L^{\frac{1}{7}}} = \frac{0.031}{(1.124\times10^7)^{\frac{1}{7}}} = 0.00305$$ (+4 points)

The area of the top and side surfaces of the truck is

$$A = A_{top} + 2A_{side} = (9 \text{ ft}\times 20 \text{ ft}) + 2(8 \text{ ft}\times 20 \text{ ft}) = 500 \text{ ft}^2$$

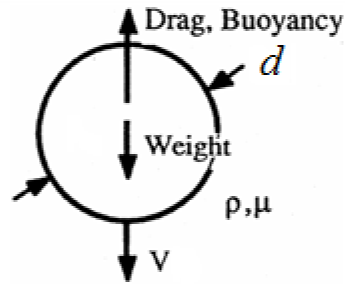
Drag force:

$$D = C_f\cdot\left(\frac{1}{2}\rho V^2 A\right)$$

$$= (0.00305)\left(\frac{1}{2}\right)(0.07350)(95.34)^2(500)\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = 15.82 \text{ lbf}$$ (+2 points)

EXAM3 Solutions

Problem 6: Blunt body drag (Chapter 9)



Information and assumptions

- $W = 0.250$ kg; $d = 0.0735$ m
- Lake depth $h = 100$ m
- $C_D = 0.5$
- $\rho = 1000\ \text{kg/m}^3$; $\nu = 1.12 \times 10^{-6}\ \text{m}^2\text{/s}$

Find

- The sphere falling speed and time

Solution

(a) Falling velocity

From a vertical force balance for the sphere, Weight = Buoyancy + Drag;

$$mg = \gamma \forall + C_D \cdot \frac{1}{2}\rho V^2 A$$

or

$$mg = \rho g \cdot \frac{4\pi}{3}\left(\frac{d}{2}\right)^3 + C_D \cdot \frac{1}{2}\rho V^2\left(\frac{\pi}{4}d^2\right)$$ (+7 points)

Solving for V,

$$V = \sqrt{\frac{8g}{\pi\rho C_D d^2}\left(m - \rho\frac{\pi}{6}d^3\right)}$$

$$= \sqrt{\frac{(8)(9.81)}{\pi(1000)(0.5)(0.0735)^2}\left(0.25 - \frac{\pi(1000)(0.0735)^3}{6}\right)}$$

(+2 points)

$$= 0.624\ \text{m/s}$$

(b) Falling time

$$t = \frac{h}{V} = \frac{100\ \text{m}}{0.624\ \text{m/s}} = 160\ \text{s} = 2\ \text{m}\ 40\ \text{s}$$ (+1 point)