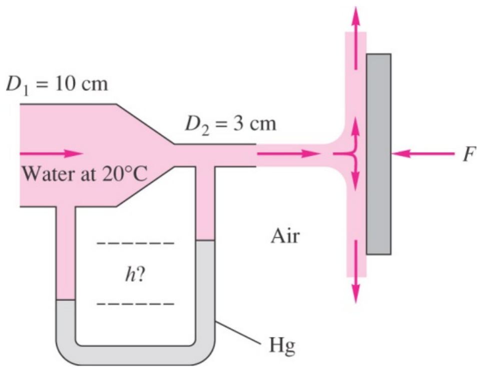
EXAM3 Solutions

Problem 1: Linear momentum equation (Chapter 5)

Information and assumptions

- ρ = 998 kg/m^3 for water at 20°C
- D_1 = 0.1 m
- D_2 = 0.03 m
- F = 70 N
- Steady, frictionless, one-dimensional flow
- ρ = 13,600 kg/m^3 for mercury



Find

- Water velocity at sections (1) and (2) and mercury manometer reading h.

Solution

(a) Momentum of the jet striking the plate,

$$-\rho A_2 V_2^2 = -F$$

$$-(998)\left(\frac{\pi}{4}\right)(0.03)^2 V_2^2 = -70 \text{ N}$$

$$\therefore V_2 = 9.96 \text{ m/s}$$

From continuity equation,

$$V_1 = \frac{A_2}{A_1} V_2 = \frac{\left(\frac{\pi}{4}\right)(0.03)^2}{\left(\frac{\pi}{4}\right)(0.1)^2}(9.96) = 0.9 \text{ m/s}$$ (+4 points)

(b) From Bernoulli equation,

$$p_2 - p_1 = \frac{1}{2}(998)(9.96^2 - 0.9^2) = 49{,}100 \text{ Pa}$$ (+3 points)

By using a manometry equation,

$$p_2 - p_1 = h(\gamma_{mercury} - \gamma_{water})$$

$$h = \frac{p_2 - p_1}{(\rho_{mercury} - \rho_{water})g}$$

or,

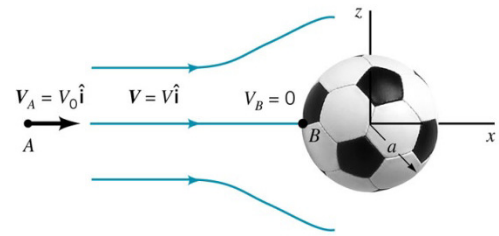
$$h = \frac{49100}{(13{,}600 - 998) \times 9.81} = 0.4 \text{ m}$$ (+3 point)

EXAM3 Solutions

Problem 2: Acceleration (Chapter 4)

Information and assumptions

- Inviscid, incompressible, steady flow
- a = 0.1 m
- $V = V_0(1 + a^3/x^3)$
- V_0 = 10 m/s
- Euler equation: $\rho a_x = -dp/dx$
- ρ = 1.23 kg/m^3



Find

- Acceleration a_x and pressure gradient dp/dx at x = -2a = -0.2 m

Solution

(a) Acceleration

$$a_x = V\frac{dV}{dx} \qquad \text{(+4 points)}$$

where, $V = V_0(1 + \frac{a^3}{x^3})$. Then,

$$a_x = V_0\left(1 + \frac{a^3}{x^3}\right)V_0\left(-3\frac{a^3}{x^4}\right) = -3V_0^2\left(1 + \frac{a^3}{x^3}\right)\frac{a^3}{x^4}$$

At x = -2a,

$$a_x = -(3)(10)^2\left(1 + \frac{(0.1)^3}{(-0.2)^3}\right)\frac{(0.1)^3}{(-0.2)^4} = -164.1\ \frac{\mathrm{m}}{\mathrm{s}^2} \qquad \text{(+4 points)}$$

(b) From Euler equation,

$$\frac{dp}{dx} = -\rho a_x = -(1.23)(-164.1) = 201.8\ \mathrm{Pa/m} \qquad \text{(+2 points)}$$

EXAM3 Solutions

Problem 3: Friction loss (Chapter 8)

Information and assumptions

- Roughness, ε = 0.12 mm
- Diameter, D = 40 mm
- Pressure drop Δp = 1.3 kPa per ℓ = 12 m
- Initial guess f = 0.0315 for iterative process
- ρ = 999 kg/m^3; ν = 1.12 × 10^{-6} m^2/s

Find

- Flow rate Q at t = 10 years

Solution

Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}$$

Where, $V_1 = V_2$ and $z_1 = z_2$. Then,

$$p_1 - p_2 = f\frac{\ell}{D}\frac{V^2}{2g}\gamma = \Delta p$$

or

$$V = \sqrt{2\frac{\Delta p}{\rho}\frac{D}{\ell}\frac{1}{f}} = \frac{0.09314}{\sqrt{f}} \qquad \text{(+5 points)}$$

Assume f = 0.0315,

$$V = \frac{0.09314}{\sqrt{0.0315}} = 0.5248\, m/s$$

$$Re = \frac{VD}{\nu} = \frac{(0.5248)(0.04)}{1.12 \times 10^{-6}} = 18743$$

Check f,

$$\frac{1}{\sqrt{f_{new}}} = -1.8\log\left[\left(\frac{0.003}{3.7}\right)^{1.11} + \frac{6.9}{18743}\right] = 5.6367$$

$$\therefore f_{new} = 0.0315 = f\ (\mathrm{O.K.}) \qquad \text{(+3 points)}$$

Flow rate

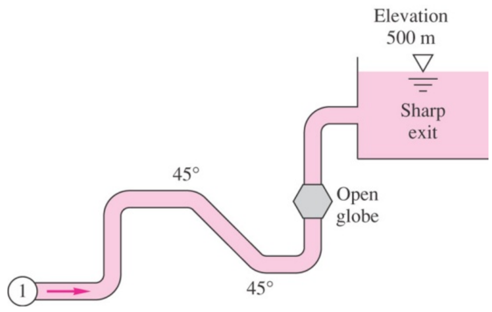
$$Q = AV = \left(\frac{\pi}{4}\right)(0.04)^2(0.5248) = 6.6 \times 10^{-4}\, m^3/s \qquad \text{(+2 points)}$$

EXAM3 Solutions

Problem 4: Minor loss (Chapter 8)

Information and assumptions

- ℓ = 1200 m
- D = 0.05 m
- z_1 = 400 m and z_2 = 500 m
- Q = 0.005 m3/s
- ρ = 998 kg/m3; μ = 0.001 kg/m·s
- f = 0.0315
- K_L =
  - 0.2 for 45° elbow
  - 0.3 for 90° elbow
  - 8.5 for open globe valve
  - 1.0 for sharp exit



Find

- Gage pressure at point 1

Solution

Energy equation

$$\frac{p_1}{\gamma}+\frac{V_1}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2}{2g}+z_2+\left(f\frac{\ell}{D}+\sum K_L\right)\frac{V^2}{2g}$$

Where $p_2=0$ and $V_2=0$. Then,

$$\frac{p_1}{\gamma}=z_2-z_1+\left(f\frac{\ell}{D}+\sum K_L\right)\frac{V^2}{2g}-\frac{V_1}{2g}$$ (+5 points)

From continuity

$$V_1=V=\frac{Q}{A}=\frac{0.005}{(\pi/4)(0.05)^2}=2.55\,\frac{\mathrm{m}}{\mathrm{s}}$$

Minor losses:

$$\sum K_L=2(0.2)+4(0.3)+8.5+1=11.1$$ (+3 points)

Then,

$$\frac{p_1}{\gamma}=500-400+\left[(0.0315)\left(\frac{1200}{0.05}\right)+11.1-1\right]\frac{(2.55)^2}{2(9.81)}=353\ \mathrm{m}$$

or

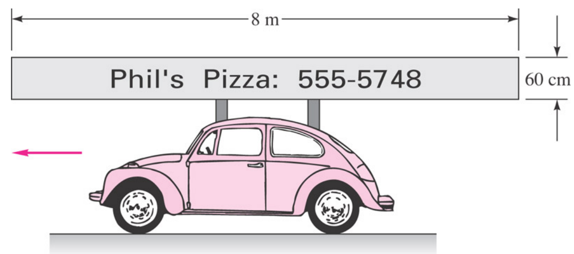
$$p_1=(998)(9.81)(353)=3.46\ \mathrm{MPa}$$ (+2 points)

EXAM3 Solutions

Problem 5: Boundary layer (Chapter 9)

Information and assumptions

- V = 29.06 m/s
- L = 8 m
- b = 0.6 m
- ρ = 1.2 kg/m3
- $\mu = 1.8 \times 10^{-5}$ kg/m·s
- Turbulent smooth-wall flow from the leading edge



Find

- Force on the sign

Solution

Reynolds number

$$Re_L = \frac{\rho V L}{\mu} = \frac{1.2(29.06)(8)}{1.8 \times 10^{-5}} = 1.55 \times 10^7 \qquad \text{(+3 points)}$$

Friction drag coefficient[1]

$$C_f = \frac{0.031}{Re_L^{\frac{1}{7}}} = \frac{0.031}{(1.55 \times 10^7)^{\frac{1}{7}}} = 0.00291 \qquad \text{(+3 points)}$$

Drag force

$$F_D = C_f \left(\frac{1}{2}\right) \rho V^2 (bL \times 2)$$
$$= 0.00291 \left(\frac{1}{2}\right)(1.2)(29.06)^2(0.6)(8)(2) = 14\ \text{N} \qquad \text{(+4 points)}$$

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[1] Alternatively,

$C_f = \frac{0.455}{(\log Re_L)^{2.58}} = 0.0028$ for turbulent smooth plate ($F_D = 13.6$ N)

$C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} = 0.0027$for "tripped" turbulent boundary layer, $Re_L \leq 10^7$ ($F_D = 13.1$ N)

EXAM3 Solutions

Problem 6: Blunt body drag (Chapter 9)

Information and assumptions

- D = 1.69 in.
- U = 200 ft/s
- ρ = 0.00238 slugs/ft^3
- ν = 1.57 $\times$ 10^{-4} ft^2/s

Find

- Drag on a standard and smooth golf balls

Solution

Reynolds number

$$Re = \frac{UD}{\nu} = \frac{(200)(1.69/12)}{1.57 \times 10^{-4}} = 1.79 \times 10^5$$

Drag

$$D = \frac{1}{2}\rho U^2 \left(\frac{\pi}{4} D^2\right) C_D$$ (+4 points)

(a) For a standard golf ball

$$C_D \approx 0.25$$

$$D = \frac{1}{2}(0.00238)(200)^2 \left(\frac{\pi}{4}\right)\left(\frac{1.69}{12}\right)^2 (0.25) = 0.185 \text{ lb}$$ (+3 points)

(b) For a smooth golf ball

$$C_D \approx 0.51$$

$$D = \frac{1}{2}(0.00238)(200)^2 \left(\frac{\pi}{4}\right)\left(\frac{1.69}{12}\right)^2 (0.51) = 0.378 \text{ lb}$$ (+3 points)