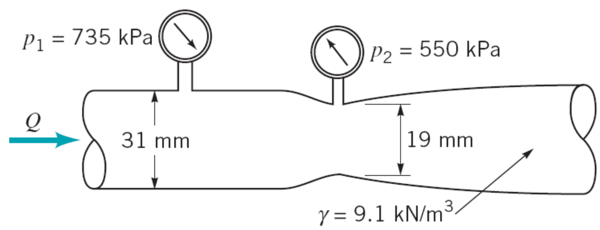
EXAM #3 December 16, 2008

Problem 1: Venturi meter (Chapter 3)



(a) Bernoulli equation:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

where, $z_1 = z_2$ (+4 points)

(b) Continuity equation:

$$A_1V_1 = A_2V_2$$

or

$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

(+2 points)

(c) By solving the equations for V_2,

$$V_2 = \sqrt{\frac{2g\,\dfrac{(p_1-p_2)}{\gamma}}{1-\left(\dfrac{D_2}{D_1}\right)^4}} = \sqrt{\frac{2(9.81)\,\dfrac{(735-550)\times 10^3}{9.1\times 10^3}}{1-\left(\dfrac{19}{31}\right)^4}} = 21.5\,\frac{\mathrm{m}}{\mathrm{s}}$$

Thus,

$$Q = A_2V_2 = \frac{\pi}{4}D_2^2V_2 = \frac{\pi}{4}\left(\frac{19}{1000}\right)^2(21.5) = \mathbf{6.10\times 10^{-3}\,\frac{m^3}{s}}$$

(+4 points)

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Problem 2: Similarity (Chapter 7)

$$\frac{P}{\rho \Omega^3 D^5} = \phi\left(\frac{Q}{\Omega D^3}\right)$$

(a) Volume flow:

$$\frac{Q_p}{\Omega_p D_p^3} = \frac{Q_m}{\Omega_m D_m^3}$$

(+2 points)

Then,

$$Q_m = Q_p\left(\frac{\Omega_m}{\Omega_p}\right)\left(\frac{D_m}{D_p}\right)^3 = 12\left(\frac{1800}{750}\right)\left(\frac{1.0}{2.0}\right)^3 = \mathbf{3.6\,\frac{ft^3}{s}}$$

(+3 points)

(b) Pump power:

$$\frac{P_p}{\rho_p \Omega_p^3 D_p^5} = \frac{P_m}{\rho_m \Omega_m^3 D_m^5}$$

(+2 points)

Similarly,

$$P_p = P_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\Omega_p}{\Omega_m}\right)^3\left(\frac{D_p}{D_m}\right)^5 = 0.082\left(\frac{1.94}{0.00234}\right)\left(\frac{750}{1800}\right)^3\left(\frac{2.0}{1.0}\right)^5 = \mathbf{157\ hp}$$

(+3 points)

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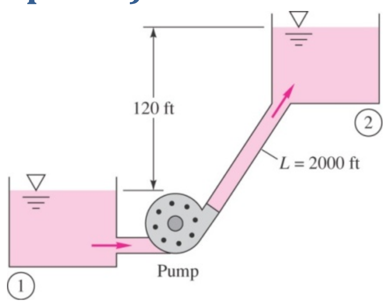
Problem 3: Head Loss (Chapter 8)



(a) Energy equation:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_t+h_L$$

where p_1 = p_2, $V_1 \approx V_2 \approx 0$, and h_t = 0 (+3 points)

(b) Head loss:

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

where,

L = 2000 ft

D = 0.5 ft

$$V=\frac{Q}{A}=\frac{3}{\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\right)^2}=15.3\,\frac{\text{ft}}{\text{s}},$$

f = 0.0228 from the given equation with

$$Re=\frac{\rho VD}{\mu}=\frac{(1.94)(15.3)(0.5)}{2.09\times10^{-5}}=7.1\times10^5 \quad \text{and} \quad \frac{\epsilon}{D}=1.7\times10^{-3}$$

or,

$$h_L=0.0228\left(\frac{2000}{0.5}\right)\frac{(15.3)^2}{2(32.2)}=331.5\text{ ft}$$ (+6 points)

Thus,

$$h_p=z_2-z_1+h_L=120+331.5=451.5\text{ ft}$$

(c) Pump power:

$$P=\rho gQh_p=(1.94)(32.2)(3.0)(450)=84{,}613\approx\mathbf{154\ hp}$$ (+1 point)

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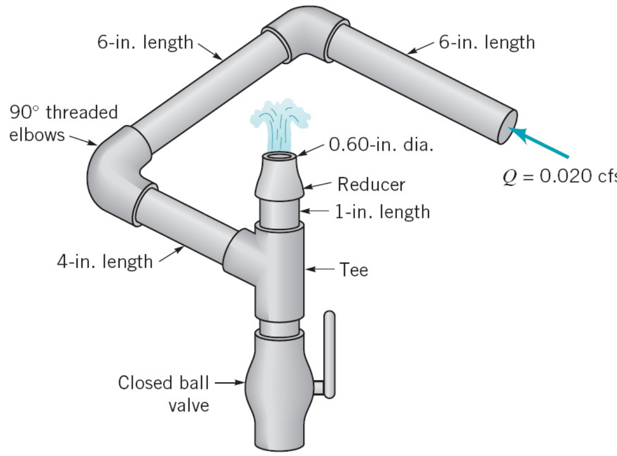
Problem 4: Minor Losses (Chapter 8)



(a) Major loss:

$$h_{L_{major}} = f\frac{L}{D}\frac{V^2}{2g}$$

where,

L = (6 + 6 + 4 +1) in. = 17 in. = 1.4167 ft,

D = 0.75 in. = 0.0625 ft,

$V = \frac{Q}{A} = \frac{0.02}{(\pi/4)(0.0625)^2} = 6.52\ \frac{\text{ft}}{\text{s}}$, and

f = 0.0376 from the given equation with

$$Re = \frac{VD}{\nu} = \frac{(6.52)(0.0625)}{1.21\times10^{-5}} = 3.37\times10^4 \quad \text{and} \quad \frac{\epsilon}{D} = 0.008$$

or $h_{L_{major}} = (0.0376)\left(\frac{17}{0.75}\right)\frac{V^2}{2g} = 0.85\frac{V^2}{2g}$ (+3 points)

(b) Minor loss:

$$h_{L_{minor}} = \sum K_L\frac{V^2}{2g}$$

where,

$$\sum K_L = 2\times\underbrace{1.5}_{90^\circ\ elbow} + \underbrace{2}_{tee} + \underbrace{0.15}_{reducer} = 5.15$$

Or $h_{L_{minor}} = 5.15\frac{V^2}{2g}$ (+6 points)

Thus,

$$\frac{h_{L_{major}}}{h_{L_{minor}}} = \frac{0.85\frac{V^2}{2g}}{5.15\frac{V^2}{2g}} = \mathbf{0.165}$$ (+1 point)

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Problem 5: Boundary Layer (Chapter 9)

(a) Local wall shear stress:

$$Re_x = \frac{\rho U x}{\mu} = \frac{(998)(5)(0.4)}{0.001} = 2 \times 10^6 \quad \text{(Turbulent)}$$

$$c_f = \frac{0.058}{Re_x^{\frac{1}{5}}} = 3.186 \times 10^{-3}$$ (+5 points)

$$\tau_w = \frac{1}{2}\rho U^2 c_f = \left(\frac{1}{2}\right)(998)(5)^2(3.186 \times 10^{-3}) = \mathbf{39.7\ Pa}$$ (+2 points)

(b) Total friction drag:

$$Re_L = \frac{\rho U L}{\mu} = \frac{(998)(5)(2)}{0.001} = 1 \times 10^7 \quad \text{(Turbulent)}$$

$$C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} = 2.946 \times 10^{-3}$$

$$D_f = \frac{1}{2}\rho U^2 (2bL) C_f = \left(\frac{1}{2}\right)(998)(5)^2(2 \times 1 \times 2)(2.946 \times 10^{-3}) = \mathbf{147\ N}$$ (+3 points)

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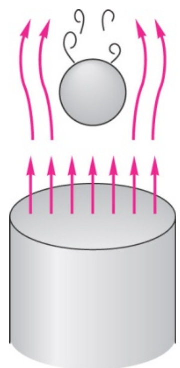
Problem 6: Drag (Chapter 9)



The ball weight must balance its drag:

$$W = \frac{1}{2}\rho V^2\left(\frac{\pi}{4}D^2\right)C_D$$ (+6 points)

At the given conditions,

$$Re = \frac{\rho V D}{\mu} = \frac{(1.225)(14.5)(0.1)}{1.78\times10^{-5}} \approx 1\times10^5$$

and from the C_D curve

$$C_D \approx 0.5$$

Thus,

$$W = \left(\frac{1}{2}\right)(1.225)(14.5)^2\left(\frac{\pi}{4}\right)(0.1)^2(0.5) = \mathbf{0.51\ N}$$

or W = **51 g** (+4 point)