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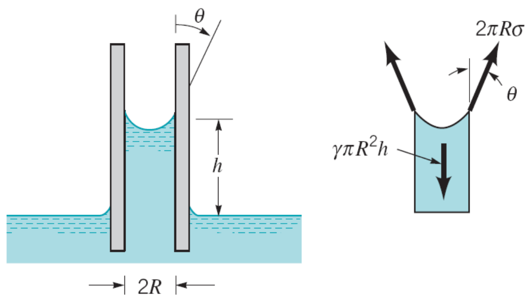
Problem 1: Surface Tension (Chapter 1)



From the vertical force balance

$$\underbrace{\gamma\pi R^2 h}_{\substack{\text{water weight}\\ \text{inside the column}}} = \underbrace{2\pi R\sigma}_{\substack{\text{surface}\\ \text{tension}}}\cos\theta$$

(+6 points)

so that

$$R = \frac{2\sigma\cos\theta}{\gamma h}$$

(+3 points)

Since $\theta = 0°$ it follows that for h = 1.0 mm,

$$R = \frac{2 \times 0.0728\ \mathrm{N/m} \times 1}{9789\ \mathrm{N/m^3} \times 0.001\ \mathrm{m}} = 0.0149\ \mathrm{m}$$

Thus, the minimum required tube diameter, D, is

$$D = 2R = 0.0298\ \mathrm{m} = \mathbf{29.8\ mm}$$

(+1 point)

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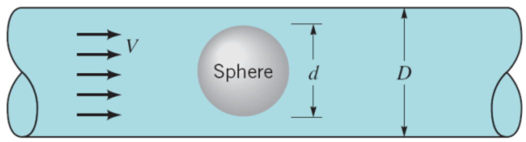
Problem 2: Similarity (Chapter 7)



(a) The similarity requirement is

$$\frac{d_m}{D_m} = \frac{d}{D}$$

so that

$$\frac{0.2 \text{ in}}{0.5 \text{ in}} = \frac{d}{2 \text{ ft}}$$

and

$$d = \mathbf{9.6 \text{ in}} = \mathbf{0.8 \text{ ft}}$$

(+4 points)

(b)The prediction equation is

$$\frac{F}{\rho V^2 D^2} = \frac{F_m}{\rho_m V_m^2 D_m^2}$$

so that

$$F = \frac{\rho}{\rho_m}\left(\frac{V}{V_m}\right)^2 \left(\frac{D}{D_m}\right)^2 F_m$$

(+4 points)

and with $\rho = \rho_m$

$$F = (1)\left(\frac{6 \text{ ft/s}}{2 \text{ ft/s}}\right)^2 \left(\frac{2 \text{ ft}}{0.5/12 \text{ ft}}\right)^2 (1.5 \times 10^{-3} \text{ lb}) = \mathbf{31.1 \text{ lb}}$$

(+2 points)

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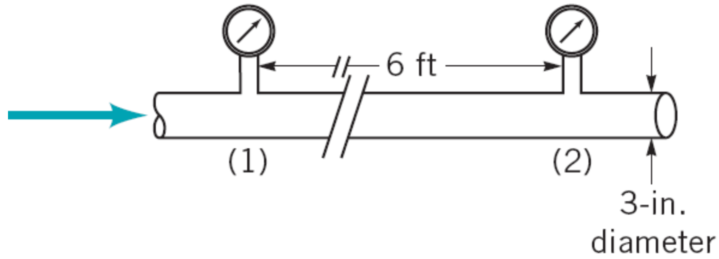
Problem 3: Head Loss (Chapter 8)



Energy equation:

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_L$$

Since $V_1 = V_2$ from continuity and $z_1 = z_2$,

$$p_1 - p_2 = h_L\gamma$$ (+4 points)

where,

$$h_L = f\frac{\ell}{D}\frac{V^2}{2g}$$

$$V = \frac{Q}{A} = \frac{0.12\ \text{ft}^3/\text{s}}{\pi/4(3/12\ \text{ft})^2} = 2.445\ \text{ft/s}$$

$$Re_D = \frac{VD}{\nu} = \frac{(2.445\ \text{ft/s})(3/12\ \text{ft})}{1.21\times10^{-5}\text{ft}^2/\text{s}} = 5.05\times10^4 > 2100 \Rightarrow \text{Turbulent flow}$$

$$\epsilon/D = \frac{0.0005\ \text{ft}}{3/12\ \text{ft}} = 2\times10^{-3}$$

$$f = \frac{1.325}{\{\ln[(\epsilon/3.7D)+(5.74/Re^{0.9})]\}^2} = 0.02673$$

or,

$$h_L = (0.02673)\left(\frac{6\ \text{ft}}{3/12\text{ft}}\right)\left[\frac{(2.445\text{ft/s})^2}{2\times32.2\text{ft/s}^2}\right] = 0.05955\ \text{ft}$$ (+5 points)

Thus,

$$\Delta p = p_1 - p_2 = h_L\gamma = 0.05955\ \text{ft}\times62.4\ \text{lb/ft}^3 = \mathbf{3.72\ lb/ft^2}$$ (+1 point)

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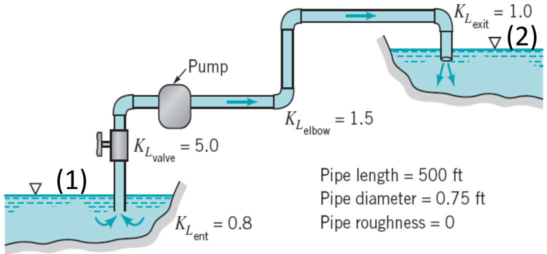
Problem 4: Minor Losses (Chapter 8)



Bernoulli equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f + \sum_i K_{L_i} \frac{V^2}{2g}$$

where $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $h_f = f \frac{\ell}{D} \frac{V^2}{2g}$

$$h_p = z_2 - z_1 + \left(f \frac{\ell}{D} + \sum_i K_{L_i} \right) \frac{V^2}{2g}$$

(+6 points)

So that with $\sum_i K_{L_i} = 0.8 + 4 \times 1.5 + 5.0 + 1.0 = 12.8$

$$V^2 = [h_p - (z_2 - z_1)] \frac{2g}{f\ell/D + \sum_i K_{L_i}}$$

$$= [250\text{ ft} - 200\text{ ft}] \left(\frac{2 \times 32.2\text{ ft/s}^2}{0.0121 \times 500\text{ ft}/0.75\text{ ft} + 12.8} \right) = 154.31\text{ ft}^2/\text{s}^2$$

Thus,

$$V = 12.4\text{ ft/s}$$

(+3 points)

The pump power required is

$$\dot{W}_s = \gamma Q h_p = \gamma \frac{\pi D^2}{4} V h_p$$

$$= (62.4\text{ lb/ft}^3) \left(\frac{\pi}{4} \right) (0.75\text{ ft})^2 (12.4\text{ ft/s})(250\text{ ft})$$

$$= (8.55 \times 10^4\text{ ft} \cdot \text{lb/s}) \left(\frac{1\text{ hp}}{550\text{ ft} \cdot \text{lb/s}} \right) = \mathbf{155\ hp}$$

(+1 point)

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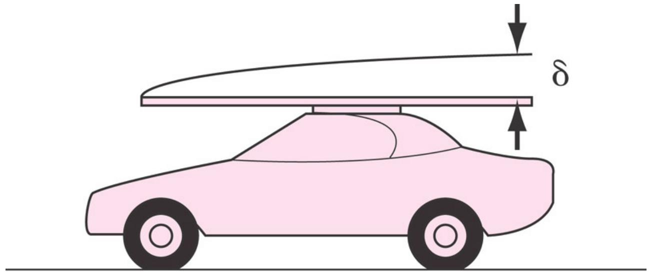
Problem 5: Boundary Layer (Chapter 9)



(a) For a turbulent boundary layer flow,

$$\frac{\delta}{L} = \frac{0.16}{(Re_L)^{1/7}} \quad \text{or} \quad \frac{\delta}{L} = \frac{0.370}{(Re_L)^{1/5}}$$

(+2 points)

where,

$$Re_L = \frac{UL}{\nu} = \frac{(35\times1.467\text{ft/s})(8\text{ ft})}{1.57\times10^{-4}\text{ft}^2/\text{s}} = 2.62\times10^6$$

Thus,

$$\delta = \frac{0.16\times8\text{ ft}}{(2.62\times10^6)^{1/7}} = \mathbf{0.155\ ft = 1.86\ in}$$ (+2 point)

(b) Drag force over a flat plate is

$$D = C_f\frac{1}{2}\rho U^2 A$$

(+2 points)

Where,

$$C_f = \frac{0.031}{(Re_L)^{1/7}} = \frac{0.031}{(2.6\times10^6)^{1/7}} = 3.754\times10^{-3} \quad \text{or} \quad C_f = \frac{0.0720}{(Re_L)^{1/5}}$$

$$A = 4\text{ ft}\times8\text{ ft} = 32\text{ ft}^2$$

Thus,

$$D = (3.754\times10^{-3})\frac{1}{2}(2.38\times10^{-3}\text{ slug/ft}^3)(51.345\text{ ft/s})^2(32\text{ ft}^2) = \mathbf{0.377\ lb}$$

(+4 points)

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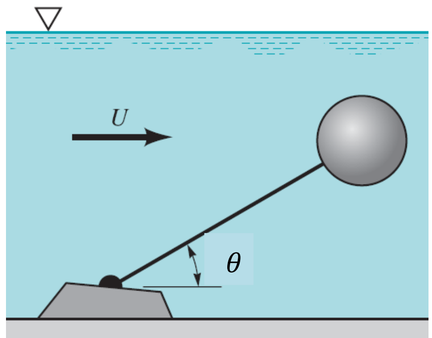
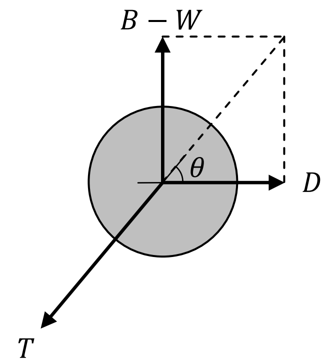
Problem 6: Drag (Chapter 9)





$\sum F_x = 0 : T\cos\theta = D$

$\sum F_y = 0 : T\sin\theta = B - W$

$$\tan\theta = \frac{B-W}{D}$$

where,

$B = \gamma \forall = \gamma \frac{4}{3}\pi R^3 = (998\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\left(\frac{4\pi}{3}\right)\left(\frac{0.1\text{m}}{2}\right)^3 = 5.126\text{ N}$ (+1 point)

$W = \gamma_{cork}\forall = \left(\frac{\gamma_{cork}}{\gamma}\right)\gamma\forall = SG \cdot B = 0.21 \times 5.126\text{ N} = 1.077\text{ N}$ (+1 point)

$D = \frac{1}{2}\rho U^2 A C_D$ (+3 points)

$Re_D = \frac{UD}{\nu} = \frac{(1.12\text{m/s})(0.1\text{ m})}{1.12\times10^{-6}\text{m}^2/\text{s}} = 1 \times 10^5$

From figure 7, $C_D = 0.5$ at $Re_D = 10^5$ (+3 points)

Thus

$D = \frac{1}{2}(998\,\text{kg/m}^3)(1.12\,\text{m/s})^2\left(\frac{\pi}{4}\right)(0.1\text{ m})^2(0.5) = 2.458\text{ N}$ (+1 point)

Hence,

$$\theta = \tan^{-1}\frac{B-W}{D} = \tan^{-1}\frac{5.126\text{ N} - 1.077\text{ N}}{2.458\text{ N}} = \mathbf{58.7}^\circ$$

(+1 point)