057:20 **TEST # 3** **Fall 2006**

Prob. 1

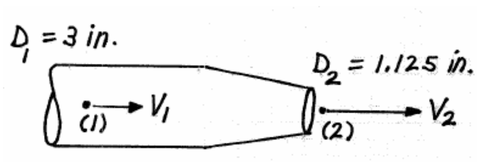
Information and assumptions

Provided in problem statement

Find

Determine pressure just upstream of the nozzle.

Solution



Bernoulli equation:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \qquad (+6)$$

With:

$$z_1=z_2, \quad p_2=0 \qquad (+1)$$

and:

$$Q=250\times 0.002228=0.557\ ft^3/s$$

$$V_1=\frac{Q}{A_1}=\frac{0.557}{\frac{\pi}{4}\left(\frac{1.125}{12}\right)^2}=80.7\ ft/s \qquad (+1)$$

$$V_2=\frac{Q}{A_2}=\frac{0.557}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2}=11.3\ ft/s \qquad (+1)$$

Then:

$$p_1=\frac{\gamma}{2g}\left(V_2^2-V_1^2\right)=\frac{\rho}{2}\left(V_2^2-V_1^2\right)$$

$$=\frac{1}{2}(1.94)\left(80.7^2-11.3^2\right)=6192\,lbf/ft^2 \qquad (+1)$$

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Prob. 2

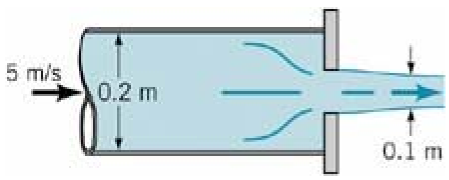
Information and assumptions

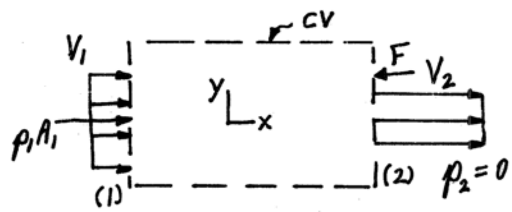
Provided in problem statement

Find

Determine the force needed to hold the plate against the pipe.

Solution





The x-component of the momentum equation for the control volum shown is:

$$\int u\rho \mathbf{V}\cdot\mathbf{n}dA = \sum F_x$$

Or

$$V_1\rho\left(-V_1\right)A_1 + V_2\rho\left(V_2\right)A_2 = p_1A_1 - F \qquad (+6)$$

Thus

$$F = p_1A_1 + \rho V_1^2A_1 - \rho V_2^2A_2 = p_1A_1 + \dot{m}\left(V_1 - V_2\right) \qquad (+1)$$

Where: $\dot{m} = \rho V_1A_1 = 999\times\frac{\pi}{4}\times0.2^2\times5 = 157\,kg/s$

Continuity gives $V_1A_1 = V_2A_2$

$$V_2 = \frac{A_1}{A_2}V_1 = \left(\frac{d_1}{d_2}\right)^2V_1 = \left(\frac{0.2}{0.1}\right)^2\times5 = 20\,m/s \qquad (+1)$$

In addition Bernoulli equation gives $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$ with $p_2 = 0$

$$p_1 = \frac{\rho}{2}\left(V_2^2 - V_1^2\right) = \frac{1}{2}\times999\times\left(20^2 - 5^2\right) = 1.87\times10^5\,N/m^2 \qquad (+1)$$

Therefore

$$F = 1.87\times10^5\times\frac{\pi}{4}\times0.2^2 + 157\times\left(5-20\right) = 3520N \qquad (+1)$$

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Prob. 3

Information and assumptions

Provided in problem statement

Find

Corresponding velocity and wave resistance of the prototype .

Solution

Dynamic similarity by equating Froude numbers:

$$Fr_m = Fr_p \qquad (+4)$$

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}}$$

$$V_p = \sqrt{\frac{gL_p}{gL_m}}V_m = \sqrt{\frac{L_p}{L_m}}V_m = \sqrt{\frac{16}{1}} \times 3 = 12\, m/s \qquad (+2)$$

$$\frac{F_p}{F_m} = \left(\frac{L_p}{L_m}\right)^3 \qquad (+3)$$

$$F_p = \left(\frac{L_p}{L_m}\right)^3 F_m = \left(\frac{16}{1}\right)^3 \times 12 = 49152 N \qquad (+1)$$

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Prob. 4

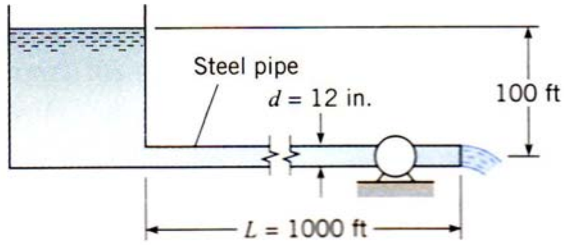
Information and assumptions

Provided in problem statement

Find

Power delivered by the turbine

Solution



Energy equation from the reservoir water surface to the jet at the end of the pipe:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_T+\sum h_L$$

$$0+0+z_1=0+\frac{V^2}{2g}+z_2+h_T+\left(K_e+f\frac{L}{D}\right)\frac{V^2}{2g} \qquad (+3)$$

$$z_1-z_2=h_T+\left(1+0.5+f\frac{L}{D}\right)\frac{V^2}{2g} \qquad (+1)$$

But:

$$V=\frac{Q}{A}=\frac{5}{\frac{\pi}{4}\times 1^2}=6.37\ ft/s$$

$$\frac{V^2}{2g}=0.630\,ft$$

$$\text{Re}=\frac{VD}{\nu}=\frac{6.37\times 1}{1.21\times 10^{-5}}=5.26\times 10^5 \qquad (+2)$$

From Moody diagram $f=0.0153$ for $k_s/D=0.00015$ (+2)

$$100=h_T+\left(1+0.5+0.0153\times\frac{1000}{1}\right)\times 0.629$$

$$h_T=89.4\,ft \qquad (+1)$$

So power delivered by the turbine

$$p=\eta Q\gamma h_T=80\%\times 5\times 62.4\times 89.4=22314\,ft\cdot lbf/s=40.57\,hp \qquad (+1)$$

Prob. 5

Information and assumptions

Provided in problem statement

Find

Determine the boundary layer thickness

Solution

Calculate the Reynolds number at the end of the plate:

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{15.5 \times 10.6}{1.57 \times 10^{-4}} = 1.05 \times 10^6 > 5 \times 10^5 \qquad (+3)$$

So the boundary layer is transitional. (+1)

Suppose the boundary layer is laminar everywhere

$$\delta = \frac{5x}{\sqrt{\mathrm{Re}_x}} = \frac{5 \times 10.6}{\sqrt{1.06 \times 10^6}} = 0.0515\,ft = 0.618in \qquad (+3)$$

Suppose the boundary layer is turbulent everywhere

$$\delta = \frac{0.16x}{\left(\mathrm{Re}_x\right)^{1/7}} = \frac{0.16 \times 10.6}{\left(1.06 \times 10^6\right)^{1/7}} = 0.234\,ft = 2.80in \qquad (+3)$$

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Prob. 6

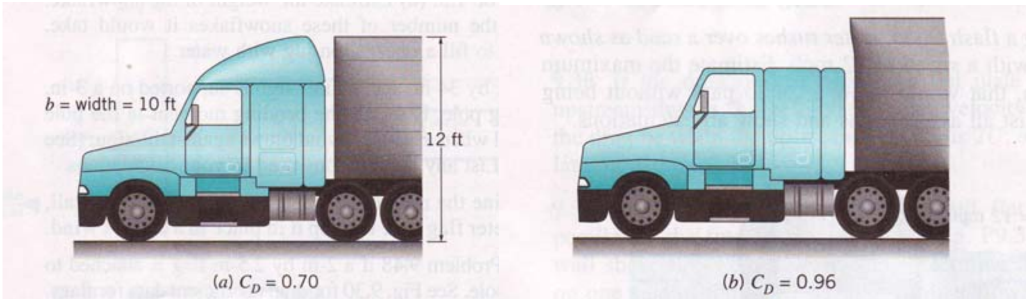
Information and assumptions

Provided in problem statement

Find

Determine a reduction of how many horsepower needed at a highway speed of 65 mph.

Solution



(a) $C_D = 0.70$ (b) $C_D = 0.96$

Drag:

$$D = C_D \frac{1}{2} \rho U^2 A \qquad (+4)$$

Drag reduction:

$$\Delta D = \left(C_{Db} - C_{Da}\right) \frac{1}{2} \rho U^2 A \qquad (+3)$$

Power

$$P = DU \qquad (+1)$$

With $U = 65 mph = 95.3\ ft/s$

Power reduction:

$$\Delta P = \left(\Delta D\right) U = \left(C_{Db} - C_{Da}\right) \frac{1}{2} \rho U^3 A$$

$$= \left(0.96 - 0.70\right) \times \frac{1}{2} \times 0.00238 \times 95.3^3 \times 10 \times 12$$

$$= 32135\ ft \cdot lb/s$$

$$= 58.4 hp \qquad (+2)$$