Review for Exam 2

Chapter 5: Mass, Bernoulli, and Energy equation

- **Flow rate and conservation of mass**

general case

$$Q = \int_{CS} \underline{V} \cdot \underline{n} dA$$

$$= \int_{CS} |\underline{V}| \cos\theta dA$$

$$\dot{m} = \int_{CS} \rho(\underline{V} \cdot \underline{n}) dA$$

average velocity: $\overline{V} = \dfrac{Q}{A}$

General Form of Continuity Equation

$$\frac{dM}{dt} = 0 = \frac{d}{dt}\int_{CV} \rho d\forall + \int_{CS} \rho \underline{V} \cdot \underline{dA}$$

or

$$\underbrace{\int_{CS} \rho \underline{V} \cdot \underline{dA}}_{} = \underbrace{-\frac{d}{dt}\int_{CV} \rho d\forall}_{}$$

net rate of outflow of mass across CS — rate of decrease of mass within CV

Simplification of Continuity Equation

1. Steady flow: $-\frac{d}{dt}\int_{CV} \rho d\forall = 0$

2. $\underline{V}$ = constant over discrete $\underline{dA}$ (flow sections):

$$\int_{CS} \rho \underline{V} \cdot \underline{dA} = \sum_{CS} \rho \underline{V} \cdot \underline{A}$$

3. Incompressible fluid (ρ = constant)

$$\int_{CS} \underline{V} \cdot \underline{dA} = -\frac{d}{dt}\int_{CV} d\forall \qquad \text{conservation of volume}$$

4. Steady One-Dimensional Flow in a Conduit:

$$\sum_{CS} \rho \underline{V} \cdot \underline{A} = 0$$

$$-\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0$$

for ρ = constant $\qquad Q_1 = Q_2$

- **Bernoulli Equations, Application, and Limitation**

Assume the flow is

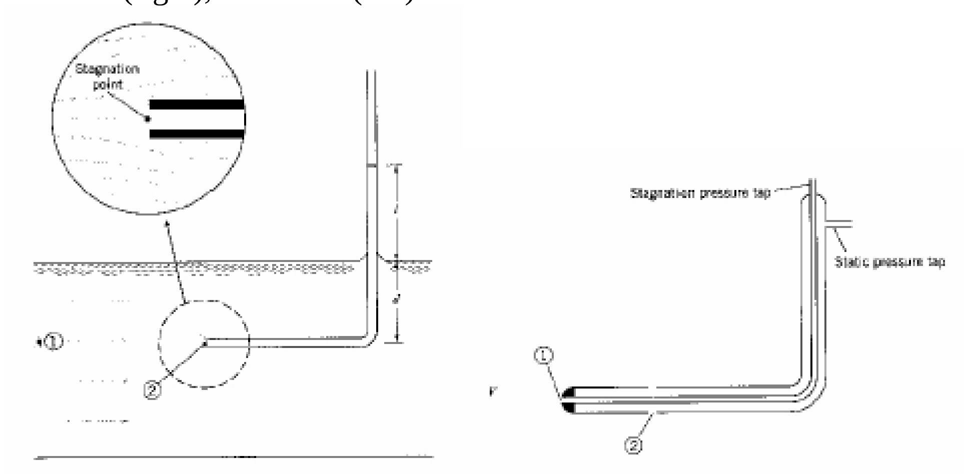
- Irrotational
- Inviscid
- Incompressible

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

Bernoulli equation for steady state

Application:

Stagnation tube (right), Pitot tube (left)



- **Energy Equation**

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{CV} \rho \left(\frac{V^2}{2} + gz + u \right) d\forall + \int_{Cs} \rho \left(\frac{V^2}{2} + gz + u \right) \underline{V} \cdot \underline{dA}$$

rate of heat transfer to sysem ($\dot{Q}$)

rate of work done by system ($\dot{W}$)

rate of change of energy in CV

flux of energy out of CV (ie, across CS)

- **Simplified form of energy equation for steady one dimensional incompressible flow**

Simplification:

- No acceleration normal to the stream lines, pressure is hydrostatically distributed.
- Internal energy u is considered as constant.
- Shaft work defined as $\dot{W}_S = \dot{W}_t - \dot{W}_p$
- Head loss defined as $h_L = \dfrac{u_2 - u_1}{g} - \dfrac{\dot{Q}}{g\dot{m}}$

The final form of simplified energy equation:

$$\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_t+h_L$$

$$h_p=\dot{W}_p/\dot{m}g$$

$$h_t=\dot{W}_t/\dot{m}g$$

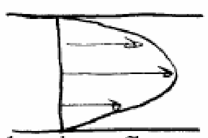
$$\alpha=\frac{1}{A\bar{V}^2}\int_A V^3\,dA=\text{kinetic energy correction factor}$$

$$\bar{V}=\frac{1}{A}\int_A V\,dA=Q/A$$

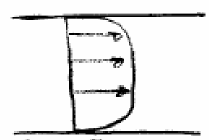
V_1 & V_2 are average velocities

note that: $\alpha = 1$ if V is constant across the flow section

$\alpha > 1$ if V is nonuniform



laminar flow $\alpha = 2$



turbulent flow $\alpha = 1.05 \sim 1$ may be used

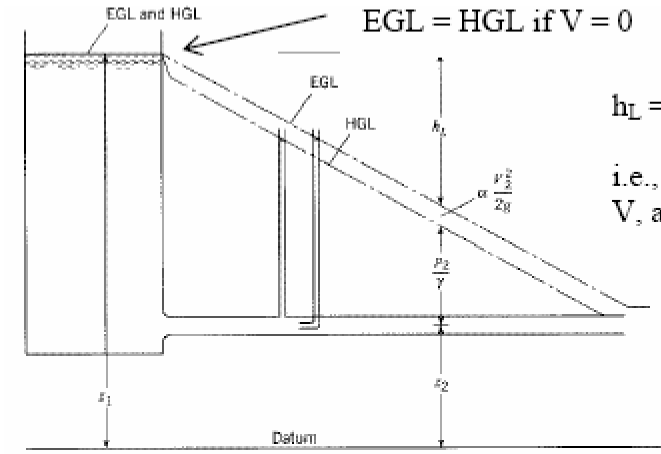
- **<u>HGL and EGL</u>**

$$\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_t+h_L$$

Define $\text{HGL}=\frac{p}{\gamma}+z$, $\text{EGL}=\frac{p}{\gamma}+z+\alpha\frac{V^2}{2g}$ } point-by-point application is graphically displayed

<u>HGL corresponds to pressure tap measurement + z</u>

<u>EGL corresponds to stagnation tube measurement + z</u>



EGL = HGL if V = 0

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

i.e., linear variation in L for D, V, and f constant

f = friction factor
f = f(Re)

Chapter 6: Momentum Analysis of Flow System

- **Momentum Equation**

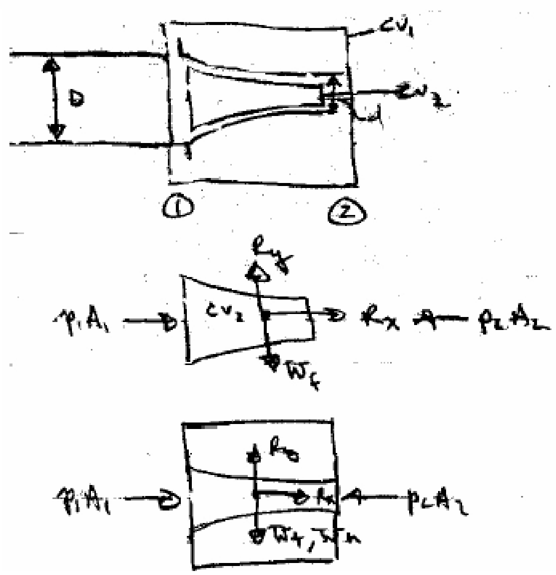
$$\sum \mathbf{F} = \frac{d}{dt}\int_{CV} \mathbf{V}\rho d\forall + \int_{CS} \mathbf{V}\rho\mathbf{V}_R \cdot d\mathbf{A}$$

$\sum \mathbf{F}$: sum of all external forces acting on CS, i.e., pressure force, forces transmitted through solids, etc.

$\mathbf{V}_R$: relative velocity = $\mathbf{V}$-$\mathbf{V}_{CS}$

Special form for steady flow with uniform flow across discrete CS

$$\sum \mathbf{F} = \sum \rho\mathbf{V}(\mathbf{V}\cdot\mathbf{A})$$



(R_x,R_y) = reaction force on fluid

(R_x,R_y) = reaction force on nozzle

Important features for momentum equation:

1. Vector equations
2. Carefully define CV to include all external body and surface faces
3. Velocity must be referenced to a non-accelerating inertial frame.
4. Steady vs Unsteady flow
5. Uniform vs Nonuniform flow
6. Always use gage pressure
7. Pressure condition at a jet exit

Chapter 7: Dimensional Analysis and Modeling

- Buckingham Π theorem

$$F\left(A_1, \cdots A_n\right) = 0$$

$$\Rightarrow f\left(\pi_1, \cdots \pi_{\underbrace{n-\hat{m}}_{r}}\right) = 0 \quad \hat{m} = m \text{ usually}$$

significant reduction in number of variables which reduces number of experiments or calculations required

- Methods for determining Π_i's

1. Functional Relationship Method

Identify functional relationships $F(A_i)$ and $f(\Pi_j)$by first determining A_i's and then evaluating Π_j's

a. Inspection	intuition
b. Step-by-step Method	text
c. Exponent Method	class

2. Nondimensionalize governing differential equations and initial and boundary conditions

- **Common Dimensionless Parameters for Fluid Flow Problems**

Re, Fr, We, Ma, C_p, etc

- **Nondimensionalization of the Basic Equation**

$$\nabla \cdot \mathbf{V} = 0$$

$$\frac{D\mathbf{V}}{Dt} = -\nabla p + Re^{-1}\nabla^2 \mathbf{V}$$

$$\mathbf{V} = \mathbf{V}/U_0$$
$$\mathbf{X} = \mathbf{X}/L$$
$$t = tU_0/L$$
$$p = (p + \rho g z)/\rho U_0^2$$

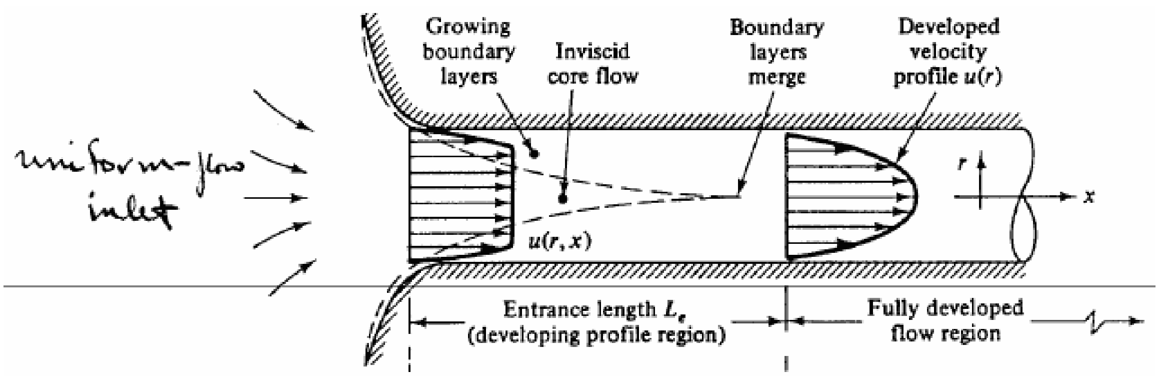
- **Similarity and Model testing**

$$\Pi_{i\ model} = \Pi_{i\ prototype} \qquad i = 1, r = n - \hat{m}\ (m)$$

Similarity is classified as:

1. Geometric Similarity (similar length scale) : $\alpha = L_m / L_p$
2. Kinematic Similarity (similar length and time scale)
3. Dynamic Similarity (similar length, time, and force scales)
   (Re& Fr scaling etc)

Chapter 8: Flow in Pipes



Entrance vs. fully developed flow: $L_e/D = f(\text{Re})$

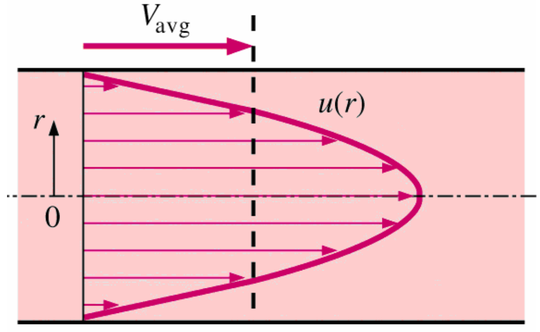
Laminar vs. turbulent flow: $\text{Re}_{crit} \approx 2000$

Laminar Flow in Pipes

Velocity profile for laminar pipe flow

$$u(r)=\frac{r^2-r_0^2}{4\mu}\frac{d}{dx}(p+\gamma z),$$

$$u(r)>0 \text{ for } \frac{d}{dx}(p+\gamma z)<0, \text{ i.e., favorable pressure gradient}$$



Laminar flow

Head loss and friction factor for laminar pipe flow (These are exact solutions!!):

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

$$\Delta h=\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)$$

$$h_L=\frac{p_1-p_2}{\gamma}+(z_1-z_2)=-\Delta h$$

$$h_L=\frac{L}{\gamma}\left[-\frac{d}{ds}(p+\gamma z)\right] \qquad L = \text{length of pipe} = ds$$

$$=\frac{L}{\gamma}\left[\frac{8\mu\overline{V}}{r_o^2}\right]=-\Delta h\alpha\overline{V} \qquad h_L=L\left[-\frac{d}{ds}\left(\frac{p}{\gamma}+z\right)\right]=L\left(-\frac{dh}{ds}\right)$$

or $\quad h_f=h_L=\dfrac{32\mu L\overline{V}}{\gamma D^2}$ $\qquad h_f$ = head loss due to friction

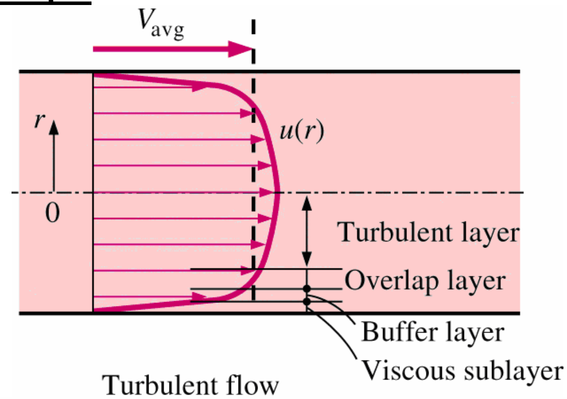
exact solution

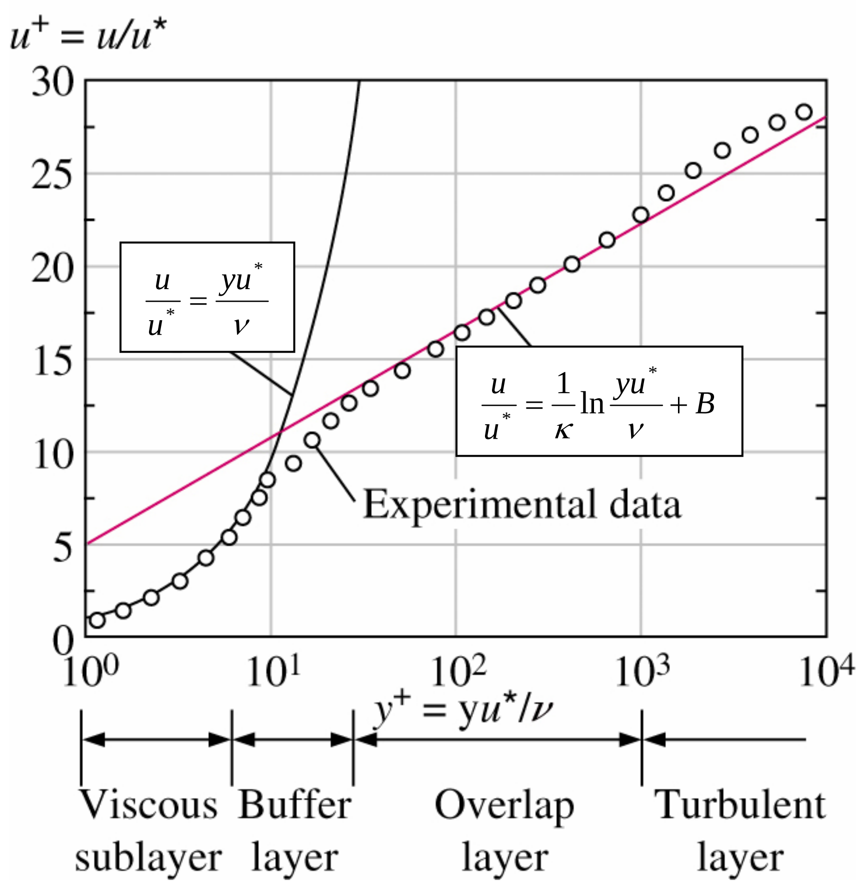
$$h_L=\frac{32\mu LV}{\gamma D^2}=f\frac{L}{D}\frac{V^2}{2g}$$

$$\tau_0=\frac{D}{4}\frac{\gamma h_L}{L}\Rightarrow h_L=\frac{4L\tau_0}{D\gamma}=4L\frac{1}{8}\rho V^2 f/D\gamma=f\frac{L}{D}\frac{V^2}{2g}$$

$$\therefore\ f=\frac{64}{\text{Re}} \qquad \text{Re}=\frac{VD}{\nu}$$

- **<u>Turbulent Flow in Pipes</u>**





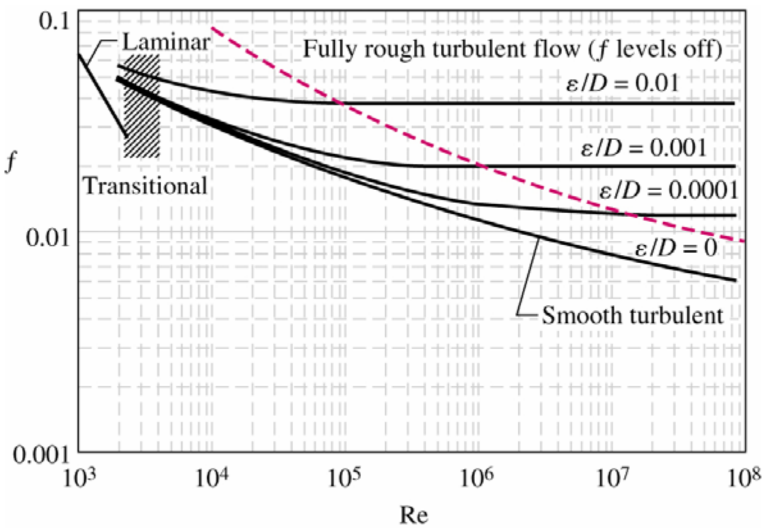
Head losses and friction factor for turbulent pipe flow:

Friction factor:

$1/\sqrt{f} = 1.14 - 2\log\left(k_s/D + 9.35/\mathrm{Re}\sqrt{f}\right)$, as shown in Moody diagram

k_s/D = roughness parameter

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = h_1 - h_2,\ h_1 = \frac{p_1}{\gamma} + z_1,\ h_2 = \frac{p_2}{\gamma} + z_2$$



Moody-Diagram

- **Types of problems for turbulent pipe flow**

1. Determine head loss

$$f = f\left(\text{Re}, k_s/D\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = h_1 - h_2$$

2. Determine flow rate

$$V = \underbrace{\left[\frac{2gh_L}{LD}\right]^{1/2}}_{known\ from\ data} f^{-1/2}$$

guess $f \Rightarrow V \Rightarrow \text{Re} \Rightarrow f$, repeat to converge

3. Determine pipe size

$$D = \underbrace{\left[\frac{8LQ^2}{\pi^2 gh_L}\right]^{1/5}}_{known\ from\ data} f^{1/5}$$

guess $f \Rightarrow D \Rightarrow \text{Re}\,\&\, k_s/D \Rightarrow f$, repeat to converge

- **Minor losses**

$$\frac{p_1}{\gamma}+z_1+\alpha_1\frac{V_1^2}{2g}+h_p=\frac{p_2}{\gamma}+z_2+\alpha_2\frac{V_2^2}{2g}+h_t+h_L+\sum_i h_{m_i}$$

$$h_{m_i}=\left(K\frac{V^2}{2g}\right)_i$$

(K: minor loss coefficient→Depending on the shape of the pipe inlet/exit/curvature)