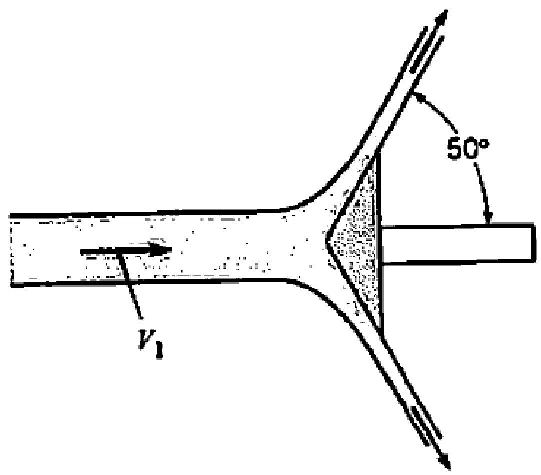
11/09/2008 Midterm 2 Review problem
057:020 Fall 2008

1. RTT: Mass and momentum conservation

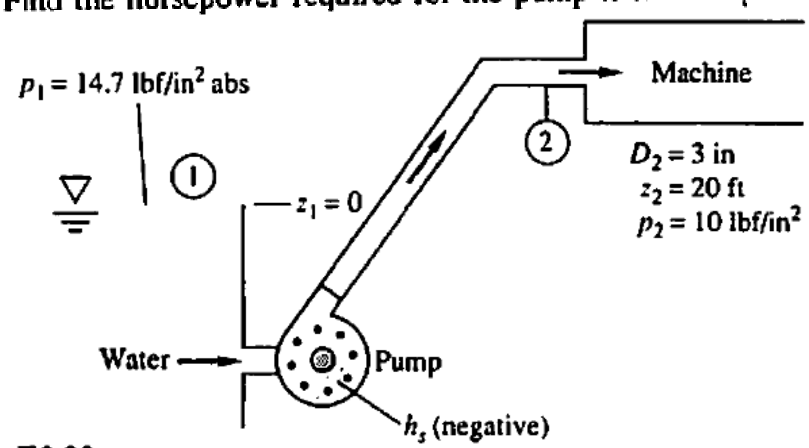
The water in this jet has a speed of 30 m/s to the right and is deflected by a cone that is moving to the left with a speed of 13 m/s. The diameter of the jet is 10 cm. Determine the external horizontal force needed to move the cone. Assume negligible friction between the water and the vane.



2. RTT: Energy conservation

The pump in Fig. E3.20 delivers water (62.4 lbf/ft^3) at 1.5 ft^3/s to a machine at section 2, which is 20 ft higher than the reservoir surface. The losses between 1 and 2 are given by

$h_f = KV_2^2/(2g)$, where $K \approx 7.5$ is a dimensionless loss coefficient (see Sec. 6.7). Take $\alpha \approx 1.07$. Find the horsepower required for the pump if it is 80 percent efficient.



E3.20

11/09/2008 Midterm 2 Review problem
057:020 Fall 2008

3. Differential analysis

Two immiscible, incompressible, viscous fluids having the same densities but different viscosities are contained between two infinite, horizontal, parallel plates (Fig. P6.81). The bottom plate is fixed and the upper plate moves with a constant velocity U. Determine the velocity at the interface. Express your answer in terms of U, μ_1, and μ_2. The motion of the fluid is caused entirely by the movement of the upper plate; that is, there is no pressure gradient in the x direction. The fluid velocity and shearing stress are continuous across the interface between the two fluids. Assume laminar flow.

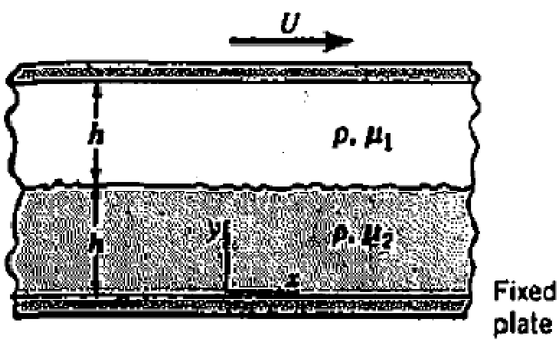


FIGURE P6.81

4. Dimensional analysis

A 1/49 scale model of a proposed dam is used to predict prototype flow conditions. If the design flood discharge over the spillway is 15,000 m^3/s, what water flow rate should be established in the model to simulate this flow? If a velocity of 1.2 m/s is measured at a point in the model, what is the velocity at a corresponding point in the prototype?

11/09/2008 Midterm 2 Review problem
057:020 Fall 2008

Answer

1. RTT: Mass and momentum conservation

Information and assumptions

provided in problem statement

Find

external horizontal force needed to move cone.

Solution

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone.

Velocity analysis

$$\begin{aligned} v_1 &= V_1 = 43 \text{ m/s} \\ v_2 &= 43 \text{ m/s} \end{aligned}$$

x-momentum

$$\begin{aligned} F_x &= \dot{m}(v_{2x} - v_1) \\ F_x &= 1,000 \times \pi \times (0.05)^2 \times 43 \times (27.64 - 43) = -5,187 \text{ N} \\ F_x &= \underline{\underline{5.19 \text{ kN (acting to left)}}} \end{aligned}$$

11/09/2008 Midterm 2 Review problem
057:020 Fall 2008

2. RTT: Energy conservation

- *Assumptions:* Steady flow, negligible viscous work, large reservoir ($V_1 \approx 0$).
- *Approach:* First find the velocity V_2 at the exit, then apply the steady flow energy equation.
- *Solution steps:* Use BG units, $p_1 = 14.7(144) = 2117$ lbf/ft^2 and $p_2 = 10(144) = 1440$ lbf/ft^2. Find V_2 from the known flow rate and the pipe diameter:

$$V_2 = \frac{Q}{A_2} = \frac{1.5\ \text{ft}^3/\text{s}}{(\pi/4)(3/12\ \text{ft})^2} = 30.6\ \text{ft/s}$$

The steady flow energy equation (3.71), with a pump (no turbine) plus $z_1 \approx 0$ and $V_1 \approx 0$, becomes

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 - h_p + h_f, \quad h_f = K\frac{V_2^2}{2g}$$

or

$$h_p = \frac{p_2 - p_1}{\gamma} + z_2 + (\alpha_2 + K)\frac{V_2^2}{2g}$$

- *Comment:* The pump must balance four different effects: the pressure change, the elevation change, the exit jet kinetic energy, and the friction losses.
- *Final solution:* For the given data, we can evaluate the required pump head:

$$h_p = \frac{(1440 - 2117)\ \text{lbf/ft}^2}{62.4\ \text{lbf/ft}^3} + 20 + (1.07 + 7.5)\frac{(30.6\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = -11 + 20 + 124 = 133\ \text{ft}$$

With the pump head known, the delivered pump power is computed similar to the turbine in Example 3.19:

$$P_{pump} = \dot{m} w_s = \gamma Q h_p = \left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right)\left(1.5\ \frac{\text{ft}^3}{\text{s}}\right)(133\ \text{ft})$$

$$= 12450\ \frac{\text{ft}-\text{lbf}}{\text{s}} = \frac{12{,}450\ \text{ft}-\text{lbf/s}}{550\ \text{ft}-\text{lbf/(s}-\text{hp)}} = 22.6\ \text{hp}$$

If the pump is 80 percent efficient, then we divide by the efficiency to find the input power required:

$$P_{input} = \frac{P_{pump}}{\text{efficiency}} = \frac{22.6\ \text{hp}}{0.80} = 28.3\ \text{hp} \qquad \textit{Ans.}$$

11/09/2008 Midterm 2 Review problem
057:020 Fall 2008

3. Differential analysis

For the specified conditions, $v=0$, $w=0$, $\frac{\partial p}{\partial x}=0$, and $g_x=0$, so that the x-component of the Navier-Stokes equations (Eq. 6.127a) for either the upper or lower layer reduces to

$$\frac{d^2u}{dy^2}=0 \qquad (1)$$

Integration of Eq. (1) yields

$$u = Ay + B$$

which gives the velocity distribution in either layer.
In the upper layer at $y=2h$, $u=U$ so that

$$B_1 = U - A_1(2h)$$

where the subscript 1 refers to the upper layer.
For the lower layer at $y=0$, $u=0$ so that

$$B_2 = 0$$

where the subscript 2 refers to the lower layer. Thus,

$$u_1 = A_1(y-2h) + U$$

and

$$u_2 = A_2 y$$

At $y=h$, $u_1 = u_2$ so that

$$A_1(h-2h) + U = A_2 h$$

or

$$A_2 = -A_1 + \frac{U}{h} \qquad (2)$$

(con't)

11/09/2008 Midterm 2 Review problem
057:020 Fall 2008

Since the velocity distribution is linear in each layer the shearing stress

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu \frac{du}{dy}$$

is constant throughout each layer. For the upper layer

$$\tau_1 = \mu_1 A_1$$

and for the lower layer

$$\tau_2 = \mu_2 A_2$$

At the interface $\tau_1 = \tau_2$ so that

$$\mu_1 A_1 = \mu_2 A_2$$

or

$$\frac{A_1}{A_2} = \frac{\mu_2}{\mu_1} \qquad (3)$$

Substitution of Eq. (3) into Eq. (2) yields

$$A_2 = -\frac{\mu_2}{\mu_1} A_2 + \frac{U}{h}$$

or

$$A_2 = \frac{U/h}{1 + \mu_2/\mu_1}$$

Thus, velocity at the interface is

$$u_2\,(y = h) = A_2 h = \frac{U}{1 + \frac{\mu_2}{\mu_1}}$$

11/09/2008 Midterm 2 Review problem
057:020 Fall 2008

4. Dimensional analysis

Solution The Froude-number criterion will be used. Therefore,

$$\mathrm{Fr}_m = \mathrm{Fr}_p$$

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

However,

$$g_m = g_p$$

Thus we have

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

We multiply both sides of this equation by the area ratio A_m/A_p:

$$\frac{V_m A_m}{V_p A_p} = \frac{A_m}{A_p}\sqrt{\frac{L_m}{L_p}}$$

The left-hand side of the equation is the discharge ratio, and $A_m/A_p = L_m^2/L_p^2$. Hence we obtain

$$\frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)^{5/2}$$

$$Q_m = Q_p\left(\frac{1}{49}\right)^{5/2} = 15{,}000\frac{1}{16{,}800} = 0.89\ \mathrm{m}^3/\mathrm{s}$$

From the fourth equation in this example,

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

Consequently,

$$V_p = V_m\sqrt{\frac{L_p}{L_m}} = 1.2 \times 7 = 8.4\ \mathrm{m/s}$$

At the given point in the prototype, we would have a velocity of 8.4 m/s.