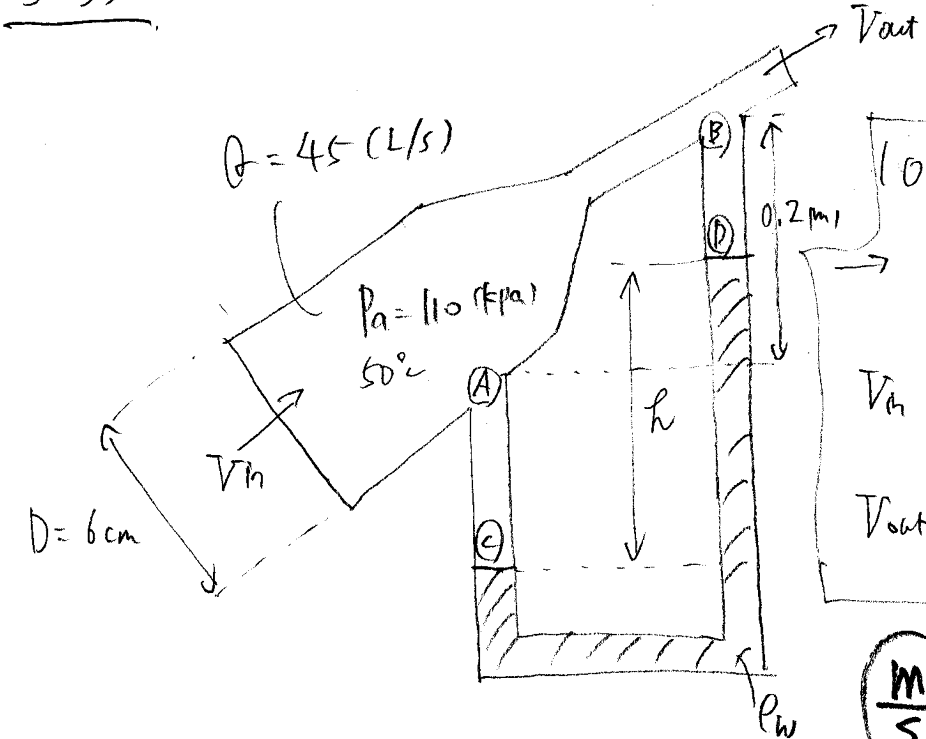
5-55




Velocity at Inlet & Exit

$1000\ (L/s) = 1\ [m^3/s]$

$45\ (L/s) = 4.5 \times 10^{-3}\ (m^3/s)$

$$V_{in} = \frac{Q}{A_{in}} = \frac{4.5 \times 10^{-3}}{\left(\frac{0.06}{2}\right)^2 \pi} = 15.9\ (m/s)$$

$$V_{out} = \frac{Q}{A_{out}} = \frac{4.5 \times 10^{-3}}{\left(\frac{0.04}{2}\right)^2 \pi} = 35.8\ (m/s)$$

$$\left(\frac{m}{s}\right) = \frac{\frac{m^3}{s}}{m^2}$$

Use Bernoulli eqn.

$$A\text{-}B: \quad \frac{1}{2}\rho_a V_{in}^2 + P_A + \rho_a g Z_A = \frac{1}{2}\rho_a V_{out}^2 + P_B + \rho_a g Z_B \quad \text{— ①}$$

$$C\text{-}D: \quad P_A + \rho_W g Z_C = P_B + \rho_W g Z_D \quad \text{— ②}$$

From ②

$$P_A - P_B = \rho_w g (Z_D - Z_C) = \rho_w g h \quad \text{— ③}$$

From ①

$$P_A - P_B = \rho_a g (Z_B - Z_A) + \frac{1}{2}\rho_a (V_{out}^2 - V_{in}^2) \quad \text{— ④}$$

Equate ③ and ④

$$\rho_w g h = \frac{1}{2}\rho_a (V_{out}^2 - V_{in}^2) + \rho_a g (Z_B - Z_A)$$

$$\rightarrow h = \frac{\rho_a}{\rho_f g} \left\{ \frac{1}{2}(V_{out}^2 - V_{in}^2) + g(Z_B - Z_A) \right\} \quad \text{— ⑤}$$

We need to get $\rho_a$ since we are given the pressure.

$$\rightarrow P_a = \rho_a RT \rightarrow \rho_a = \frac{P_a}{RT} = \frac{110 \times 10^3}{0.287 \times (50 + 273)} = 1.186$$

$Z_B - Z_A = 0.2\ (m)$

$\rho_f = 1000\ (kg/m^3)$

are known.

→ Plug these value into ⑤

$h = 0.0627\ (m)$ //

↓

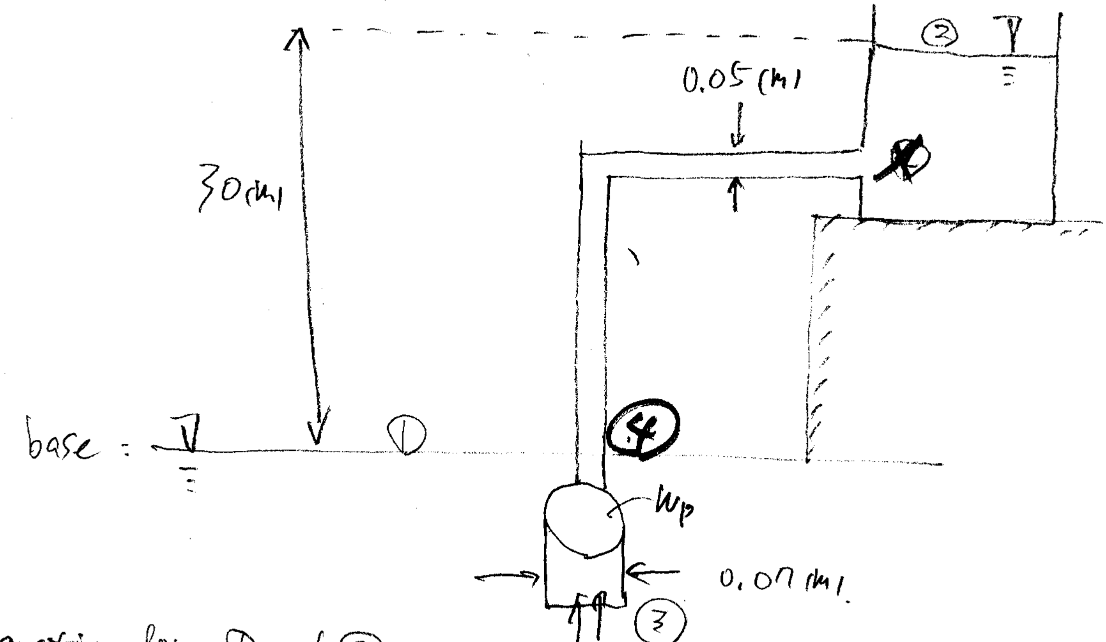
Use ① to get

$P_A - P_B = \rho_w g h$

$= 614\ (Pa)$ //

5-69

$W_p = 3$ (kW)
$= 3000$ (W)
$= 3000 \left(\frac{J}{s}\right)$
$= 3000 \left(\frac{N \cdot m}{s}\right)$
$= 3000 \left(\frac{kg \cdot m^2}{s^3}\right)$




(a) Start from energy equation between ① and ②

$$\dot m\left(\frac{\cancel{P_1}}{\rho} + \alpha\frac{V_1^2}{2} + \cancel{g z_1}\right) + \dot W_{pump} = \dot m\left(\frac{\cancel{P_2}}{\rho} + \alpha\frac{V_2^2}{2} + g z_2\right) + \cancel{\dot W_{turb}} + \dot E_{mech.loss}$$

①, ② : Open to atmosphere → $P_1 , P_2 = 0$

$\dot W_{turb} = 0$ ∵ no - turbine ,

$$\to \boxed{\alpha\frac{\dot m V_1^2}{2} + \dot W_{pump} = \alpha\frac{\dot m V_2^2}{2} + \dot m g z_2 + \dot E_{mech.loss}}$$

$V_1 , V_2 = 0$ : Free - surface is large enough.

$$\therefore \dot W_{pump,u} = \dot W_{pump} - \dot E_{mech.loss} = \dot m g z_2$$

Given that $\dot W_{pump,u} = 3000 \times 0.7 = 2100 \left[\frac{kg \cdot m^2}{s^3}\right]$

$$\therefore \dot m = \frac{\dot W_{pump,u}}{g z_2} = \frac{2100}{9.8 \times 30} = 7.142\ [kg/s]$$

$$\therefore Q = \frac{1}{\rho} \cdot \dot m = \frac{1}{1000} \times 7.142 = \underline{7.142 \times 10^{-3}\ (m^3/s)}\ //$$

(b) Energy eqn between ③ and ④

$$\underbrace{\dot m\left(\frac{P_3}{\rho} + \alpha\frac{V_3^2}{2} + g z_1\right) + \dot W_{pump}}_{\text{Inlet}} = \underbrace{\dot m\left(\frac{P_4}{\rho} + \alpha\frac{V_4^2}{2} + g z_2\right) + \dot W_{turb} + \dot E_{mech.loss}}_{\text{Exit}} \quad — ☆$$

At ③ : $(0.035)^2 \pi \times V_3 = 7.142 \times 10^{-3} \rightarrow V_3 = 1.856$

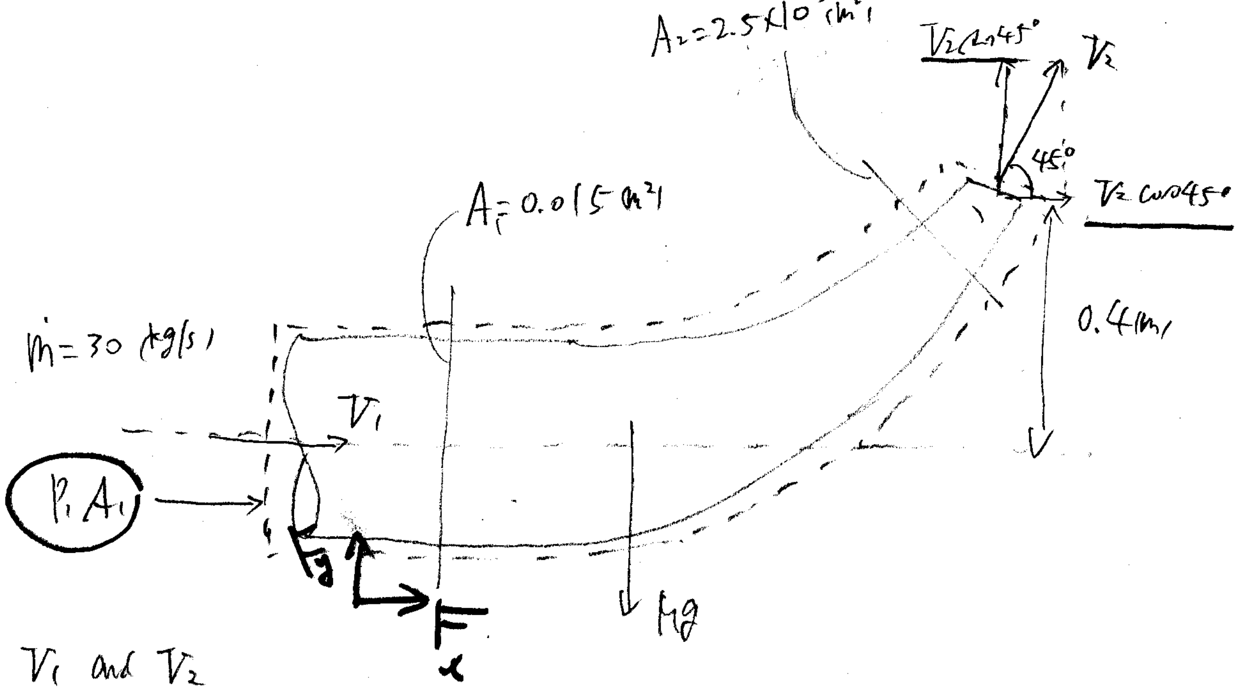
At ④ : $(0.025)^2 \pi \times V_4 = 7.142 \times 10^{-3} \rightarrow V_4 = 3.637$

Also, $\dot{W}_{pump} - \dot{E}_{mech.loss} = \underline{2100}$ ( from (a) )

Plug those values into ⊛

$\underline{P_3 - P_4 = 289.21 \text{ (kPa)}}$ //

6-25




Obtain $V_1$ and $V_2$

$$\dot{m}\left[\frac{kg}{s}\right] = \rho\left[\frac{kg}{m^3}\right] \times Q\left[\frac{m^3}{s}\right] \rightarrow Q = \frac{\dot{m}}{\rho} = \frac{30}{1000} = 0.03\left[\frac{m^3}{s}\right]$$

$$\rightarrow \left| \begin{array}{l} Q = A_1 V_1 \rightarrow V_1 = \frac{Q}{A_1} = \frac{0.03}{0.015} = 2 \text{ m/s} \\ Q = A_2 V_2 \rightarrow V_2 = \frac{Q}{A_2} = \frac{0.03}{2.5\times10^{-3}} = 12 \text{ m/s} \end{array} \right.$$

Use Bernoulli equation to get pressure at inlet.

$$\underbrace{P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1}_{\text{inlet}} = \underbrace{\cancel{P_2} + \frac{1}{2}\rho V_2^2 + \rho g z_2}_{\text{Exit}}$$

$$\rightarrow P_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho g(z_2 - z_1) = 73920 \text{ [Pa]}$$

Start from steady state linear momentum eqn.

$$\Sigma \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

→ Decompose it into horizontal and vertical components

horizontal : $F_x + P_1 A_1 = \beta \dot{m} V_2 \cos 45° - \beta \dot{m} V_1$

vertical : $F_y - Mg = \beta \dot{m} V_2 \sin 45°$ with $\beta = 1.03$

$$\rightarrow F = \sqrt{F_x^2 + F_y^2} = 1.18 \text{ (kN)}$$

# Dimensional Analysis

Fr must be the same between model and prototype.

$\rightarrow \; Fr|_{model} = Fr|_{prototype}$

$$\rightarrow \; \frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \;\rightarrow\; V_p = \frac{\sqrt{gL_p}}{\sqrt{gL_m}} V_m = \sqrt{\frac{L_p}{L_m}} V_m$$

Given that $L_p = 25 L_m$ $\quad \rightarrow V_p = \sqrt{\frac{25 L_m}{L_m}} V_m = 5 \times 5 = 25$ (ft/s)

$C_D$ (Drag coefficient) must be the same between model and prototype.

$\rightarrow \; C_D|_{model} = C_D|_{prototype}$

$$\rightarrow \; \frac{F_m}{\frac{1}{2}\rho_m V_m^2 A_m} = \frac{F_p}{\frac{1}{2}\rho_p V_p^2 A_p}$$

$\rightarrow$ Assume $\rho_m \simeq \rho_p$ then

$$\frac{F_p}{F_m} = \frac{V_p^2 A_p}{V_m^2 A_m}$$

Replace $A_p = L_p^2$, $A_m = L_m^2$, then
(area)

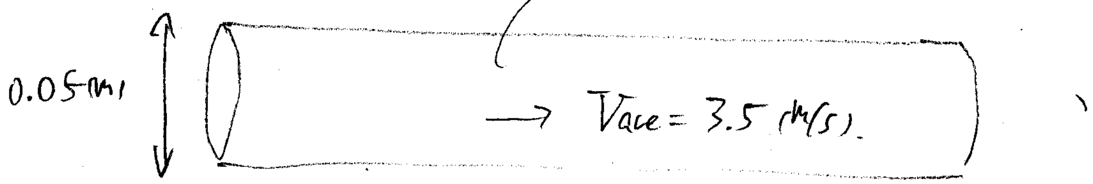
$$\frac{F_p}{F_m} = \left(\frac{V_p}{V_m}\right)^2 \left(\frac{L_p}{L_m}\right)^2$$

$$= 25 \times (25)^2 = (25)^3$$

$$F_p = (25)^3 F_m$$

8-46

40°c, Glycerin : $\rho = 1252\ (kg/m^3)$, $\mu = 0.27\ (kg/m\cdot s)$




Check the Re first to determine if the flow is laminar or turbulent.

$$Re = \frac{\rho V D}{\mu} = \frac{1252 \times 3.5 \times 0.05}{0.27} = 811.48 : \text{laminar } (\because Re < 2300)$$

$$\therefore \Delta P = \frac{32 \mu L V_{ave}}{D^2} = \frac{32 \times 0.27 \times 10 \times 3.5}{(0.05)^2} = 120960\ Pa$$

$$= 120.96\ (kPa)$$

Or

$$f = \frac{64}{Re} = \frac{64}{811.48}$$

then,

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{ave}^2}{2}$$

Yields the same result!!