# Problem review 2 (11/03/2006)

5.33 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.33. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the $x$ and $y$ components of the force that the pipe exerts on the tee.

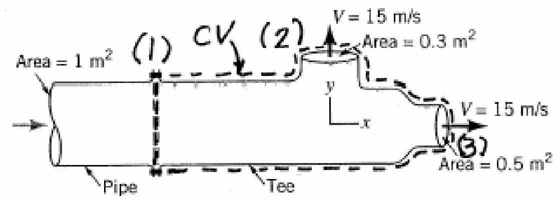



FIGURE P5.33

5.114 Water is pumped through a 4-in.-diameter pipe as shown in Fig. P5.114*a*. The pump characteristics (pump head versus flowrate) are given in Fig. P5.114*b*. Determine the flowrate if the head loss in the pipe is $h_L = 8V^2/2g$.

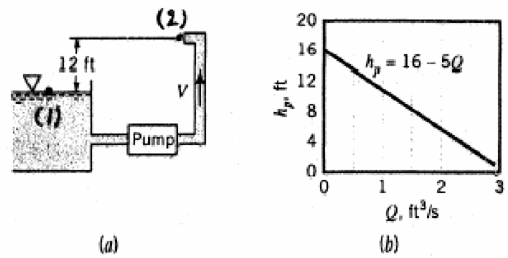



FIGURE P5.114

6.78 A fluid of density $\rho$ flows steadily *downward* between the two vertical infinite, parallel plates shown in the figure for Problem 6.77. The flow is fully developed and laminar. Make use of the Navier–Stokes equation to determine the relationship between the discharge and the other parameters involved, for the case in which the change in pressure along the channel is zero.

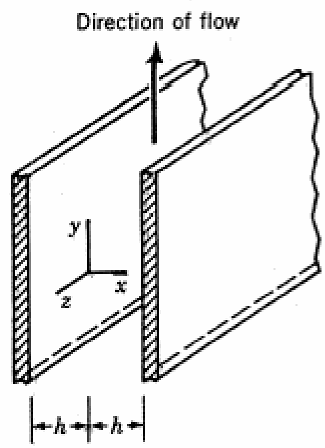



FIGURE P6.77

7.47 A thin layer of spherical particles rests on the bottom of a horizontal tube as shown in Fig. P7.47. When an incompressible fluid flows through the tube, it is observed that at some critical velocity the particles will rise and be transported along the tube. A model is to be used to determine this critical velocity. Assume the critical velocity, $V_c$ to be a function of the pipe diameter, $D$, particle diameter, $d$, the fluid density, $\rho$, and viscosity, $\mu$, the density of the particles, $\rho_p$, and the acceleration of gravity, $g$. **(a)** Determine the similarity requirements for the model, and the relationship between the critical velocity for model and prototype (the prediction equation). **(b)** For a length scale of $\frac{1}{2}$ and a fluid density scale of 1.0, what will be the critical velocity scale (assuming all similarity requirements are satisfied)?

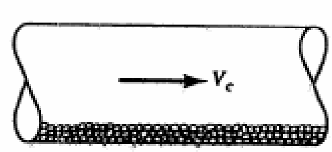



FIGURE P7.47

5.33

5.33 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.33. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the $x$ and $y$ components of the force that the pipe exerts on the tee.

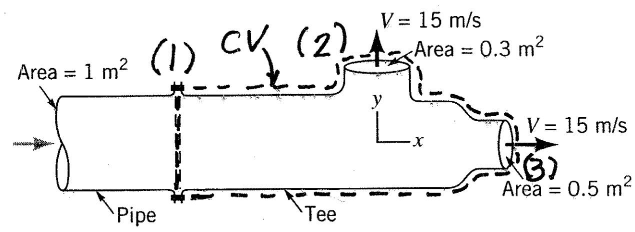


FIGURE P5.33

Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee we use the x-component of the linear momentum equation.

$$-V_1\rho V_1 A_1 + V_3\rho V_3 A_3 = p_1 A_1 - p_3 A_3 - p_{atm}(A_1 - A_3) + F_x$$

$$= (p_{1\,gage} + p_{atm})A_1 - (p_{3\,gage} + p_{atm})A_3 - p_{atm}(A_1 - A_3) + F_x$$

$$= p_{1\,gage} A_1 + F_x \qquad (1)$$

To get $V_1$ we use conservation of mass

$$Q_1 = Q_2 + Q_3$$

or $A_1 V_1 = A_2 V_2 + A_3 V_3$

so $V_1 = \dfrac{A_2 V_2 + A_3 V_3}{A_1} = \dfrac{(0.3\,m^2)(15\,\frac{m}{s}) + (0.5\,m^2)(15\,\frac{m}{s})}{1\,m^2} = 12\,\frac{m}{s}$

To estimate $p_{1\,gage}$ we use Bernoulli's equation for flow between (1) and (2)

$$\frac{p_{1\,gage}}{\rho} + \frac{V_1^2}{2} = \frac{p_{2\,gage}}{\rho} + \frac{V_2^2}{2}$$

$$p_{1\,gage} = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \left(999\,\frac{kg}{m^3}\right)\frac{\left[\left(15\,\frac{m}{s}\right)^2 - \left(12\,\frac{m}{s}\right)^2\right]}{2}\left(\frac{1\,N\cdot s^2}{kg\cdot m}\right)$$

$$p_{1\,gage} = 40{,}500\,\frac{N}{m^2}$$

Now using Eq. (1) we get:

$$\left[-\left(12\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(12\,\frac{m}{s}\right)(1\,m^2) + \left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.5\,m^2)\right]\left(\frac{1\,N\cdot s^2}{kg\cdot m}\right) =$$

$$\left(40{,}500\,\frac{N}{m^2}\right)(1\,m^2) + F_x$$

or $-72{,}000\,N = F_x$

so $\underline{\underline{F_x = 72{,}000\,N}} \leftarrow$

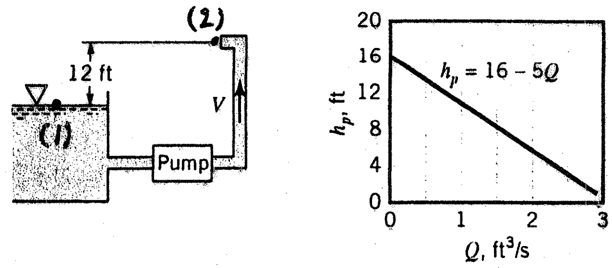
For the y component of the force exerted by the pipe on the tee we use the y component of the linear momentum equation to get

$$V_2\rho V_2 A_2 = F_y$$

$$\left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.3\,m^2) = \underline{\underline{67{,}400\,N}} \uparrow = F_y$$

5.114

**5.114** Water is pumped through a 4-in.-diameter pipe as shown in Fig. P5.114*a*. The pump characteristics (pump head versus flowrate) are given in Fig. P5.114*b*. Determine the flowrate if the head loss in the pipe is $h_L = 8V^2/2g$.



(*a*) (*b*)

**FIGURE P5.114**

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\text{, where } p_1 = p_2 = 0,\ z_1 = 0,\ z_2 = 12\text{ ft},$$
$V_1 = 0$, and $V_2 = Q/A_2$

Thus,

$$h_s - h_L = z_2 + \frac{V_2^2}{2g}\text{, with}$$

$$h_s = h_p = 16 - 5Q \text{ and } h_L = 8\frac{V_2^2}{2g} = 8\frac{Q^2}{2gA_2^2}$$

Therefore,

$$16 - 5Q - \frac{4Q^2}{gA_2^2} = 12 + \frac{Q^2}{2gA_2^2}$$

or

(1) $$\left(\frac{9}{2gA_2^2}\right)Q^2 + (5)Q - 4 = 0\text{, where } g \sim \frac{\text{ft}}{\text{s}^2},\ A_2 \sim \text{ft}^2\text{, and } Q \sim \frac{\text{ft}^3}{\text{s}}$$

Using the given data, Eq. (1) becomes

$$\left[\frac{9}{2(32.2)\left(\frac{\pi}{4}\left(\frac{4}{12}\right)^2\right)^2}\right]Q^2 + 5Q - 4 = 0$$

or

(2) $$18.35\,Q^2 + 5Q - 4 = 0$$

The positive root of Eq. (2) is $\underline{\underline{Q = 0.350\ \frac{\text{ft}^3}{\text{s}}}}$

(The negative root of Eq. (2) has no physical meaning.)

6.78

**6.78** A fluid of density $\rho$ flows steadily *downward* between the two vertical infinite, parallel plates shown in the figure for Problem 6.77. The flow is fully developed and laminar. Make use of the Navier–Stokes equation to determine the relationship between the discharge and the other parameters involved, for the case in which the change in pressure along the channel is zero.

See solution for Problem 6.83 to obtain

$$q = -\frac{2}{3}\frac{\mathcal{P} h^3}{\mu}$$

where $q$ is the discharge per unit width and $\mathcal{P} = \frac{\partial p}{\partial y} + \rho g$. Thus,

$$\frac{\partial p}{\partial y} + \rho g = -\frac{3}{2}\frac{\mu q}{h^3}$$

or

$$\frac{\partial p}{\partial y} = -\frac{3}{2}\frac{\mu q}{h^3} - \rho g$$

For $\frac{\partial p}{\partial y} = 0$

$$\underline{\underline{q = -\frac{2}{3}\frac{\rho g h^3}{\mu}}}$$

(Note: The negative sign indicates that the direction of flow must be downward to create a zero pressure gradient.)

7.47

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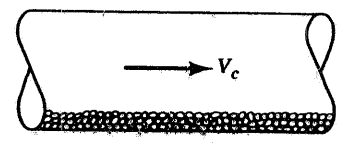


**FIGURE P7.47**

(a)
$$V_c = f(D, d, \rho, \mu, \rho_p, g)$$

$V_c \doteq LT^{-1}$ $D \doteq L$ $d \doteq L$ $\rho \doteq FL^{-4}T^2$ $\mu \doteq FL^{-2}T$ $\rho_p \doteq FL^{-4}T^2$ $g \doteq LT^{-2}$

From the pi theorem, $7-3=4$ pi terms required, and a dimensional analysis yields

$$\frac{\rho V_c D}{\mu} = \phi\left(\frac{d}{D}, \frac{\rho}{\rho_p}, \frac{g d^3 \rho^2}{\mu^2}\right)$$

Thus, the similarity requirements are

$$\underline{\underline{\frac{d_m}{D_m} = \frac{d}{D}}} \qquad \underline{\underline{\frac{\rho_m}{\rho_{pm}} = \frac{\rho}{\rho_p}}} \qquad \underline{\underline{\frac{g_m d_m^3 \rho_m^2}{\mu_m^2} = \frac{g d^3 \rho^2}{\mu^2}}}$$

The prediction equation is

$$\underline{\underline{\frac{\rho V_c D}{\mu} = \frac{\rho_m V_{cm} D_m}{\mu_m}}}$$

(b) If all similarity requirements are satisfied, the prediction equation indicates that

$$\frac{V_{cm}}{V_c} = \frac{\rho}{\rho_m}\frac{\mu_m}{\mu}\frac{D}{D_m} = (1.0)\left(\frac{\mu_m}{\mu}\right)(2) = 2\,\frac{\mu_m}{\mu} \qquad (1)$$

From the third similarity requirement (with $g = g_m$),

$$\frac{\mu_m}{\mu} = \sqrt{\left(\frac{d_m}{d}\right)^3\left(\frac{\rho_m}{\rho}\right)^2} = \sqrt{\left(\frac{1}{2}\right)^3(1.0)^2} = \sqrt{\frac{1}{8}}$$

Thus, from Eq.(1)

$$\frac{V_{cm}}{V_c} = 2\sqrt{\frac{1}{8}} = \underline{\underline{0.707}}$$