# Chapter 5 Mass, Momentum, and Energy Equations

## 1. Reynolds Transport Theorem (RTT)

where, , , fluid velocity, velocity, and

where is outward normal vector, (- inlet, + outlet)

### For a fixed control volume, ():

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter |  |  | RTT Equation |
| Mass |  | 1 |  |
| Momentum |  |  |  |
| Energy |  |  |  |

## 2. Conservation of Mass – The Continuity Equation

### Special cases:

1. Steady flow:
2. Incompressible fluid ( =constant):
3. = constant over discrete :
4. Steady one-dimensional flow in a conduit: ⇒

⇒ if = constant, or

### Some useful definitions:

* Mass flux (mass flow rate) (if = constant , )
* Volume flux (flow rate) (if = constant, )
* Average velocity

## 3. Newton’s Second Law - Momentum Equation

where = vector sum of all external forces acting on including body forces (ex: gravity force) and surface forces (ex: pressure force, and shear forces, etc.)

### Special cases:

1. Steady flow:
2. Uniform flow across :

### Examples:

|  |  |  |  |
| --- | --- | --- | --- |
| Flow type |  |  | Continuity Eq. or  Bernoulli Eq. |
| Deflecting vane |  | x-component:  y-component: |  |
| Nozzle |  | x-component:  y-component: |  |
| Bend |  | x-component:  y-component: |  |
| Sluice gate |  | x-component:  y-component: | () |

## 4. First Law of Thermodynamics - Energy Equation

where, and

or

### Simplified Form of the Energy Equation (steady, one-dimensional pipe flow):

where ,, and .

For non-uniform flows,

* pump head
* turbine head
* head loss
* : kinetic energy correction factor ( for uniform flow across )
* in energy equation refers to average velocity

### Hydraulic and Energy Grade Lines

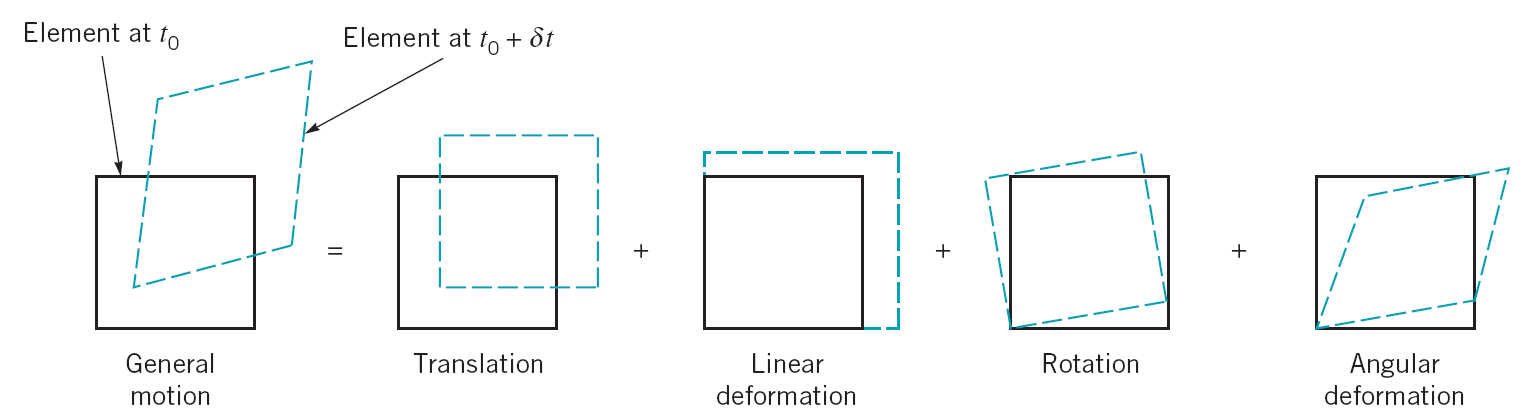
* Hydraulic Grade Line:
* Energy Grade Line:



# Chapter 6 Differential Analysis of Fluid Flow

## 1. Fluid Element Kinematics

Fluid element motion consists of translation, linear deformation, rotation, and angular deformation.



* Linear deformation(dilatation): ⇒ if the fluid is incompressible,
* Rotation(vorticity): ⇒ if the fluid is irrotational,
* Angular deformation is related to shearing stress:

## 2. Mass conservation

For a steady and incompressible flow:

## 3. Momentum conservation

For Newtonian incompressible fluid the shear stress is propotional to the rate of strain, .

## 4. Navier-Stokes Equations

### 1) Cartesian coordinates

Continuity:

Momentum:

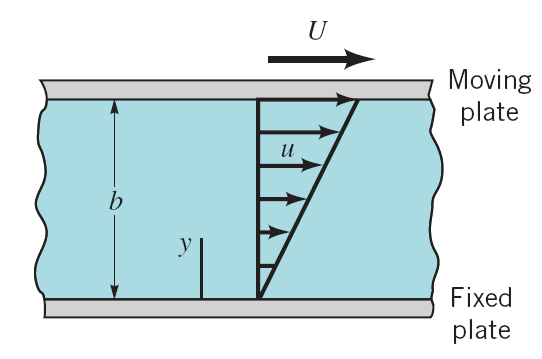
### 2) Cylindrical coordinates:

Continuity:

Momentum:

## 4. Exact solutions of NS Equations

### Ex 1) Couette Flow (without pressure gradient)

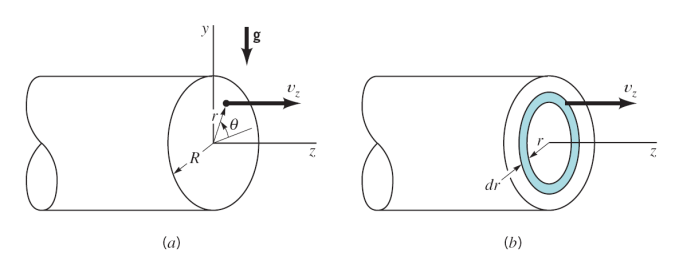
Assumptions: laminar, steady, 2-D, incompressible, ignore gravity, no pressure gradient

* Continuity:
* Momentum:
* B.C.: ,

⇒

Shear stress at the bottom wall:

## Ex 2) Circular pipe (with constant pressure gradient)

Assumptions: laminar, steady, incompressible, fully-developed, constant pressure gradient

* Continuity:
* z-Momentum:
* B.C.: , ,

⇒

1. Flow rate:
2. Mean velocity:
3. Maximum velocity:

# Chapter 7 Dimensional Analysis and Modeling

## 1. Buckingham Pi Theorem

For any physically meaningful equation involving variables, such as

with minimum number of reference dimensions , the equation can be rearranged into product of pi terms.

### Example – Exponent method:

where, ; ; ; ; . Then, the number of pi terms = .

It follows that

(for )

(for )

(for )

so that , , , and therefore

It follows that

Similarly for ,

Then,

## 2. Common Dimensionless Parameters for Fluid Flow Problems.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Variable | velocity | density | gravity | viscosity | Surface  tension | compressibility | Pressure change | Length |
| Symbol |  |  |  |  |  |  |  |  |
| Unit (SI) |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| Dimensionless Groups | Symbol | Definition | Interpretation |
| Reynolds number |  |  |  |
| Froude number |  |  |  |
| Weber number |  |  |  |
| Mach number |  |  |  |
| Euler number |  |  |  |

## 3. Similarity and Model Testing

If all relevant dimensionless parameters have the same corresponding values for model and prototype, flow conditions for a model test are completely similar to those for prototype.

### Model Testing

1) Fr similarity

⇒ Froude scaling, where

2) Re similarity

⇒