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1. Mass and momentum conservation

The water jet in Fig. P3.40 strikes normal to a fixed plate. Neglect gravity and friction, and compute the force *F* in newtons required to hold the plate fixed.

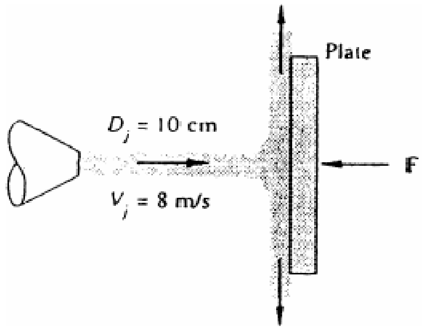

Fig. P3.40

2. Energy conservation

Turbines convert the energy contained within a fluid into mechanical energy or shaft work. Turbines are often used in power plants with generators to produce electricity. One such installation is in a dam as shown in Figure 3.20. Water is permitted to flow through a passageway to the turbine, after which the water drains downstream. For the data given in the figure determine the power available to turbine when the discharge at the outlet is 30 m^3/s.

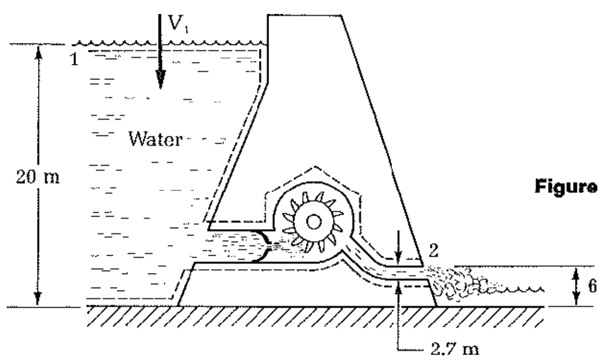

Figure 3.20.

3. Differential analysis

An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in Fig.6.92. The two plates move in opposite directions with constant velocities, U_1 and U_2, as shown. The pressure gradient in the *x* direction is zero and the only body force is due to the fluid weight. Use the Navier-Stokes equations to derive an expression for the velocity distribution between the plates. Assume laminar.

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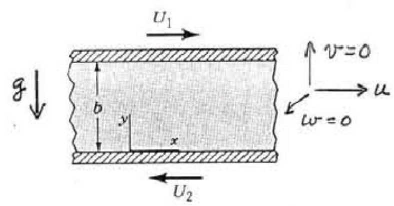

FIGURE P6.92

4. Dimensional analysis

Molasses is stored in a tank 15 m in diameter. It leaves the tank under the action of gravity through a pipe at the tank bottom and flows to a bottling machine. It is imperative that no air be allowed in the system, which is a possibility when the level of molasses in the tank is so low that a vortex forms over the discharge pipe (Fig. 4.5). A study is made of the system with a one-fifth scale model using water as the fluid medium. Measurements made on the model indicate that when h = 8 cm a vortex forms. What is the corresponding depth on the prototype? Take the density of molasses to be 1300 kg/m^3.

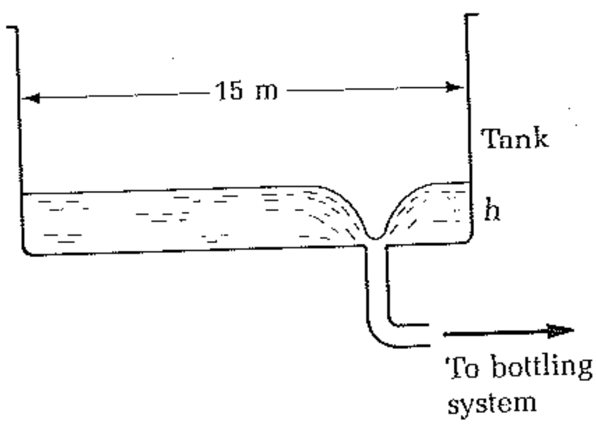

Figure 4.5. S
for Example

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Solutions

1.

$$\sum F_x = -F = \dot{m}_{up}u_{up} + \dot{m}_{down}u_{down} - \dot{m}_j u_j$$
$$= -\dot{m}_j u_j, \quad \dot{m}_j = \rho A_j V_j$$
$$\text{Thus} \quad F = \rho A_j V_j^2 = (998)\pi(0.05)^2(8)^2 \approx \mathbf{500\ N} \leftarrow \quad Ans.$$

2.

$$-\frac{dW}{dt} = \dot{m}\left[\left(\frac{p_2}{\rho} + \frac{V_2^2}{2g_c} + \frac{gz_2}{g_c}\right) - \left(\frac{p_1}{\rho} + \frac{V_1^2}{2g_c} + \frac{gz_1}{g_c}\right)\right]$$

$$\dot{m} = \rho A_1 V_1 = \rho A_2 V_2 = \rho Q = (1\ 000\ \text{kg/m}^3)(30\ \text{m}^3/\text{s}) = 30\ 000\ \text{kg/s}$$
$$p_1 = p_2 = p_{atm}$$
$$V_2 = \frac{Q}{A_2} = \frac{30}{\pi(2.7)^2/4}\ \text{m/s} = 5.24\ \text{m/s}$$

Compared to the velocity at 2, $V_1 \approx 0$. Moreover, $z_1 = 20$ m and $z_2 = 6$ m. By substitution,

$$-\frac{dW}{dt} = \left[\frac{(5.24)^2}{2} + 9.81(6) - 9.81(20)\right](30\ 000\ \text{kg/s})$$
$$= (-123.6\ \text{N-m/kg})(30\ 000\ \text{kg/s})$$

or

$$\frac{dW}{dt} = 3\ 708\ 336\ \text{N-m/s} = +3.7\ \text{MW}$$

3.

For the specified conditions, $v = 0$, $w = 0$, $\frac{\partial p}{\partial x} = 0$, and $g_x = 0$, so that the x-component of the Navier-Stokes equations (Eq. 6.127a) reduces to

$$\frac{d^2u}{dy^2} = 0 \qquad (1)$$

Integration of Eq. (1) yields

$$u = C_1 y + C_2 \qquad (2)$$

For $y = 0$, $u = -U_2$ and therefore from Eq. (2)

$$C_2 = -U_2$$

For $y = b$, $u = U_1$ so that

$$U_1 = C_1 b - U_2$$

or

$$C_1 = \frac{U_1 + U_2}{b}$$

Thus,

$$u = \left(\frac{U_1 + U_2}{b}\right) y - U_2$$

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4.

$$\frac{V_M}{\sqrt{gl_M}} = \frac{V}{\sqrt{gl}} \qquad \frac{V^2}{V_M^2} = \frac{\ell}{\ell_M}$$

Because the scale ratio is 1/5,

$$\frac{V}{V_M} = \sqrt{5}$$

and

$$\frac{A}{A_M} = \frac{\ell^2}{\ell_M^2} = 25$$

The mass flow rates in the discharge pipes are given by

$$\dot{m} = \rho AV$$
$$\dot{m}_M = \rho_M A_M V_M$$

Dividing, we obtain

$$\frac{\dot{m}}{\dot{m}_M} = \frac{\rho}{\rho_M}\frac{A}{A_M}\frac{V}{V_M}$$

It is known that the discharge pipe mass flow rate in model or in prototype is a function of pressure at the tank bottom, which in turn (from hydrostatics) is a function of height. Therefore

$$\frac{\dot{m}}{\dot{m}_M} = \frac{\rho h}{\rho_M h_M} = \frac{\rho}{\rho_M}\frac{A}{A_M}\frac{V}{V_M}$$

By substitution we obtain

$$\frac{h}{h_M} = (25)(\sqrt{5}) = 55.9$$

In the model, a vortex is formed when $h_M = 8$ cm. Therefore in the prototype

$$h = 55.9(8\text{ cm}) = 447.2\text{ cm}$$
$$h = 4.47\text{ m}$$