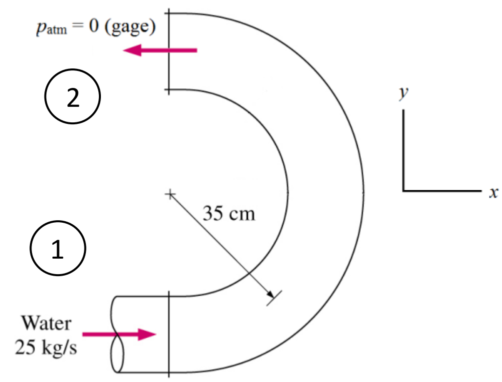
EXAM2 Solutions

Problem 1: Momentum equation (Chapter 5)

Information and assumptions

- $\rho = 1000\ kg/m^3$
- $\dot{m} = 25\ kg/s$
- $A = 10\ cm$
- $z_2 - z_1 = 70\ cm$
- Frictionless flow
- Weight of water and pipe is negligible
- Pressure at section (2), $p_2 = 0$



Find

- Determine inlet and outlet velocities V_1 and V_2
- Determine gage pressure at the inlet p_1
- The horizontal component of the anchoring force, F_x

Solution

(a) Continuity:

$$\dot{m}_1 = \dot{m}_2$$

Where $\dot{m} = V\rho A$
Therefore

$$V_1 = V_2 = V$$

Solving for V

$$V = \frac{\dot{m}}{\rho A}$$

(+2.5 point)

$$V = \frac{25}{(1000)\left(\frac{\pi(0.1)^2}{4}\right)} = 3.18\ m/s$$

(+0.5 point)

(b) Bernoulli equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

where, p_2 = 0, $V_1 = V_2$, and z_2 - z_1 = 0.7 m, thus

$$p_1 = \gamma(z_2 - z_1)$$

(+2.5 point)

$$p_1 = (1000)(9.81)(0.7) = 6867\ Pa$$

(+0.5 point)

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(c) x-momentum:

$$F_x + p_1 A_1 = \dot{m}(-V_2) - \dot{m}(V_1) = -2\dot{m}V$$

or

$$F_x = -2\dot{m}V - p_1 A_1$$

(+3.5 points)

$$F_x = -(2)(25)(3.18) - (6867)\left(\frac{\pi(0.1)^2}{4}\right) = -213\ N$$

(+0.5 point)

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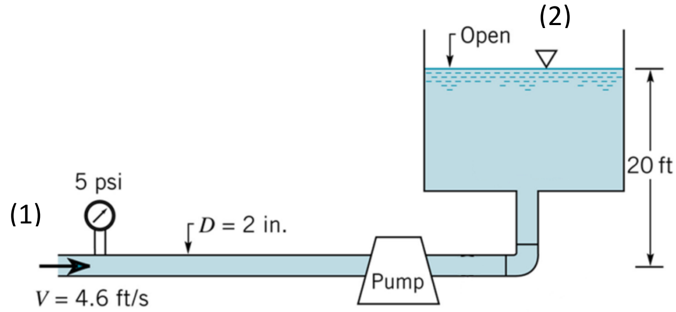
Problem 2: Energy equation (Chapter 5)

Information and assumptions

- Oil, $\gamma = 48.0\ lb/ft^3$, $\mu = 2.0 * 10^{-5}\ lb.s/ft^2$
- Head loss $h_L = 41.3\ \bar{V}^2/2g$
- Flow rate $Q = 1{,}600\ gal/min$
- kinetic energy correction factor α is 1
- $1\ hp = 550\ ft.lb/s$

Find

- Pump power $\dot{W}_p$



Solution

Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Where, $z_1 - z_2 = 20\ ft$, $p_2 = 0$, $V_2 = 0$, $h_t = 0$.

$$h_p = -\frac{p_1}{\gamma} - \frac{V_1^2}{2g} + (z_2 - z_{1)} + 41.3\frac{\bar{V}^2}{2g}$$

(+6.5 points)

$$h_p = -\frac{(5)(144)}{48} - \frac{(4.6)^2}{(2)(32.2)} + 20 + 41.3\frac{(4.6)^2}{(2)(32.2)} = 18.24\ ft$$

(+0.5 points)

Thus,

$$\dot{W}_p = \gamma Q h_p = \gamma (VA) h_p$$

(+2.5 points)

$$\dot{W}_p = (48)(4.6)\frac{(\pi)(2/12)^2}{4}(18.24) = 87.86\ \frac{ft \cdot lbf}{s} = 0.16\ hp$$

(+0.5 point)

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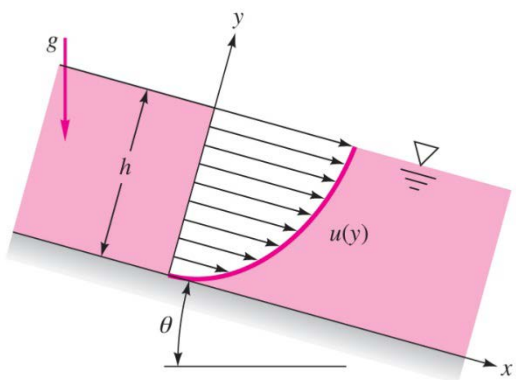
Problem 3: Exact solution of NS equation (Chapter 6)

Information and assumptions

- Pressure gradient in x-direction, $\frac{\partial p}{\partial x} = 0$
- Steady, fully-developed, one dimensional and laminar

Find

- Velocity profile function $u(y)$
- If the liquid is SAE 30W oil (ρ = 891 kg/m^3 and μ = 0.28 kg/m·s), h = 10 mm, and θ = 30°, determine the flow rate per unit width (into the paper)



Solution

(a) Navier-Stokes equation:

$$\rho\left(\underbrace{\frac{\partial u}{\partial t}}_{\substack{=0\\ steady}} + u\underbrace{\frac{\partial u}{\partial x}}_{\substack{=0\\ continuity}} + \underbrace{v}_{=0}\frac{\partial u}{\partial y} + \underbrace{w}_{=0}\frac{\partial u}{\partial z}\right) = -\underbrace{\frac{\partial p}{\partial x}}_{=0} + \rho g_x + \mu\left(\underbrace{\frac{\partial^2 u}{\partial x^2}}_{\substack{=0\\ continuity}} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\frac{\partial^2 u}{\partial z^2}}_{\substack{infinite\\ plate}}\right)$$

Thus,

$$0 + u \times 0 + 0 \times \frac{\partial u}{\partial y} + 0 \times \frac{\partial u}{\partial z} = -0 + \rho g \sin\theta + \mu \frac{\partial^2 u}{\partial y^2}$$

or

$$\frac{d^2 u}{dy^2} = -\frac{\rho g}{\mu}\sin\theta$$

Integration yields,

$$\frac{du}{dy} = -\frac{\rho g}{\mu}\sin\theta\, y + C_1$$

and

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$$u = -\frac{\rho g}{\mu}\sin\theta\frac{y^2}{2} + C_1 y + C_2$$

B.C.
$u = 0$ at $y = 0$,

$$0 = 0 + 0 + C_2$$

$\frac{du}{dy} = 0$ at $y = h$,

$$0 = -\frac{\rho g}{\mu}\sin\theta\, h + C_1$$

Thus,

$$u = \frac{\rho g}{\mu}\sin\theta\left(hy - \frac{y^2}{2}\right)$$

(+7 points)

(b) Flow rate per unit width

$$q = \int_0^h \frac{\rho g}{\mu}\sin\theta\left(hy - \frac{y^2}{2}\right)dy = \frac{\rho g h^3 \sin\theta}{3\mu}$$

(+2.5 points)

$$q = \frac{(891)(9.81)(0.01)^3(\sin 30°)}{(3)(0.28)} = 0.0052\, m^2/s$$

(+0.5 point)

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Problem 4: Dimensional analysis (Chapter 7)

Information and assumptions

- $D = f(d, \rho, V)$
- $d_m = 0.3\,ft,\ \gamma_m = 53\frac{lb}{ft^3}, V_m = 5\,ft/s, D_m = 1.4\,lb$
- $d = 2\,ft,\ \gamma = 1.94\,slug/ft^3, V = 3\,ft/s$

Find

- Find dimensionless parameters.
- Find drag on the prototype.

Solution

(a)

$$D = f(d, \rho, V)$$

where $D \doteq F$, $d \doteq L$, $\rho \doteq FL^{-4}T^2$, and $V \doteq LT^{-1}$

Thus $k - r = 4 - 3 = 1$ pi term required. By choosing d, ρ, V as the repeating variables,

$$\Pi = \mathrm{D}\mathrm{d}^{\mathrm{a}}\rho^b V^c$$

and in terms of dimensions

$$(F)(L)^a(FL^{-4}T^2)^b(LT^{-1})^c \doteq F^0L^0T^0$$

To be dimensionless it follows that

$$\begin{aligned} F&: \quad 1 + b &= 0 \\ L&: \quad a - 4b + c &= 0 \\ T&: \quad 2b - c &= 0 \end{aligned}$$

therefore, a = -2, b = -1, c = -2. The pi term then becomes

$$\frac{D}{\rho V^2 d^2} = \Pi_1 = C_D$$

(+7 point)

(b)

$$\frac{D}{\rho V^2 d^2} = \frac{D_m}{\rho_m V_m^2 d_m^2}$$

or

$$D = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{d}{d_m}\right)^2 D_m$$

(+2.5 point)

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$$\therefore D = \left(\frac{1.94}{53/32.2}\right)\left(\frac{3}{5}\right)^2\left(\frac{2}{0.3}\right)^2(1.4) = 26.4\ lb$$

(+0.5 point)