EXAM2 Solutions

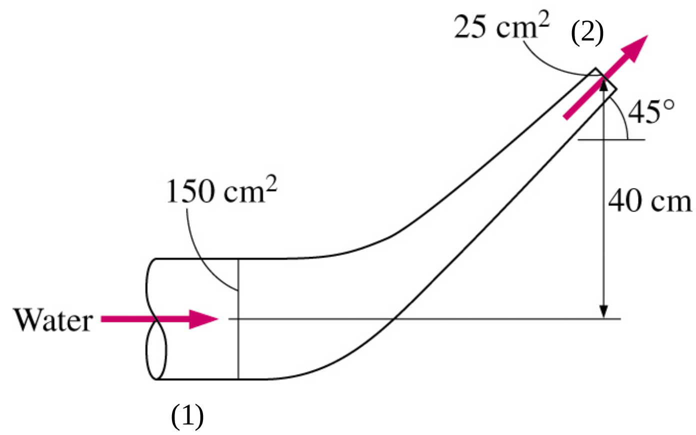
Problem 1: Momentum equation (Chapter 5)

Information and assumptions

- $\rho = 998\, kg/m^3$
- Flow rate, $Q = 0.03\, m^3/s$
- Frictionless flow
- Pressure at section (2), $p_2 = 0$

Find

- Determine the mass flow rate and water velocities at sections (1) and (2)
- The pressure at section (1)
- The horizontal component of the anchoring force, F_{Ax}



Solution

Given:
$Q = 0.03\, m^3/s;\ A_1 = 150\, cm^2 = 0.015\, m^2;\ A_2 = 0.0025\, m^2;\ p_2 = 0(gage)$

(a) Continuity:

$$\dot{m} = \rho Q = \left(998\frac{kg}{m^3}\right)\left(0.03\frac{m^3}{s}\right) = \mathbf{30\, kg/s}$$

(+1 point)

$$V_1 = \frac{Q}{A_1} = \frac{0.03\, m^3/s}{0.015\, m^2} = \mathbf{2\, m/s}; \qquad V_2 = \frac{Q}{A_2} = \frac{0.03\, m^3/s}{0.0025\, m^2} = \mathbf{12\, m/s}$$

(+2 points)

(b) Bernoulli equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

$$p_1 = p_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) + \gamma(z_2 - z_1)$$

(+2 points)

$$= (0) + \frac{1}{2}\left(998\frac{kg}{m^3}\right)\left(\left(12\frac{m}{s}\right)^2 - \left(2\frac{m}{s}\right)^2\right) + \left(9790\frac{N}{m^3}\right)(0.4\, m)$$

$$\therefore \mathbf{p_1 = 74\, kPa\, (gage)}$$

(+1 point)

(b) x-momentum:

$$F_{Ax} + p_1 A_1 - p_2 A_2 = \left(-\underbrace{\rho V_1 A_1}_{\dot{m}}\right)(V_1) + \left(\underbrace{\rho V_2 A_2}_{\dot{m}}\right)(V_2 \cos 45^\circ)$$

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or

$$F_{Ax} = \dot{m}(V_2 \cos 45° - V_1) - p_1 A_1$$

(+3 points)

$$= \left(30 \frac{kg}{s}\right)\left(\left(12 \frac{m}{s}\right)\cos 45° - \left(2 \frac{m}{s}\right)\right) - \left(74{,}000 \frac{N}{m^2}\right)(0.015\ m^2)$$

Thus,

$$\boldsymbol{F_{Ax} = 915\ N}$$

(+1 point)

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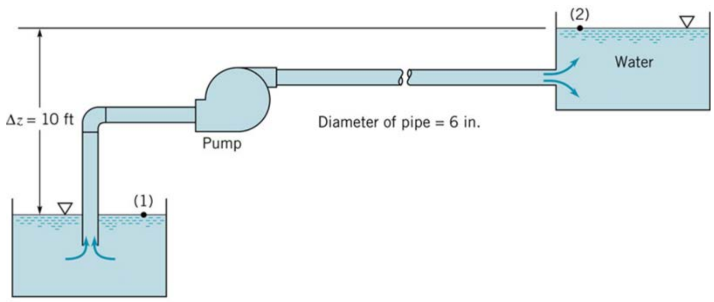
Problem 2: Energy equation (Chapter 5)

Information and assumptions

- Water, $\gamma = 62.4\ lb/ft^3$
- Head loss $h_L = 11\ \bar{V}^2/2g$
- Flow rate $Q = 1{,}600\ gal/min$
- $7.48\ gal = 1\ ft^3$
- $1\ hp = 550\ ft.lb/s$
- $g = 32.2\ ft^2/s$

Find

- The average velocity $\bar{V}$
- The head loss h_L
- Pump power $\dot{W}_p$



Solution

(a) Average velocity

$$Q = \left(1600\,\frac{gal}{min}\right)\Big/\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right) = 3.565\ ft^3/s$$

$$\bar{V} = \frac{Q}{A} = \frac{3.565\ ft^3/s}{\frac{\pi}{4}\left(\frac{6}{12}\ ft\right)^2} = \mathbf{18.16\ ft/s}$$

(+2 points)

(b) Head loss

$$h_L = 11\frac{\bar{V}^2}{2g} = (11)\times\frac{\left(18.16\frac{ft}{s}\right)^2}{2\times\left(32.2\frac{ft}{s^2}\right)} = \mathbf{56.3\ ft}$$

(+2 points)

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(c) Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

where, $p_1 = p_2 = 0$; $V_1 = V_2 = 0$; $z_2 - z_1 = 10\ ft$; $h_t = 0$; $h_L = 56.3\ ft$.

(+4 points)

Thus,

$$0 + 0 + 0 + h_p = 0 + 0 + (10\ ft) + 0 + (56.3\ ft)$$

or

$$h_p = 10\ ft + 56.3\ ft = \mathbf{66.3\ ft}$$

(+1 point)

Pump power:

$$\dot{W}_p = \gamma Q h_p = \left(62.4\frac{lb}{m^3}\right)(3.565\ \ ft^3/s)(66.3\ ft)\left(\frac{1hp}{550\ ft \cdot \frac{lb}{s}}\right)$$

$$\therefore \dot{\boldsymbol{W}}_{\boldsymbol{p}} = \mathbf{26.8\ hp}$$

(+1 point)

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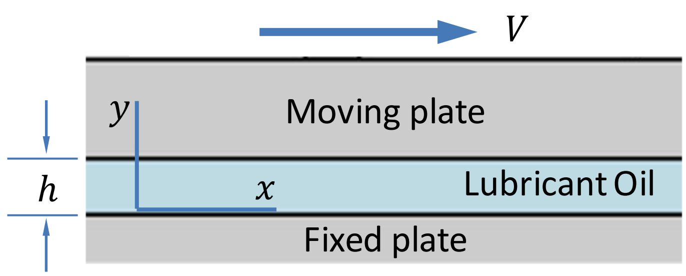
Problem 3: Exact solution of NS equation (Chapter 6)

Information and assumptions

- Pressure gradient in x-direction, $\frac{\partial p}{\partial x} = 0$
- Steady, fully-developed, one dimensional and laminar

Find

- Velocity distribution between the plates
- The shear stress on the bottom fixed wall surface if $\mu = 0.38\ N.s/m^2, V = 0.1 m/s$ and $h = 0.1\ mm$



Solution

(a) Navier-Stokes equation:

$$\underbrace{\frac{\partial u}{\partial t}}_{\substack{=0\\ steady}} + u\underbrace{\frac{\partial u}{\partial x}}_{\substack{=0\\ continuity}} + \underbrace{v}_{=0}\frac{\partial u}{\partial y} + \underbrace{w}_{=0}\frac{\partial u}{\partial z} = -\underbrace{\frac{\partial p}{\partial x}}_{=0} + \mu\left(\underbrace{\frac{\partial^2 u}{\partial x^2}}_{\substack{=0\\ continuity}} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\frac{\partial^2 u}{\partial z^2}}_{\substack{infinite\\ plate}}\right)$$

Assume the flow is steady ($\partial u/\partial t = 0$), one dimensional ($v = w = 0$ and $\partial u/\partial z = 0$), and fully-developed ($\partial u/\partial x = 0$), and with no pressure gradient, $\frac{\partial p}{\partial x} = 0$, then

$$\mathbf{0 = \mu\frac{\partial^2 u}{\partial y^2}}$$

(+5 points)

Integrate with respect to y

$$u = C_1 y + C_2$$

By using the boundary conditions $u = 0$ at $y = 0$ and $u = V$ at $y = h$,

$$\boldsymbol{u(y) = \frac{V}{h}y}$$

(+3 points)

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(b) Shear stress at the bottom wall:

$$\tau_w = \mu \left.\frac{du}{dy}\right)_{y=0} = \mu \frac{V}{h}$$

or

$$\tau_w = (0.38\ N \cdot s/m^2)\left(\frac{0.1\ m/s}{0.0001\ m}\right) = \boldsymbol{380\ Pa}$$

(+2 points)

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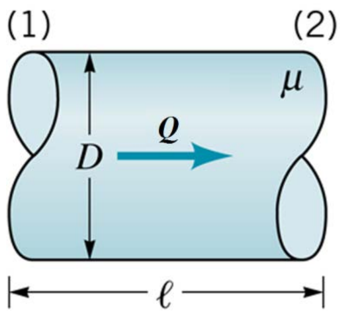
Problem 4: Dimensional analysis (Chapter 7)

Information and assumptions

- $\Delta p_\ell = f(Q, D, \mu)$

Find

- Develop a dimensionless parameters.



$$\Delta p_\ell = (p_1 - p_2)/\ell$$

Solution

$$\Delta p_\ell = f(Q, D, \mu)$$

where,

$$\Delta p_\ell \doteq ML^{-2}T^{-2};\ Q \doteq L^3T^{-1};\ D \doteq L;\ \mu \doteq ML^{-1}T^{-1}$$

(+3 points)

From the Buckingham Pi theorem, $k - r = 4 - 3 = 1$ pi term is needed. By choosing Q, D, and μ as the repeating variables,

$$\Pi = \Delta p_\ell Q^a D^b \mu^c$$

(+3 points)

and in terms of dimensions

$$(ML^{-2}T^{-2})(L^3T^{-1})^a(L)^b(ML^{-1}T^{-1})^c \doteq M^0L^0T^0$$

To be dimensionless it follows that

$$\begin{aligned} M&: & 1 + c &= 0 \\ L&: & -2 + 3a + b - c &= 0 \\ T&: & -2 - a - c &= 0 \end{aligned}$$

therefore, a = -1, b = 4, c = -1. The pi term then becomes

(+3 points)

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$$\therefore \boldsymbol{\Pi} = \frac{\boldsymbol{\Delta p_\ell D^4}}{\boldsymbol{\mu Q}}$$

(+1 point)

Alternate approach:

$$\Delta p_\ell = f(Q, D, \mu)$$

where,

$$\Delta p_\ell \doteq FL^{-3};\ Q \doteq L^3T^{-1};\ D \doteq L;\ \mu \doteq FL^{-2}T$$

From the Buckingham Pi theorem, $k - r = 4 - 3 = 1$ pi term is needed. By choosing Q, D, and μ as the repeating variables,

$$\Pi = \Delta p_\ell Q^a D^b \mu^c$$

and in terms of dimensions

$$(FL^{-3})(L^3T^{-1})^a(L)^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

To be dimensionless it follows that

$$\begin{aligned} F&: \quad 1 + c &= 0 \\ L&: \quad -3 + 3a + b - 2c &= 0 \\ T&: \quad -a + c &= 0 \end{aligned}$$

therefore, a = -1, b = 4, c = -1. The pi term then becomes

$$\therefore \Pi = \frac{\Delta p_\ell D^4}{\mu Q}$$