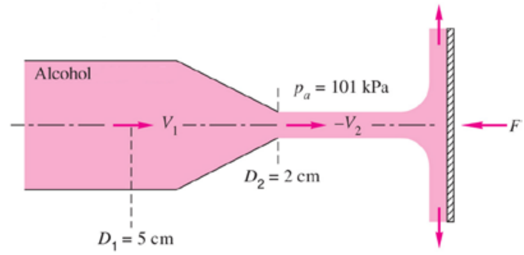
EXAM2 Solutions

Problem 1: Momentum equation (Chapter 5)

Information and assumptions



- p_1 = 760 kPa; $p_2 = p_a$ = 101 kPa
- D_1 = 5 cm; D_2 = 2 cm
- ρ = 788.42 kg/m^3 for alcohol
- No losses in the nozzle flow

Find

- Alcohol jet velocity V_2 and the force F required to hold the plate stationary

Solution

(a) As no losses in the nozzle flow, one can apply the Bernoulli equation between the sections 1 and 2 to find V_2. By knowing that $V_1 = (A_2/A_1)V_2 = (D_2/D_1)^2 V_2$ from the continuity equation and that $z_1 = z_2$, the Bernoulli equation can be written as

$$\frac{p_1}{\rho} + \frac{1}{2}\left(\left(\frac{D_2}{D_1}\right)^2 V_2\right)^2 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 \qquad \text{(+3 points)}$$

or

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (D_2/D_1)^4]}}$$

Thus,

$$V_2 = \sqrt{\frac{2(760{,}000 \text{ Pa} - 101{,}000 \text{ Pa})}{\left(788.42 \dfrac{\text{kg}}{\text{m}^3}\right)[1 - (0.02 \text{ m}/0.05 \text{ m})^4]}} = 41.42 \text{ m/s} \qquad \text{(+2 point)}$$

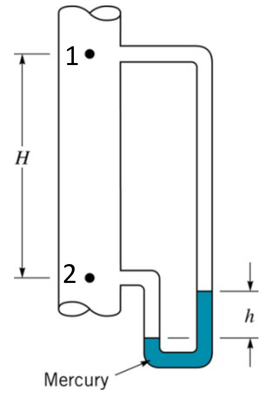
(b) For a control volume that encompasses the jet from section 2 and the plate, a momentum analysis gives

$$-F = -\dot{m}(V_2) + 0 = -\rho A_2 V_2^2 \qquad \text{(+4 points)}$$

Thus,

$$F = \left(788.42 \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\pi}{4}\right)(0.02 \text{ m})^2 \left(41.42 \frac{\text{m}}{\text{s}}\right)^2 = \mathbf{425\ N} \qquad \text{(+1 point)}$$

EXAM2   Solutions



Problem 2: Energy equation (Chapter 5)

Information and assumptions

- H = 50 cm; h = 5 cm
- SG = 13.6 for mercury

Find

- Head loss h_L between the two points and the flow direction

Solution

(a) Assume a downward flow and apply the energy equation between the two points. By knowing that $V_1 = V_2$ from the continuity equation and without a pump or a turbine, the energy equation can be written as

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + h_L \qquad (+4 \text{ points})$$

With $z_1 - z_2 = H$ from the given condition, the head loss

$$h_L = \frac{(p_1 - p_2)}{\gamma} + H \qquad (+3 \text{ points})$$

However, the manometer equation yields

$$p_1 - p_2 = \gamma h(1 - SG) - \gamma H$$

where, SG = 13.6 is the specific gravity for mercury, and thus,

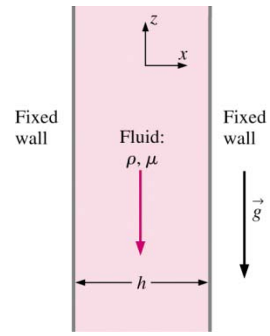
$$h_L = h(1 - SG) = (0.05\text{ m})(1 - 13.6) = -0.63\text{ m} \qquad (+2 \text{ points})$$

(b) The flow is ***upward*** as a negative loss is not physically possible (+1 point).

EXAM2 Solutions

Problem 3: Exact solution of NS equation (Chapter 6)

Information and assumptions



- Steady, incompressible, parallel, laminar flow
- Two infinite, vertical walls
- No forced pressure ($\partial p/\partial z = 0$)
- Fluid falls by gravity alone (g_z = -g)
- Flow is purely 2-D ($v = 0$ and $\partial/\partial y = 0$) and parallel ($u = 0$)
- h = 2 mm; ρ = 1,260 kg/m^3; μ = 1.49 N·s/m^2

Find

- (a) An expression for w and (b) w at x = 0

Solution

(a) For steady flow, $\partial/\partial t$ = 0. As the flow is laminar and parallel, u = v = 0. In this case $\partial w/\partial z$ = 0 from the continuity equation. For 2D flow with infinite walls, $\partial w/\partial y$ = 0 as well. With these conditions and $\partial p/\partial z = 0$ and $g_z = -g$, the Navier-Stokes equation reduces to

$$\frac{\partial^2 w}{\partial x^2} = \frac{\rho g}{\mu} \qquad \text{(+5 points)}$$

By integrating the equation twice with respect to x,

$$w = \frac{\rho g}{2\mu}x^2 + C_1 x + C_2 \qquad \text{(+2 points)}$$

where C_1 and C_2 are the integral constants. By applying the boundary conditions, $w = 0$ at $x = \pm h/2$, those constants are found to be

$$C_1 = 0; \quad C_2 = -\frac{\rho g}{8\mu}h^2$$

Thus the velocity distribution can be written as

$$w = \frac{\rho g}{2\mu}x^2 - \frac{\rho g}{8\mu}h^2$$

or

$$w = \frac{\rho g}{2\mu}\left(x^2 - \left(\frac{h}{2}\right)^2\right) \qquad \text{(+2 points)}$$

(b) At $x = 0$,

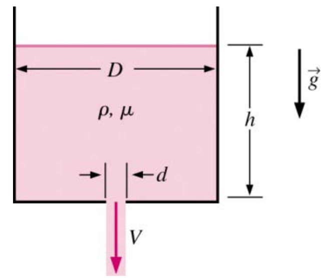
$$w = \frac{\left(1260\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{2\left(1.49\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)}\left(0 - \left(\frac{0.002\ \text{m}}{2}\right)^2\right) = \mathbf{-4.15\ mm/s} \qquad \text{(+1 point)}$$

EXAM2   Solutions

Problem 4: Dimensional analysis (Chapter 7)

Information and assumptions

- Very small exit hole compare to the tank (d << D)
- V is independent of d, D, ρ, or μ
- V depends only on g and h



Find

- (a) Dimensionless relationship for V as function of g and h and (b) the factor at which V increase when h is doubled.

Solution

(a) Given that $V = f(g, h)$, where $V \doteq LT^{-1}$, $g \doteq LT^{-2}$, and $h \doteq L$, so that $k – r$ = 3 – 2 = 1. Thus only one pi parameter is needed. By using the exponent method,

$$\Pi = V g^a h^b = (LT^{-1})(LT^{-2})^a (L)^b = M^0 L^0 T^0$$ (+5 points)

Thus,

$$\Pi = \frac{V}{\sqrt{gh}} = \text{constant}$$ (+3 points)

(b) When h is doubled, or $h_2/h_1 = 2$,

$$\frac{V_1}{\sqrt{gh_1}} = \frac{V_2}{\sqrt{gh_2}}$$

or,

$$V_2 = \sqrt{\frac{h_2}{h_1}} \cdot V_1 = \sqrt{2} \cdot V_1$$ (+2 points)

Thus, V increases by a factor of $\sqrt{2}$.