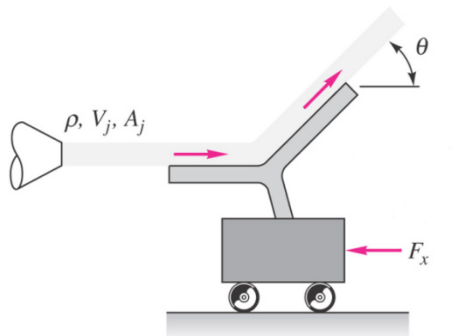
EXAM2   Solutions

Problem 1: Linear momentum equation (Chapter 5)

Information and assumptions

- ρ = 999 kg/m^3
- V_j = 30 m/s
- A_j = 0.01 m^2
- θ = 30°
- Frictionless cart
- Jet velocity magnitude remains constant along the vane surface



Find

- The restrain force F_x

Solution

For a control volume that surrounds the vane and cart, the continuity equation gives,

$$\rho V_{in} A_{in} = \rho V_{out} A_{out}$$

Since $V_{in} = V_{out} = V_j$, thus

$$A_{in} = A_{out} = A_j \qquad \text{(+1 point)}$$

The linear momentum equation in the horizontal direction is

$$\int_{CS} u\rho \underline{V} \cdot \hat{n} dA = \sum F_{CV} \qquad \text{(+2 points)}$$

or

$$-V_j \rho V_j A_j + (V_j \cos\theta)\rho V_j A_j = -F_x$$

Thus,

$$F_x = \rho A_j V_j^2 (1 - \cos\theta) \qquad \text{(+5 points)}$$

For ρ = 999 kg/m^3, V_j = 30 m/s, A_j = 0.01 m^2, and θ = 30°,

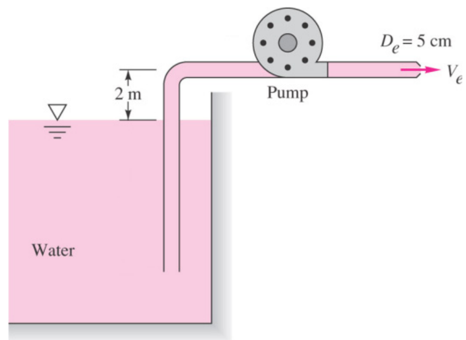
$$F_x = \left(999 \frac{\text{kg}}{\text{m}^3}\right)(0.01\ \text{m}^2)\left(30 \frac{\text{m}}{\text{s}}\right)^2 (1 - \cos 30°) = 1205\ \text{N} \qquad \text{(+2 points)}$$

EXAM2 Solutions

Problem 2: Energy equation (Chapter 5)

Information and assumptions

- Q = 220 m^3/h
- ρ = 999 kg/m^3
- Friction head loss h_L = 5 m
- Δz = 2 m
- D_e = 5 cm
- α = 1



Find

- Pump power in kW delivered to the water

Solution

Energy equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$ (+2 points)

Let "1" be at the reservoir surface and "2" be at the nozzle exit. Then,

$$p_1 = p_2 = 0 \text{ (gage)}$$

$$z_2 - z_1 = 2\ \mathrm{m}$$

$$V_1 \approx 0$$

$$V_2 = \frac{Q}{A} = \frac{Q}{\pi (D_e/2)^2} = \frac{(220\ \mathrm{m}^3/\mathrm{h})(\mathrm{h}/3600\mathrm{s})}{\pi (0.025\ \mathrm{m})^2} = 31.12\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$h_L = 5\ \mathrm{m}$$ (+4 points)

The energy equation with $\alpha_1 = \alpha_2 = 1$ becomes

$$h_p = \frac{V_2^2}{2g} + (z_2 - z_1) + h_L$$

or

$$h_p = \frac{(31.12\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} + 2\ \mathrm{m} + 5\ \mathrm{m} = 56.4\ \mathrm{m}$$ (+2 points)

The pump power is

$$P = \dot{m} h_p = \rho g Q h_p$$

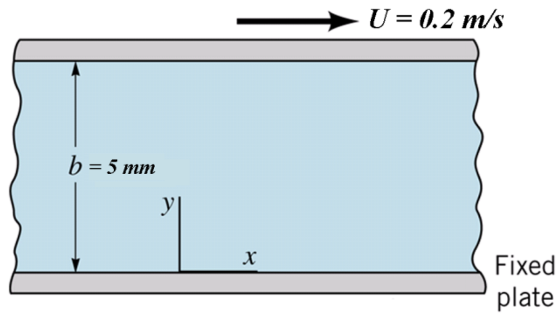
$$= \left(999\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{220\,\frac{\mathrm{m}^3}{\mathrm{h}}}{3600\,\frac{\mathrm{s}}{\mathrm{h}}}\right)(56.4\ \mathrm{m}) = 33{,}778\ \mathrm{W} = 33.8\ \mathrm{kW}$$ (+2 points)

EXAM2 Solutions

Problem 3: Exact solution (Chapter 6)

Information and assumptions

- Steady, paralle, viscous flow
- No pressure gradient in the flow direction
- U = 0.2 m/s
- b = 5 mm



Find

- Velocity distribution across the plates and flow rate between the plates per unit depth

Solution

(a) For a steady flow ($\partial/\partial t$ =0) with $v = w = 0$ and for a zero pressure gradient in the flow direction ($\partial p/\partial x = 0$), the Navier-Stokes equations (also by using the continuity equation, $\partial u/\partial x = 0$) reduce to

$$\frac{\partial^2 u}{\partial y^2} = 0$$ (+5 points)

So that

$$u = c_1 y + c_2$$

By using the boundary conditions, $u = 0$ at $y = 0$ and $u = U$ at $y = b$,

$$c_1 = \frac{U}{b}$$

$$c_2 = 0$$

Therefore

$$u = \frac{U}{b} y = \frac{0.2 \ \text{m/s}}{0.005 \ \text{m}} = 40 \cdot y \ \ (\text{m/s})$$ (+3 points)

(b) The flow rate per unit depth is

$$q = \int_0^b u dy$$

Thus,

$$q = \int_0^b \frac{U}{b} y dy = \frac{Ub}{2}$$

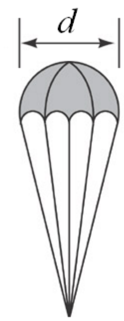
$$= \frac{(0.2 \ \text{m/s})(0.005 \ \text{m})}{2} = 5 \times 10^{-4} \ \ \text{m}^2/\text{s}$$ (+2 points)

EXAM2 Solutions

Problem 4: Dimensional analysis (Chapter 7)

Information and assumptions

- $D = f(\rho, V, d)$
- For model: D = 17 lb, d = 1 ft, V = 4 ft/s
- For prototype: d = 30 ft
- For water: ρ = 1.94 slugs/ft^3
- For air: $\rho = 2.38 \times 10^{-3}$ slugs/ft^3



Find

- Pi parameter and the drag D of the prototype

Solution

(a) From the pi theorem, 4 -3 = 1 pi term required.

$$\Pi = D \cdot \rho^a \cdot V^b \cdot d^c \qquad \text{(+2 points)}$$

$$(F)(FL^{-4}T^2)^a(LT^{-1})^b(L)^c = F^0L^0T^0$$

or

$$(MLT^{-1})(ML^{-3})^a(LT^{-1})^b(L)^c = M^0L^0T^0$$

$$\therefore \Pi = \frac{D}{\rho V^2 d^2} \qquad \text{(+3 points)}$$

(b) For similarity between model and prototype,

$$\frac{D}{\rho V^2 d^2} = \frac{D_m}{\rho_m V_m^2 d_m^2} \qquad \text{(+3 points)}$$

where, the subscript m stands for 'model'. Then,

$$D = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{d}{d_m}\right)^2 D_m$$

$$= \left(\frac{2.38 \times 10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}}{1.94\ \frac{\text{slugs}}{\text{ft}^3}}\right)\left(\frac{10\ \frac{\text{ft}}{\text{s}}}{4\ \frac{\text{ft}}{\text{s}}}\right)^2\left(\frac{30\ \text{ft}}{1\ \text{ft}}\right)^2 (17\ \text{lb}) = 117\ \text{lb} \qquad \text{(+2 points)}$$