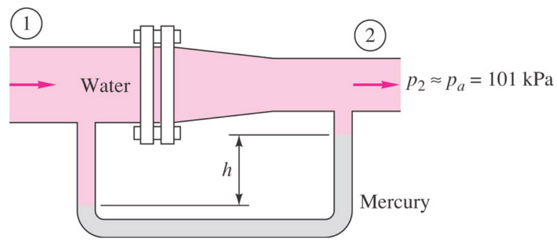
# EXAM2    Solutions

## Problem 1: Momentum equation (Chapter 5)

### Information and assumptions

- $D_1$ = 8 cm
- $D_2$ = 5 cm
- $p_2$ = 1 atm
- $V_1$ = 5 m/s
- $h$ = 58 cm
- $\rho_{mercury}$ = 13,546 Kg/m$^3$
- $\rho_{water}$ = 998 Kg/m$^3$




### Find

- The total horizontal force resisted by the flange bolts

### Solution

a) Manometer:

$$p_1 - p_2 = (\gamma_{mercury} - \gamma_{water})h = (\rho_{mercury} - \rho_{water})g \cdot h$$

$$= (13{,}546 - 998)\left(\frac{\text{Kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.58\text{ m}) = 71{,}300\text{ Pa (gage)}$$

$\therefore p_1 = 71{,}300$ Pa (gage) and $p_2 = 0$ Pa (gage) (+2 points)

b) Conservation of mass:

$$Q_1 = Q_2 = Q,\ \text{or}\ \left(5\frac{m}{s}\right)\left(\frac{\pi}{4}\right)(0.08\ m)^2 = V_2\left(\frac{\pi}{4}\right)(0.05\ m)^2$$

Solve for $V_2$

$$V_2 = 12.8\ m/s$$

$$Q = V_2 A_2 = \left(12.8\frac{m}{s}\right)\left(\frac{\pi}{4}\right)(0.05\ m)^2 = 0.0251\frac{m^3}{s}$$ (+3 points)

c) Momentum equation:

$$\sum F_x = -F_{bolt} + p_{1,gage}A_1 = (\rho_{water}Q)(V_2 - V_1)$$

$$F_{bolt} = \left(71{,}300\frac{N}{m^2}\right)\left(\frac{\pi}{4}\right)(0.08\ m)^2 - \left(998\frac{Kg}{m^3}\right)\left(0.0251\frac{m^3}{s}\right)(12.8 - 5.0)\left(\frac{m}{s}\right)$$

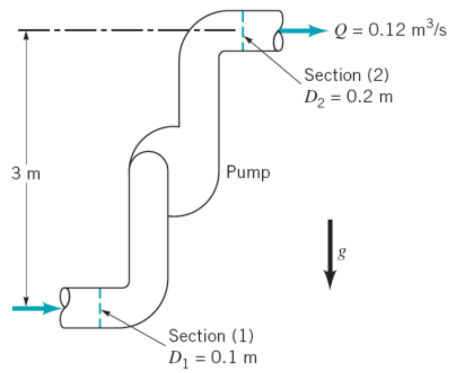
$\therefore F_{bolt} = \mathbf{163\ N}$ (+5 points)

# EXAM2 Solutions

## Problem 2: Energy equation (Chapter 5)

### Information and assumptions

- $SG = 0.68$
- $Q = 0.12\ \text{m}^3/\text{s}$
- $h_L = 0.3\,V_1^2/2g$
- $\dot{W}_p = 20$ kW
- $\rho = 999\ \text{Kg/m}^3$ for water at 20°C
- $D_1 = 0.1$ m; $D_2 = 0.2$ m
- $z_2 - z_1 = 3$ m
- $g = 9.81\ \text{m/s}^2$




### Find

- The difference in pressures between sections (1) and (2)

### Solution

a) Continuity equation

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\pi D_2^2/4} = \frac{0.12\ \text{m}^3/\text{s}}{(\pi/4)(0.2\ \text{m})^2} = 3.82\ \text{m/s}$$

$$V_1 = V_2\frac{A_2}{A_1} = V_2\left(\frac{D_2}{D_1}\right)^2 = (3.82\ \text{m/s})\left(\frac{0.2\ \text{m}}{0.1\ \text{m}}\right)^2 = 15.28\ \text{m/s}$$ (+2 points)

b) Pump head

$$h_p = \frac{\dot{W}_p}{\dot{m}g} = \frac{\dot{W}_p}{(SG\cdot\rho)gQ} = \frac{20{,}000\ \text{N}\cdot\text{m/s}}{(0.68\times 999\text{Kg/m}^3)(9.81\text{m/s}^2)(0.12\text{m}^3/\text{s})} = 25.01\ \text{m}$$ (+2 points)

c) Head loss

$$h_L = 0.3\frac{V_1^2}{2g} = \frac{(0.3)(15.28\text{m/s})^2}{(2)(9.81\text{m/s}^2)} = 3.57\ \text{m}$$ (+2 points)

d) Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \underbrace{h_t}_{=0} + h_L$$

$$p_1 - p_2 = (SG\cdot\rho)\cdot g\left[\frac{V_2^2 - V_1^2}{2g} + (z_2 - z_1) - h_p + h_L\right]$$

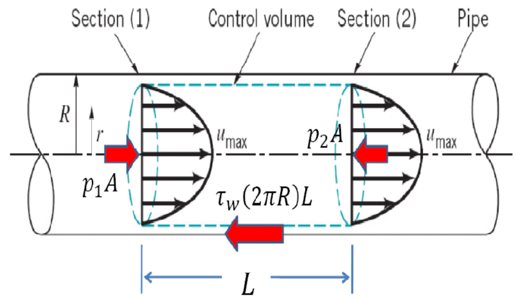
$$= (0.68)\left(999\frac{\text{Kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left[\frac{(3.82\text{m/s})^2 - (15.28\text{m/s})^2}{(2)(9.81\text{m/s}^2)} + 3\ \text{m} - 25.01\ \text{m} + 3.57\ \text{m}\right]$$

$$= \mathbf{-197\ kPa}$$ (+4 points)

# EXAM2 Solutions

## Problem 3: Exact solution (Chapter 6)

### Information and assumptions




- $\rho$ = 1,260 Kg/m3
- $\mu$ = 1.50 N·s/m2
- $D$ = 0.075 m
- $u(r) = \frac{u_{\max}}{R^2}(R^2 - r^2)$
- $u_{\max}$ = 1.0 m/s
- $L$ = 10 m

### Find

- Wall shear stress $\tau_w$, pressure drop $\Delta p$, and head loss $h_L$

### Solution

(a) Wall shear stress

$$\tau_w = \mu\left(-\frac{du}{dr}\right)_{r=R} = \mu\frac{u_{\max}}{R^2}(2r)_{r=R} = \frac{2\mu u_{\max}}{R}$$

$$= \frac{(2)(1.5\ \mathrm{N{\cdot}s/m^2})(1\ \mathrm{m/s})}{(0.075/2)\mathrm{m}} = \mathbf{80.0\,\frac{N}{m^2}}$$ (+3 points)

(b) Pressure difference

$$(p_1 - p_2)(\pi R^2) - \tau_w(2\pi R L) = 0 \qquad (\because V_1 = V_2)$$

$$\Delta p = p_1 - p_2 = \tau_w\frac{2L}{R} = \left(80.0\,\frac{\mathrm{N}}{\mathrm{m^2}}\right)\frac{(2)(10\ \mathrm{m})}{(0.075/2)\mathrm{m}} = \mathbf{42,667\ Pa}$$ (+4 points)

(c) Head loss

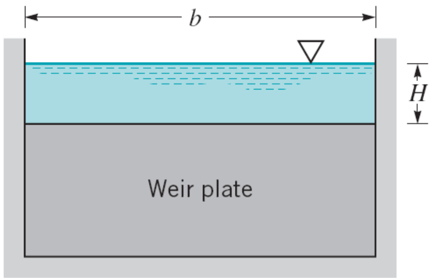
$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L \qquad (\because \alpha_1 V_1 = \alpha_2 V_2,\ z_1 = z_2,\ h_p = h_t = 0)$$

$$h_L = \frac{1}{\gamma}(p_1 - p_2) = \frac{\Delta p}{\rho g} = \frac{42{,}667\ \mathrm{N/m^2}}{(1{,}260\ \mathrm{Kg/m^3})(9.81\ \mathrm{m/s^2})} = \mathbf{3.45\ m}$$ (+3 points)

# EXAM2 Solutions

## Problem 4: Dimensional analysis (Chapter 7)

### Information and assumptions




- $Q = f(H, b, g)$
- $Q \doteq L^3T^{-1}$
- $H \doteq L$
- $b \doteq L$
- $g \doteq LT^{-2}$

### Find

- A suitable set of dimensionless variables

### Solution

$$Q = f(H, b, g)$$

where,

$$Q \doteq L^3T^{-1}; \quad H \doteq L; \quad b \doteq L; \quad g \doteq LT^{-2}$$

From the Pi theorem, $k - r = 4 - 2 = 2$ Pi terms required. (+ 5 points)

a) $\Pi_1 = QH^a g^b$

$$(L^3T^{-1})(L)^a(LT^{-2})^b = M^0L^0T^0$$

$$3 + a + b = 0 \quad \text{(for } L\text{)}$$
$$-1 - 2b = 0 \quad \text{(for } T\text{)}$$

and $a = -\frac{5}{2}$ and $b = -\frac{1}{2}$

$$\therefore \boldsymbol{\Pi_1} = \frac{\boldsymbol{Q}}{\boldsymbol{H^{\frac{5}{2}}g^{\frac{1}{2}}}}$$

b) $\Pi_2 = bH^a g^b$

$$(L)(L)^a(LT^{-2})^b = M^0L^0T^0$$

$$1 + a + b = 0 \quad \text{(for } L\text{)}$$
$$-2b = 0 \quad \text{(for } T\text{)}$$

and $a = -1$ and $b = 0$

$$\therefore \boldsymbol{\Pi_2} = \frac{\boldsymbol{b}}{\boldsymbol{H}}$$

Thus,

$$\frac{\boldsymbol{Q}}{\boldsymbol{H^{\frac{5}{2}}g^{\frac{1}{2}}}} = \boldsymbol{\phi}\left(\frac{\boldsymbol{b}}{\boldsymbol{H}}\right)$$ (+5 points)