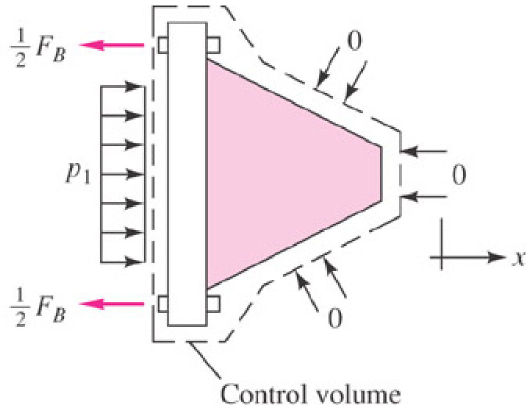
EXAM2 Solutions

Problem 1: Momentum equation (Chapter 5)

Information and assumptions

- Frictionless flow
- $D_1 = 10\ \text{cm} = 0.1\ \text{m}$
- $D_2 = 3\ \text{cm} = 0.03\ \text{m}$
- $Q = 1.5\ \text{m}^3/\text{min} = 0.025\ \text{kg/s}$
- $\rho_{water} = 1000\ \text{kg/m}^3$
- $p_2 = 0\ \text{(gage)}$



Find

- The force F_B exerted by the flange bolts

Solution

x-momentum equation:

$$-F_B + p_1 A_1 = \dot{m}(V_2 - V_1)$$

where,

$$\dot{m} = \rho Q = 1000\ \text{kg/m}^3 \times (0.025\ \text{m}^3/\text{s}) = 25\ \text{kg/s}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.025\text{m}^3/\text{s}}{\pi(0.1\ \text{m})^2/4} = 3.18\ \text{m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.025\text{m}^3/\text{s}}{\pi(0.03\ \text{m})^2/4} = 35.37\ \text{m/s}$$ (+5 points)

p_1 can be determined by using the Bernoulli equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + z_1 = p_2 + \frac{1}{2}\rho V_2^2 + z_2$$

Since the hose is horizontal, $z_1 = z_2$ and $p_2 = p_{atm} = 0$ gage

$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

$$= \frac{1}{2} \times 1000\ \text{kg/m}^3 \times [(35.37\ \text{m/s})^2 - (3.18\ \text{m/s})^2] = 620{,}462\ \text{Pa}$$ (+4 points)

Thus,

$$F_B = p_1 A_1 - \dot{m}(V_2 - V_1)$$

$$= 620462\ N/m^2 \times \frac{\pi(0.1\ m)^2}{4} - 25\ kg/s \times (35.37\ \text{m/s} - 3.18\ \text{m/s})$$

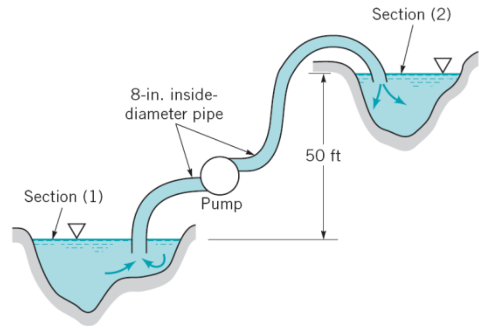
$$= \mathbf{4068\ N}$$ (+1 point)

EXAM2 Solutions

Problem 2: Energy equation (Chapter 5)

Information and assumptions

- $Q = 2.5\ \mathrm{ft}^3/\mathrm{s}$
- $\mathrm{loss} = 61\bar{V}^2/2$
- $z_2 - z_1 = 50\ \mathrm{ft}$
- $D = 8\ \mathrm{in}$
- $p_1 = p_2 = p_{atm}$
- The reservoirs are large enough $\Rightarrow V_1 \approx V_2 \approx 0$



Find

- Shaft power $\dot{W}_s$

Solution

Energy equation:

$$\cancel{\frac{p_2}{\rho}} + \cancel{\frac{V_2^2}{2}} + gz_2 = \cancel{\frac{p_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + gz_1 + w_s - \mathrm{loss}$$

where, $p_1 = p_2 = p_{atm} = 0$ (gage) and $V_1 \approx V_2 \approx 0$ since the reservoirs are large enough.

The required shaft power is

$$\dot{W}_s = \dot{m}w_s = \dot{m}[g(z_2 - z_1) + \mathrm{loss}] \qquad \text{(+6 points)}$$

where,

$$\dot{m} = \rho Q = 1.94\ \mathrm{slugs/ft^3} \times 2.5\ \mathrm{ft^3/s} = 4.85\ \mathrm{slugs/s}$$

$$\bar{V} = Q/A = \frac{4Q}{\pi D^2} = \frac{4\times 2.5\mathrm{ft^3/s}}{\pi\times(8/12\ \mathrm{ft})^2} = 7.162\ \mathrm{ft/s}$$

$$\mathrm{loss} = 61\,\bar{V}^2/2 = 61 \times \frac{(7.162\mathrm{ft/s})^2}{2} = 1564.5\ \mathrm{ft^2/s^2} \qquad \text{(+3 points)}$$

or

$$\dot{W}_s = 4.85\ \mathrm{slugs/s} \times [32.2\ \mathrm{ft/s^2} \times 50\ \mathrm{ft} + 1564.5\ \mathrm{ft^2/s^2}] = 15396\ \mathrm{slugs\cdot ft^2/s^3}$$

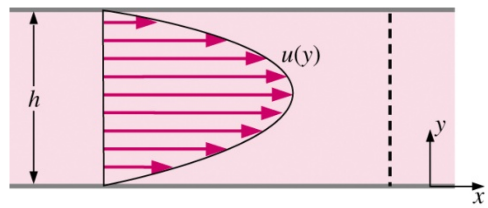
$$= 15396\ \mathrm{slugs\cdot ft^2/s^3} \times \frac{1\ \mathrm{hp}}{550\mathrm{slugs\cdot ft^2/s^3}} = \mathbf{28\ hp} \qquad \text{(+1 points)}$$

EXAM2 Solutions

Problem 3: Bernoulli equation and manometer (Chapter 3)

Information and assumptions

- Steady ($\partial/\partial t = 0$), fully developed ($\partial/\partial x = 0$)
- 2-D flow ($w = 0$, $\partial/\partial z = 0$)
- $\mu = 0.38\ \mathrm{N\cdot s/m^2}$
- $h = 0.004\ \mathrm{m}$
- $\Delta p/\ell = -dp/dx = 20{,}000\ \mathrm{Pa/m}$



Find

- Velocity distribution
- Maximum velocity u_{max}

Solution

Since flow is steady, fully developed, 2-D, the Navier-Stokes equations reduce to

$$\mu \frac{d^2u}{dy^2} = -\frac{dp}{dx}$$ (+4 points)

By integrating the equation twice with respect to y and using boundary conditions

$$u(y) = \frac{1}{2\mu}\left(\frac{dp}{dx}\right) y^2 + C_1 y + C_2$$

B.C. $u(0) = C_2 = 0$

$$u(h) = \frac{1}{2\mu}\left(\frac{dp}{dx}\right) h^2 + C_1 h = 0 \qquad \Rightarrow C_1 = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right) h,\ C_2 = 0$$

Thus the velocity profile is

$$u(y) = \frac{1}{2\mu}\left(\frac{dp}{dx}\right) y(y-h)$$ (+3 points)

The maximum velocity occurs $y = \frac{h}{2}$

$$u_{max} = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)\frac{h}{2}\left(\frac{h}{2} - h\right) = -\frac{h^2}{8\mu}\left(\frac{dp}{dx}\right)$$ (+2 points)

For the oil (SAE 30),

$$u_{max} = -\frac{(0.004\ \mathrm{m})^2}{8\times 0.38\mathrm{N\cdot s/m^2}} \times (-20000\ \mathrm{N/m^3}) = \mathbf{0.105\ m/s}$$ (+ 1 point)

EXAM2 Solutions

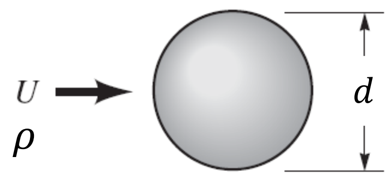
Problem 4: Dimensional analysis (Chapter 7)

Information and assumptions

- $D = f(\rho, U, d)$

Find

- A relationship for D as a function of ρ, U, d



Solution

From the statement of the problem

$$D = f(\rho, U, d)$$

And the dimensions of the variables are

$$D \doteq MLT^{-2}$$

$$\rho \doteq ML^{-3}$$

$$U \doteq LT^{-1}$$

$$d \doteq L$$

Number of pi terms = $k - r = 4 - 3 = 1$

$$\Pi = D\rho^a U^b d^c$$ (+6 points)

It follows that

$$(MLT^{-2})(ML^{-3})^a(LT^{-1})^b(L)^c = M^0L^0T^0$$

and

$$\begin{aligned} 1 + a &= 0 \quad &\text{(for } M\text{)} \\ 1 - 3a + b + c &= 0 \quad &\text{(for } L\text{)} \\ -2 - b &= 0 \quad &\text{(for } T\text{)} \end{aligned}$$

so that $a = -1$, $b = -2$, $c = -2$, and therefore (+3 points)

$$\Pi = \frac{D}{\rho U^2 d^2}$$

Thus, the drag will increase by **a factor of 4** if U is doubled. (+1 points)