057:20 TEST # 1 Fall 2006

Prob. 1

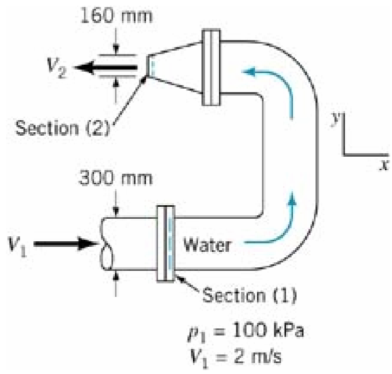
Information and assumptions

Provided in problem statement

Find

Determine the magnitude and direction of the anchoring force.

Solution



The control volume shown in the sketch above is used.
Application of the x direction linear momentum equation leads to:

$$-u_1\rho u_1 A_1 - u_2 \rho u_2 A_2 = p_1 A_1 - R_x + p_2 A_2 \qquad (+5)$$

From the conservation of mass equation:

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2 \qquad (+1)$$

$$u_2 = \left(\frac{A_1}{A_2}\right) u_1 = \frac{D_1^2}{D_2^2} u_1 \qquad (+1)$$

Then:

$$R_x = \rho u_1 A_1 \left(u_1 + u_2\right) + p_1 A_1 + p_2 A_2 \qquad (+1)$$

$$= \left[\rho u_1 \left(u_1 + \frac{D_1^2}{D_2^2} u_1\right) + p_1\right] A_1 + (0) A_2$$

$$= \left[\rho u_1 \left(u_1 + \frac{D_1^2}{D_2^2} u_1\right) + p_1\right] \left(\frac{1}{4}\pi D_1^2\right)$$

$$= \left[(999)(2)\left(2 + \frac{0.3^2}{0.16^2} 2\right) + 100000\right]\left(\frac{1}{4}\pi \times 0.3^2\right) = 8344 N \qquad (+1)$$

Application of the y direction linear momentum equation yields:

$$R_y = 0 \qquad (+1)$$

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Prob. 2

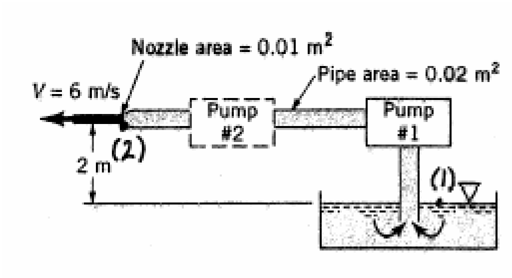
Information and assumptions

Provided in problem statement

Find

How much power must pump 2 add to the water.

Solution



Energy equation between (1) and (2):

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}+h_p-h_L=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g} \qquad (+3)$$

Where

$$p_1=0,\ p_2=0,\ V_1=0,\ z_1=0,\ z_2=2m \qquad (+2)$$

Thus

$$h_p=h_L+z_2+\frac{V_2^2}{2g} \qquad (+1)$$

The flow rate: $Q=A_2V_2=0.01\times 6=0.06\,m^3/s$

The head loss: $h_L=250Q^2=250\times 0.06^2=0.9m$

Thus

$$h_p=0.9+2++\frac{6^2}{2\times 9.81}=4.73m \qquad (+2)$$

So that

$$\dot{W}_p=\gamma Qh_p=(9800)\times 0.06\times 4.73=2780W=2.78kW \qquad (+1)$$

Therefore

$$\dot{W}_p=\dot{W}_{pump1}+\dot{W}_{pump2}=1+\dot{W}_{pump2}=2.78$$

$$\dot{W}_{pump2}=1.78kW \qquad (+1)$$

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Prob. 3

Information and assumptions

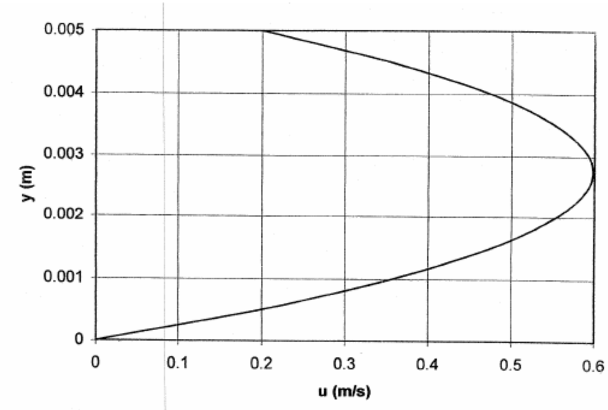
Provided in problem statement

Find

(a) Calculate the velocity at y=2.5mm.

(b) Calculate the shear stress on the bottom plate.

Solution



(a) The velocity distribution:

$$u = U\frac{y}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left(y^2 - by\right) \qquad (+3)$$

$$= 0.2\frac{0.0025}{0.005} + \frac{1}{2\times 0.38}(-60000)\left(0.0025^2 - 0.005\times 0.0025\right)$$

$$= 0.593\, m/s \qquad (+2)$$

(b) The shear stress on the bottom plate:

$$\tau = \mu\left.\frac{du}{dy}\right|_{y=0} = \mu\left.\left[\frac{U}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(2y - b)\right]\right|_{y=0} \qquad (+4)$$

$$= \left[\mu\frac{U}{b} + \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(2y - b)\right]_{y=0}$$

$$= \mu\frac{U}{b} + \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)b$$

$$= 0.38\times\frac{0.2}{0.005} + \frac{1}{2}(60000)\times 0.005$$

$$= 165.2\, N/m^2 \qquad (+1)$$

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Prob. 4

Information and assumptions

Provided in problem statement

Find

At what depth and flowrate would the model operate?

Solution

For Froude number similarity:

$$\frac{V_m}{\sqrt{gl_m}} = \frac{V_p}{\sqrt{gl_p}}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{l_m}{l_p}} \qquad (+5)$$

Since the flow rate is $Q = VA$, where A is the appropriate cross sectional area,

$$\frac{1}{250} = \frac{Q_m}{Q_p} = \frac{V_m A_m}{V_p A_p} = \sqrt{\frac{l_m}{l_p}}\frac{A_m}{A_p} = \sqrt{\frac{l_m}{l_p}}\left(\frac{l_m}{l_p}\right)^2 = \left(\frac{l_m}{l_p}\right)^{5/2}$$

$$\frac{l_m}{l_p} = 0.11 \qquad (+3)$$

So the corresponding model depth

$$l_m = 0.11 l_p = 0.11 \times 4 = 0.44\ ft \qquad (+1)$$

So the corresponding model flowrate

$$Q_m = \frac{1}{250} Q_p = \frac{1}{250} \times 700 = 2.8\ ft^3/s \qquad (+1)$$