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Prob. 1

Information and assumptions

Provided in problem statement

Find

The differential height of mercury between the two pipe sections

Solution

Take points 1 and 2 along the centerline of the pipe over two tubes of the manometer.

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{1}{4}\pi D_1^2} = \frac{1gal}{\frac{1}{4}\pi\left(\frac{4}{12}\right)^2}\frac{0.13368\,ft^3}{1gal} = 1.53\ ft/s$$

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\frac{1}{4}\pi D_2^2} = \frac{1gal}{\frac{1}{4}\pi\left(\frac{2}{12}\right)^2}\frac{0.13368\,ft^3}{1gal} = 6.13\ ft/s$$ (+2)

Bernoulli equation between points 1 and 2:

$$\frac{P_1}{\rho_w g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho_w g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

(1) $$P_1 - P_2 = \frac{1}{2}\rho_w\left(V_2^2 - V_1^2\right)$$ (+3)

Manometer equation

$$P_1 + \rho_w g\left(s+h\right) = P_2 + \rho_w gs + \rho_{\mathrm{Hg}} gh$$

(2) $$P_1 - P_2 = \left(\rho_{\mathrm{Hg}} - \rho_w\right)gh$$ (+3)

Combing eqn (1) and (2) and solving for h

$$h = \frac{\rho_w\left(V_2^2 - V_1^2\right)}{2g\left(\rho_{\mathrm{Hg}} - \rho_w\right)} = \frac{\left(V_2^2 - V_1^2\right)}{2g\left(\frac{\rho_{\mathrm{Hg}}}{\rho_w} - 1\right)}$$

$$h = \frac{\left(6.13^2 - 1.53^2\right)}{2\left(32.2\ ft/s^2\right)\left(\frac{847}{62.4} - 1\right)} = 0.0435\,ft = 0.522in$$ (+2)

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Prob. 2

Information and assumptions

Provided in problem statement

Find

(a) The average water exit velocity
(b) The horizontal resistance force

Solution

The average outlet velocity

$$V = \frac{Q}{A} = \frac{Q}{\frac{1}{4}\pi D^2} = \frac{5\,m^3/\min}{\frac{1}{4}\pi\left(0.06\right)^2} = 1768\,m/\min = 29.5\,m/s$$

(+3)

The mass flow rate

$$\dot{m} = \rho Q = \left(1000\,kg/m^3\right)\left(5\,m^3/\ \right) = 5000\,kg/\min = 83.3\,kg/s$$

(+2)

The momentum equation for steady one-dimensional flow:

$$\sum F_R = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$$ (+3)

$$F_R = \dot{m} V_e - 0 = \dot{m} V = \left(83.3\,kg/s\right)\left(29.5\,m/s\right) = 2457\,N$$

(+2)

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Prob. 3

Information and assumptions

Provided in problem statement

Find

The air velocity in the wind tunnel

Solution

Reynolds number similarity

$$\mathrm{Re}_m = \mathrm{Re}_p$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

(+7)

$$V_{air} = \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}V_p = \frac{L_w}{L_{air}}\frac{\nu_{air}}{\nu_w}V_w$$

(+2)

$$= \left(\frac{1}{1}\right)\left(\frac{1.41\times10^{-5}}{1.31\times10^{-6}}\right)(10) = 107.6\, m/s$$

(+1)

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Prob. 4

Information and assumptions

Provided in problem statement

Find

The head loss

Solution

The average velocity:

$$V = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho\left(\pi D^2/4\right)} = \frac{0.15}{(665.1)\left(\pi 0.005^2/4\right)} = 11.49\, m/s$$

(+2)

The Reynolds number:

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(665.1)(11.49)(0.005)}{2.361\times 10^{-4}} = 1.618\times 10^5$$

Therefore the flow is turbulent

(+2)

The relative roughness of the pipe

$$\varepsilon/D = \frac{1.5\times 10^{-6}}{0.005} = 3\times 10^{-4}$$

(+1)

From Moody chart: $f = 0.019$

(+1)

Head loss:

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = 0.019\times\frac{30}{0.005}\times\frac{11.49^2}{2(9.81)} = 767 m$$

(+4)