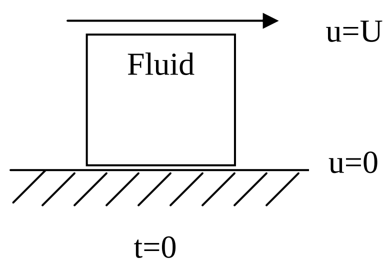
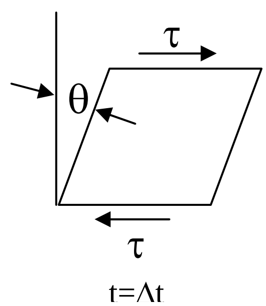
Review for Exam 1

Chapter 1: Introduction and basic concepts

- Definition of a fluid: a substance that deforms continuously when subjected to a shear stress
- No slip condition: no relative motion between fluid and boundary, i.e., fluid in contact with lower plate is stationary, whereas fluid in contact with upper plate moves at speed U





- Both liquid and gas behave as fluids!
- Flow classification:

  Hydrodynamics→Flow of fluids for which density is constant such as liquid and low-speed gases. (e.g. hydraulics, ship-hydrodynamics)

  Gas dynamics→Flow of fluids for which density is variable such as high-speed gases. (e.g. high-speed aerodynamics, gas turbines)

- Continuum hypothesis: Assumption that the fluid behaves as a continuum, i.e., the number of molecules within the smallest region of interest (a point) are sufficient that all fluid properties are point functions (single valued at a point).

- Properties of fluids
  - SI units and BG(English) units

Primary Units	SI	BG
Mass M	kg	Slug=32.2lbm
Length L	m	ft
Time t	s	s
Temperature T	°C (°K)	°F (°R)

Secondary (derived) units	Dimension	SI	BG
velocity V	L/t	m/s	ft/s
acceleration a	L/t^2	m/s^2	ft/s^2
force F	ML/t^2	N ($kg \cdot m/s^2$)	lbf
pressure p	F/L^2	Pa (N/m^2)	lbf/ft^2
density ρ	M/L^3	kg/m^3	$slug/ft^3$
internal energy u	FL/M	J/kg (N·m/kg)	BTU/lbm

  - Extensive and intensive properties

    Extensive property: Depending on total mass of system (e.g. M, W)

    Intensive property: Independent of amount of mass system (e.g. p, ρ)

  - Properties involving the mass or weight of the fluid

    Specific weight $\gamma = \rho g$
    Mass density ρ=Mass/Volume
    Specific gravity $S = \gamma / \gamma_{\text{water, T=4°C}}$

  - Variation in density: $\rho = p/RT$ for ideal gas (R: gas constant)

  - Vapor pressure and Cavitation:

    When the pressure of a liquid falls below the vapor pressure, it evaporates.

$$Ca \text{ (Cavitation number)} = \frac{p - p_v}{0.5 \rho U_\infty^2} < 0 \text{ implies cavitation}$$

- Properties involving the flow of heat

specific heats	c_p and c_v	J/kg·°K
specific internal energy	u	J/kg
specific enthalpy	$h = u + p/\rho$	J/kg

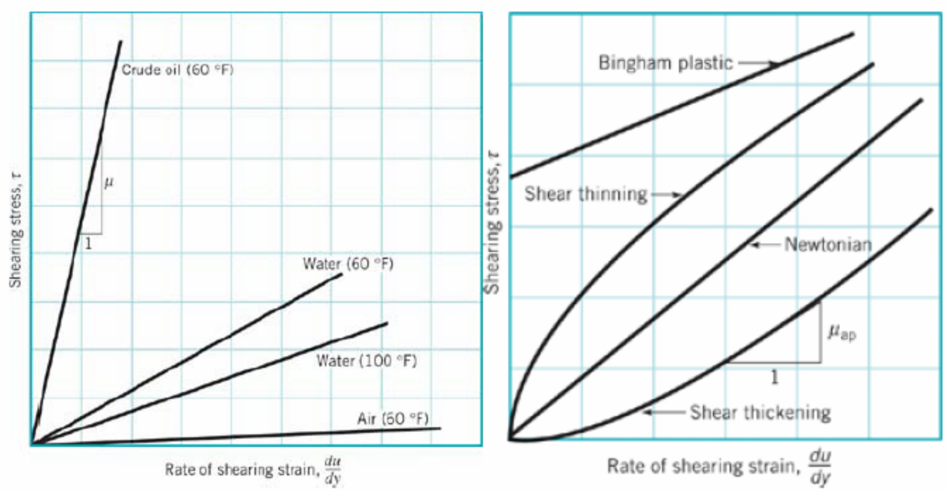
- Compressibility:

  Liquids are in general incompressible and gases are in general compressible.

- Viscosity

  Newtonian fluid: Linear relationship between shear stress and velocity gradient.

Dilatant:	$\tau\uparrow$	$dV/dy\uparrow$
Newtonian:	$\tau \propto$	dV/dy
Pseudo plastic:	$\tau\downarrow$	$dV/dy\uparrow$



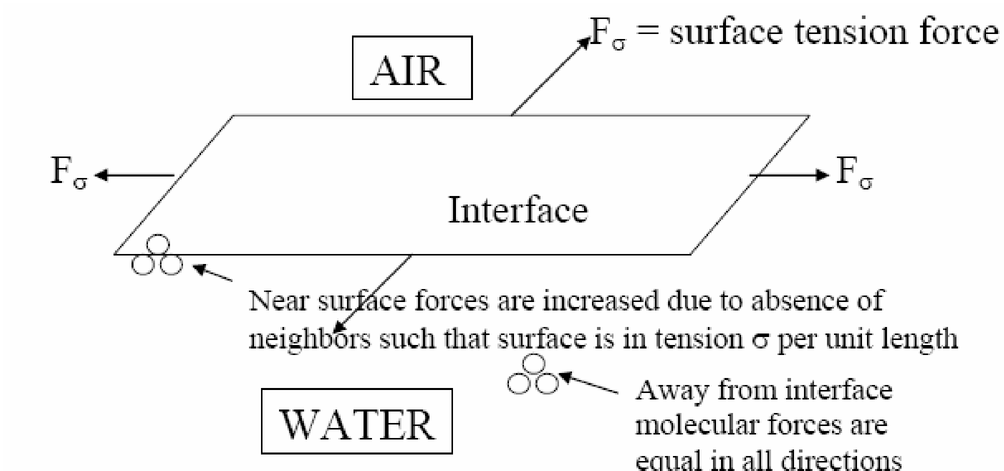
$$\tau \propto dV/dy \qquad\qquad \tau \propto \left(dV/dy\right)^n$$

μ = coefficient of viscosity = proportionality constant for Newtonian fluid

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{N/m^2}{\frac{m}{s}/m} = \frac{Ns}{m^2}$$

$$\nu = \frac{\mu}{\rho} = \frac{m^2}{s} = \text{kinematic viscosity}$$

- Surface tension and capillary effects



$\sigma_{air/water} = 0.073$ N/m

$F_\sigma = \sigma \times L$ = line force with direction normal to the cut

L = length of cut through the interface

<u>**Chapter 2: Pressure and Fluid Statics**</u>

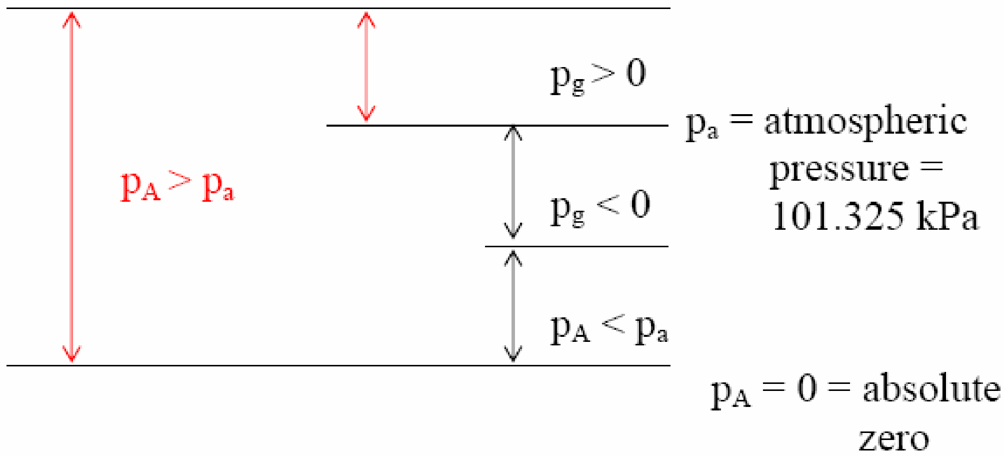
- Pressure
    - For a static fluid, only stress is the normal stress since by definition a fluid subjected to a shear stress must deform and undergo motion. Normal stresses are referred to as pressure p.
    - P is isotropic, one value at a point which is independent of direction, a scalar.

$$-p = \tau_{xx} = \tau_{yy} = \tau_{zz} \quad i = j \qquad \text{normal stresses} = -p$$

- Pressure transmission

Pascal's law: in a closed system, a pressure change produced at one point in the system is transmitted throughout the entire system.

- Absolute pressure, gage pressure, and vacuum

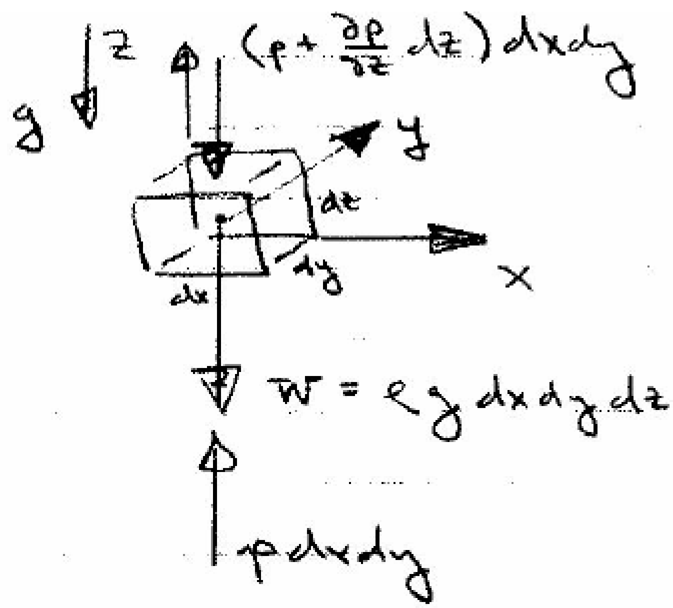


For $p_A > p_a$, $p_g = p_A - p_a$ = gage pressure

For $p_A < p_a$, $p_{vac} = -p_g = p_a - p_A$ = vacuum pressure

- Pressure variation

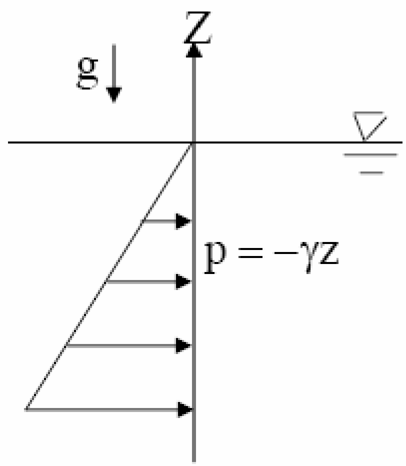
  - with elevation



$$\frac{\partial p}{\partial z} = -\rho g = -\gamma$$

For a static fluid, the pressure only varies with elevation z and is constant in horizontal xy planes.

- for a uniform-density fluid



$$p_1 + \gamma z_1 = p_2 + \gamma z_2 = \text{constant}$$

$p + \gamma z = \text{constant}$ piezometric pressure

$p = -\gamma z$ increase linearly with depth
decrease linearly with height

- for compressible fluids

$p = \rho R T$

R = gas constant = 287 J/kg ·°K dry air
p,T in absolute scale

$$\frac{dp}{dz} = -\frac{pg}{RT}$$

$$\frac{dp}{p} = \frac{-g}{R}\frac{dz}{T(z)}$$

which can be integrated for T(z) known

- in the troposphere

$$\frac{p}{p_o} = \left[\frac{T_o - \alpha(z - z_o)}{T_o}\right]^{g/\alpha R}$$

i.e., p decreases for increasing z

α = lapse rate = 6.5 °K/km

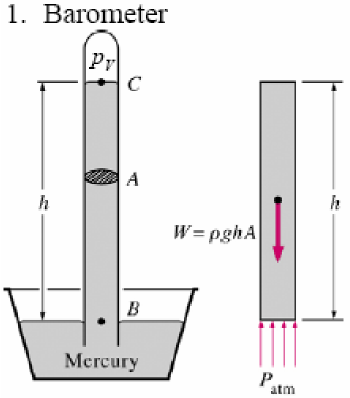
- in the stratosphere

$$p = p_o \exp[-(z - z_o)g / RT_s]$$

i.e., p decreases exponentially for increasing z.

- Pressure measurements
  Many devices are based on hydrostatics such as barometers and manometers.

1. Barometer



$p_v + \gamma_{Hg} h = p_{atm}$

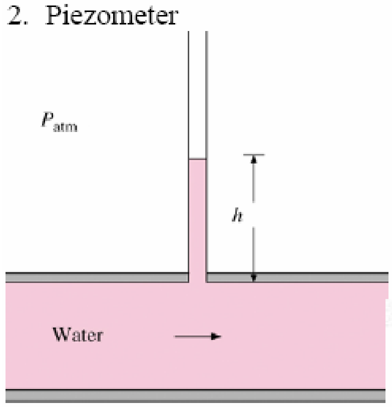
$p_{atm} = \gamma_{Hg} h$ $\qquad$ $p_v \sim 0$ i.e., vapor pressure Hg nearly zero at normal T

$h \sim 76$ cm

$\therefore$ $p_{atm} \sim 101$ kPa (or 14.6 psia)

Note: p_{atm} is relative to absolute zero, i.e., absolute pressure. $p_{atm} = p_{atm}$(location, weather)

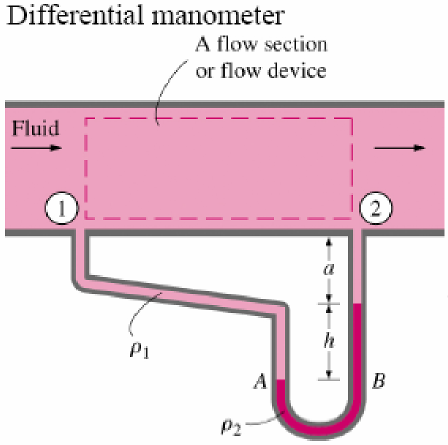
2. Piezometer



$p_{atm} + \gamma h = p_{pipe} = p$ $\qquad$ absolute

$p = \gamma h$ $\qquad$ gage

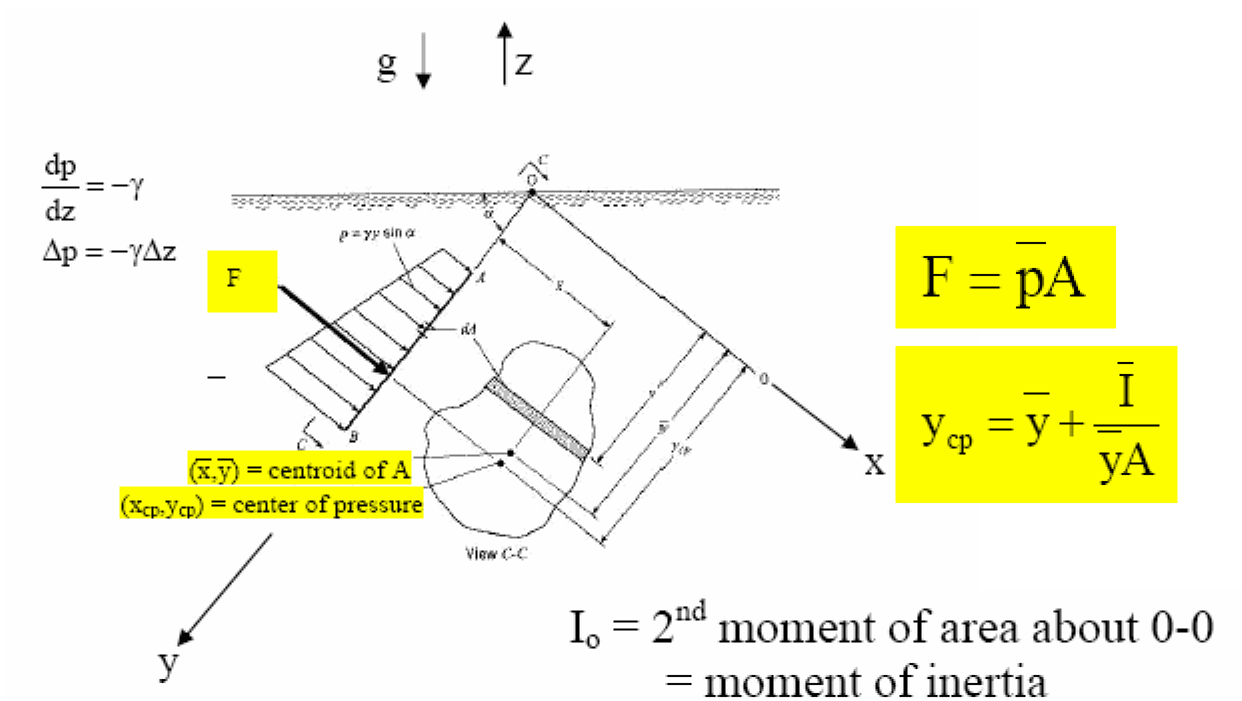
3. Differential manometer

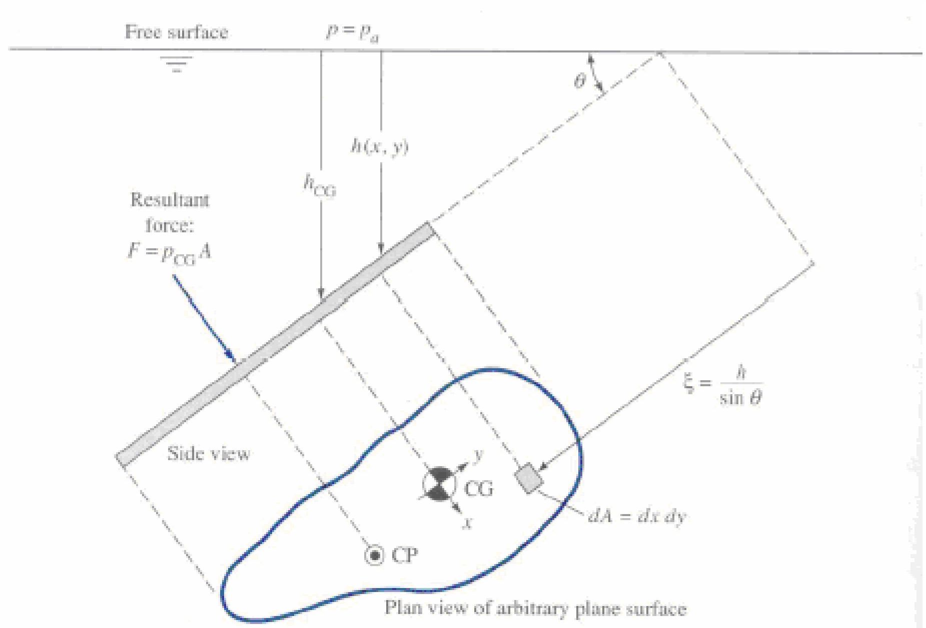


$$P_1 + \rho_1 g(a + h) - \rho_2 g h - \rho_2 g a = P_2$$

- Hydrostatic forces on plane surfaces

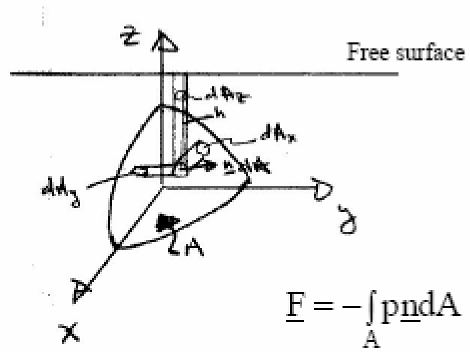
  For a static fluid, the shear stress is zero and the only stress is the normal stress, i.e. pressure.





Magnitude of resultant hydrostatic force on plane surface is product of pressure at centroid of area and area of surface.

- Hydrostatic force on curved surface



$p = \gamma h$

$\underline{F} = -\int_A p\underline{n}dA$

h = distance belc
free surface

Horizontal Components (x and y components)

$F_x = \underline{F} \cdot \hat{i} = -\int_A p\underline{n} \cdot \hat{i}dA$

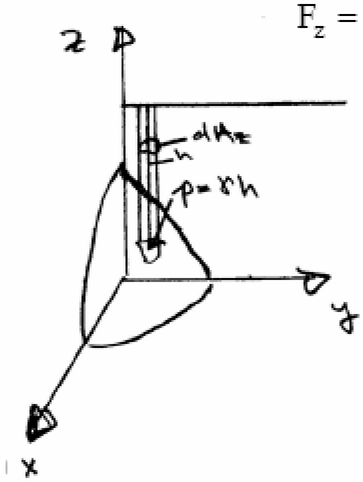
$= -\int_{A_x} pdA_x$

dA_x = projection of $\underline{n}dA$ or
plane ⊥ to x-direction

$F_y = \underline{F} \cdot \hat{j} = -\int_{A_y} pdA_y$ $dA_y = \underline{n} \cdot \hat{j}dA$

= projection $\underline{n}dA$
onto plane ⊥ to
y-direction

Vertical Components



$F_z = \underline{F} \cdot \hat{k} = -\int_A p\underline{n} \cdot \hat{k}dA$

$= -\int_{A_z} pdA_z$ $p = \gamma h$

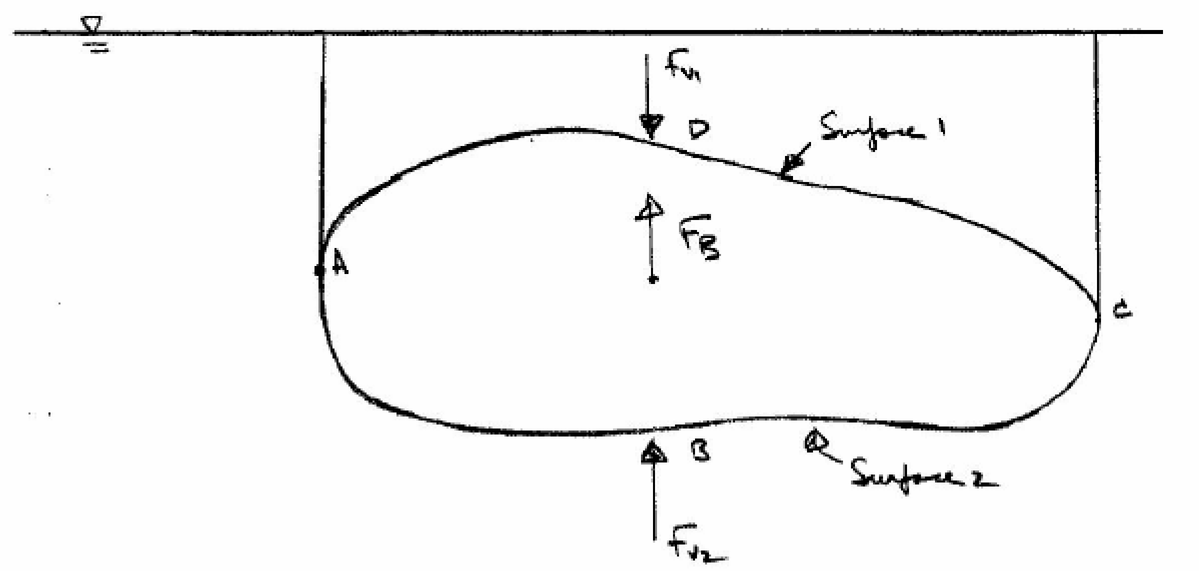
h=distance
below free
surface

$= \gamma \int_{A_z} hdA_z = \gamma V$

= weight of
fluid above
surface A

- Buoyancy

Archimedes principle



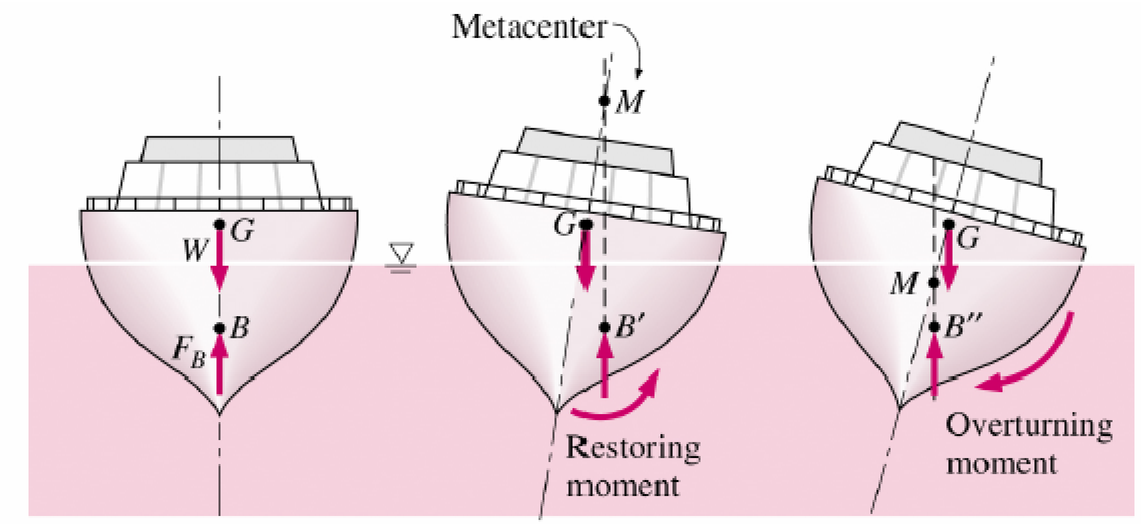
$F_B = F_{v2} - F_{v1}$

= fluid weight above Surface 2 (ABC) – fluid weight above Surface 1 (ADC)

= fluid weight equivalent to body volume $\forall$

$F_B = \rho g \forall$ $\forall$ = submerged volume

- Stability of immersed and floating bodies



(a) Stable (b) Stable (c) Unstable

The point of intersection of the lines of action of the buoyant force before and after heel is called the metacenter M and the distance GM is called the metacentric height. If GM is positive, that is, if M is above G, then the ship is stable; however, if GM is negative, the ship is unstable.

$$CM = I_{OO} / \forall$$

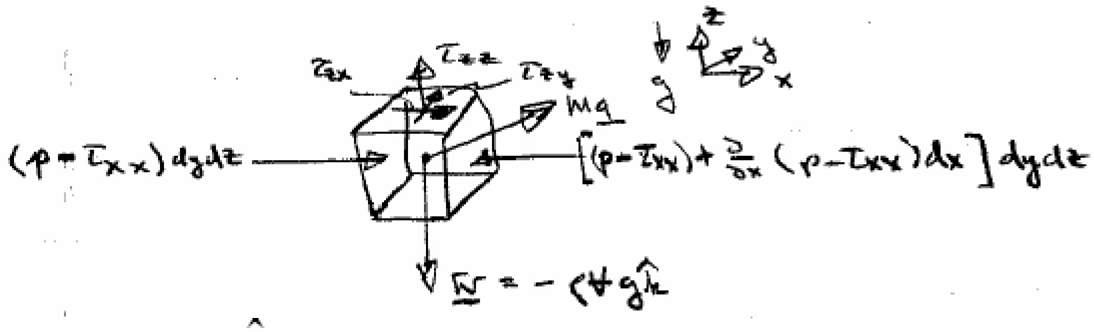
$$GM = CM - CG$$

$$GM = \frac{I_{OO}}{\forall} - CG$$

GM > 0	Stable
GM < 0	Unstable

I_{OO} = moment of inertia of waterplane area about centerplane axis

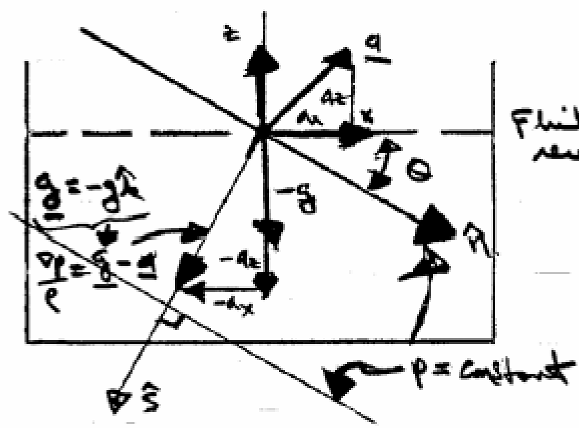
- Fluids in rigid-body motion



$$\rho \underline{a} = -\rho g \hat{k} - \nabla p$$

inertia force = body force due to gravity + surface force due to pressure gradients

- uniform linear acceleration



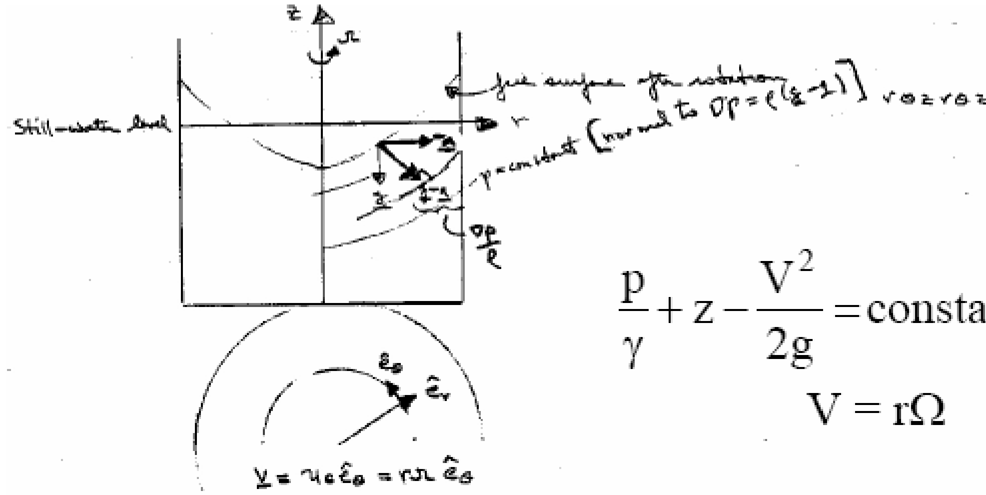
$$\frac{\partial p}{\partial x} = -\rho a_x$$

1. $a_x < 0$ p increase in +x
2. $a_x > 0$ p decrease in +x

$$\frac{\partial p}{\partial z} = -\rho \left(g + a_z \right)$$

1. $a_z > 0$ p decrease in +z
2. $a_z < 0$ and $|a_z| < g$ p decrease in +z but slower than g
3. $a_z < 0$ and $|a_z| > g$ p increase in +z

- rigid body rotation



$$\frac{p}{\gamma} + z - \frac{V^2}{2g} = \text{constant}$$

$$V = r\Omega$$

Chapter 3: Bernoulli equation

- Bernoulli equation

$$p + \frac{1}{2}\rho V^2 + \gamma z = C \quad \text{(along a streamline)}$$

Under the assumptions;

(1) Inviscid
(2) Incompressible
(3) Steady
(4) Conservative body force

- Physical interpretation: work-energy principle

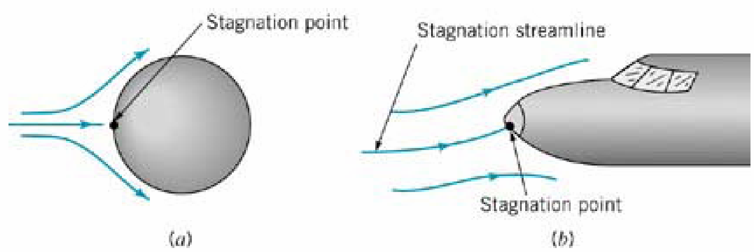
- Static, stagnation, dynamic, and total pressure

$$p+\frac{1}{2}\rho V^2+\gamma z=C \quad \text{(along a streamline)}$$

Static pressure: p

Dynamic pressure: $\frac{1}{2}\rho V^2$

Hydrostatic pressure: γz



Stagnation points on bodies in flowing fluids.

Stagnation pressure: $p+\frac{1}{2}\rho V^2$ (assume elevation effects negligible)

Total pressure: $p_T=p+\frac{1}{2}\rho V^2+\gamma z=C$ (along a streamline)

- Application of Bernoulli equation

<u>Stagnation Tube</u>

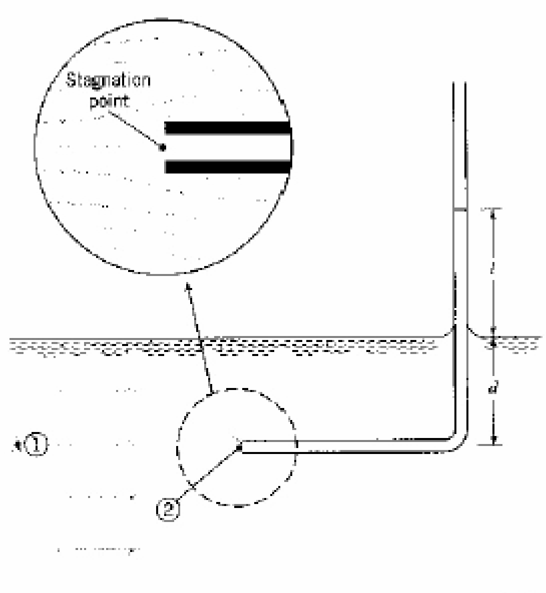


FIGURE 5.5
Stagnation tube.

Pitot Tube

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad (V_1 \to 0)$$

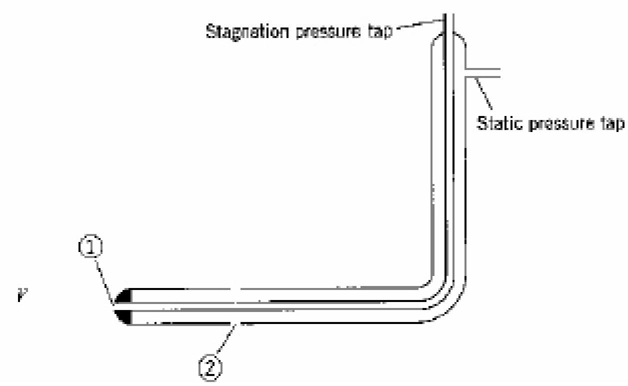
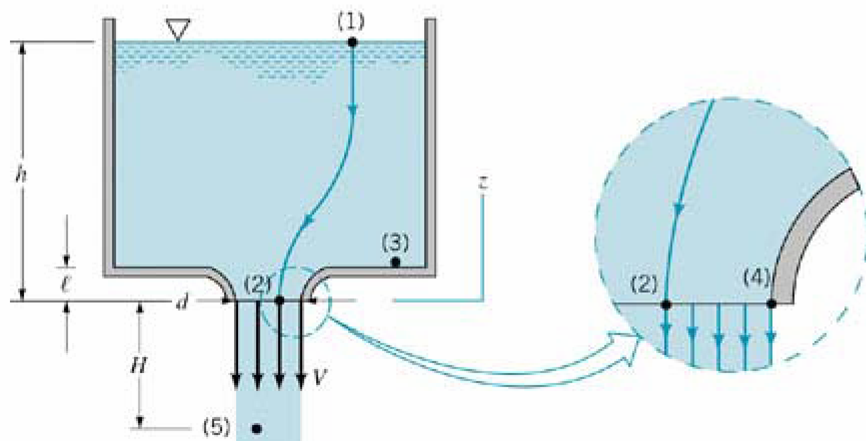


FIGURE 5.6
Pitot tube.

Free Jets



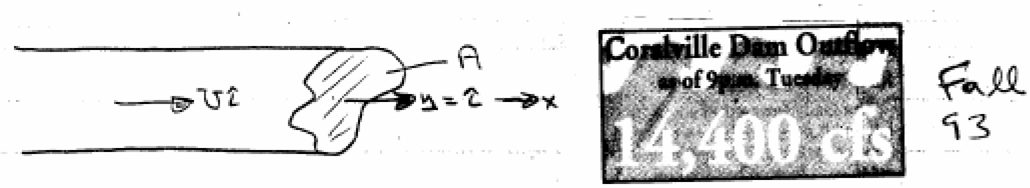
Vertical flow from a tank

Application of Bernoulli Equation between points (1) and (2) on the streamline shown gives

$$p_1+\frac{1}{2}\rho V_1^2+\gamma z_1=p_2+\frac{1}{2}\rho V_2^2+\gamma z_2$$

- Volume rate of flow

cross-sectional area oriented normal to velocity vector (simple case where V ⊥ A)

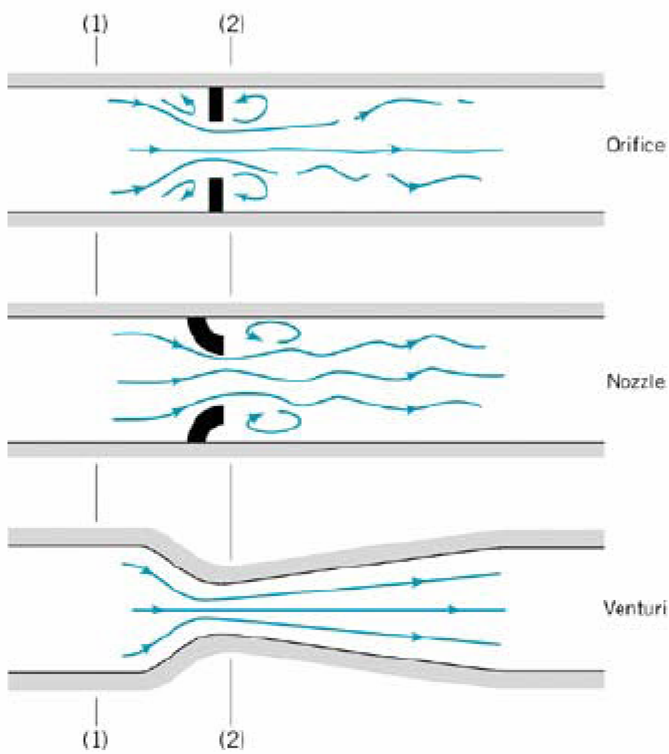


U = constant: Q = volume flux = UA [m/s × m² = m³/s]

U ≠ constant: $Q = \int_A U dA$

- Flowrate measurement

Various flow meters are governed by the Bernoulli and continuity equations.



Typical devices for measuring flowrate in pipes

- Energy grade line (EGL) and hydraulic grade line (HGL)

Define $\quad \text{HGL} = \dfrac{p}{\gamma} + z$

$\quad \text{EGL} = \dfrac{p}{\gamma} + z + \dfrac{V^2}{2g}$

} point-by-point application is graphically displayed

<u>HGL corresponds to pressure tap measurement + z</u>
<u>EGL corresponds to stagnation tube measurement + z</u>

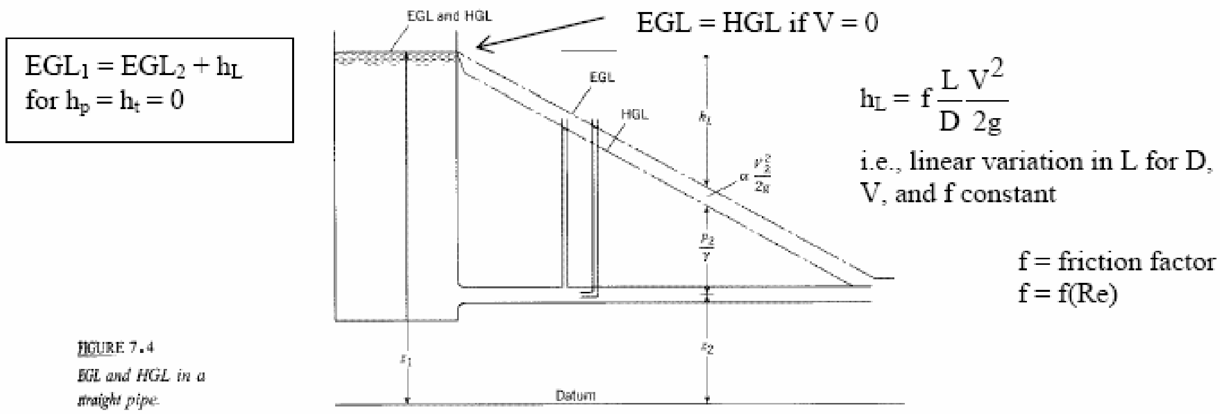


FIGURE 7.4
EGL and HGL in a straight pipe.

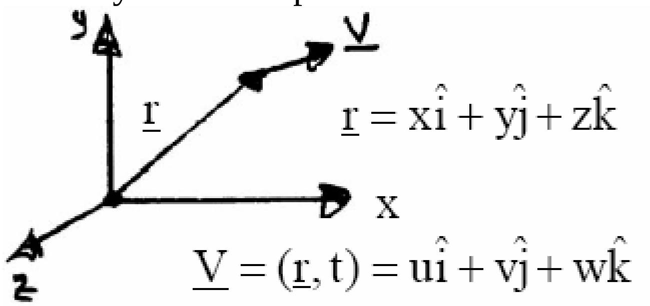
- Limitations of Bernoulli equation

The Bernoulli equation cannot be used for;

1. Compressibility effects
2. Unsteady effects
3. Rotational effects
4. viscous flow
5. flows that involve pumps or turbines

Chapter 4: Fluids Kinematics

- Velocity and description methods

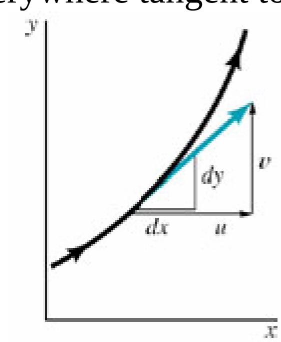


Two approaches to analyzing the velocity field:

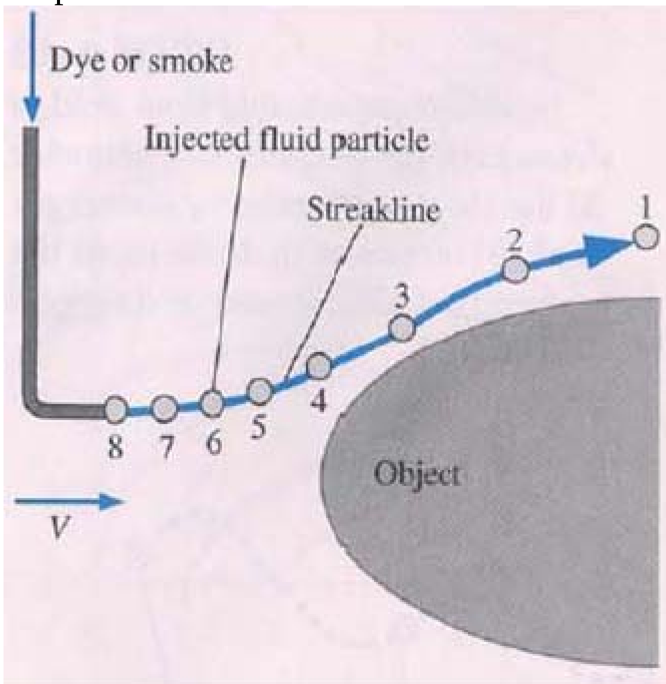
- Lagrangian: keep track of individual fluids particles (i.e., solve F = Ma for each particle): $\mathbf{V_p} = u_p\hat{i} + v_p\hat{j} + w_p\hat{k}$
- Eulerian: focus attention on a fixed point $\mathbf{x} = x\hat{i} + y\hat{j} + z\hat{k}$ in space,
  $\mathbf{V} = \mathbf{V}(\mathbf{x}, t) = u\hat{i} + v\hat{j} + w\hat{k}$ **,**
  where u = u(x,y,z,t), v = v(x,y,z,t), w = w(x,y,z,t)

- Streamlines, streaklines, and pathlines

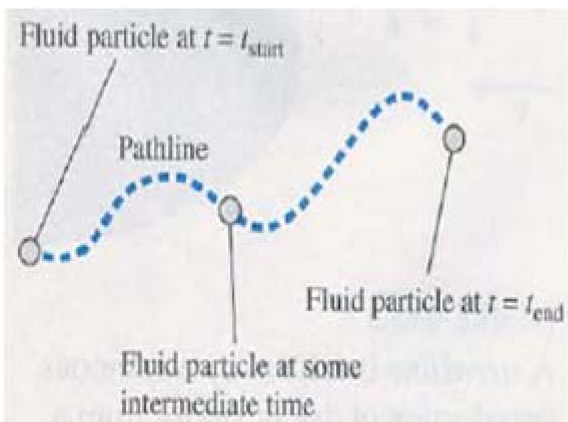
Streamline: a line that is everywhere tangent to the velocity field.



Streakline: consists of all particles in a flow that have previously passed through a common point.



Pathline: the line traced out by a given particle as it flows from one point to another.



- Acceleration Field and Material Derivative:
  Lagrangian approach: the velocity of a fluid particle is a function of time only.

$$\underline{r}_p = x_p(t)\hat{i} + y_p(t)\hat{j} + z_p(t)\hat{k}$$

$$\underline{V}_p = \frac{d\bar{r}_p}{dt} = u_p\hat{i} + v_p\hat{j} + w_p\hat{k}$$

$$\underline{a}_p = \frac{d\bar{v}_p}{dt} = \frac{d^2\bar{r}_p}{dt^2} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$a_x = \frac{du_p}{dt} \quad a_y = \frac{dv_p}{dt} \quad a_z = \frac{dw_p}{dt}$$

  Eulerian approach: the velocity is a function of both space and time.

$$\mathbf{V} = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$$

Total Acceleration = Local Acceleration + Convective Acceleration

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

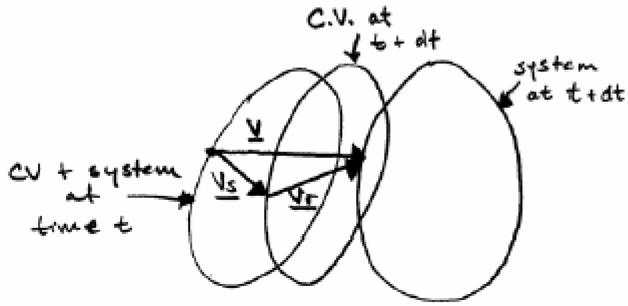
- Separation, vortices, turbulence, and flow classification:

  - One-, Two-, and Three-dimensional Flow
  - Steady vs. Unsteady Flow
  - Incompressible and Compressible Flow
  - Viscous and Inviscid Flows
  - Rotational vs. Irrotational Flow
  - Laminar vs. Turbulent Viscous Flows
  - Internal vs. External Flows
  - Separated vs. Unseparated Flow

- Basic Control-Volume Approach and RTT:

Need relationship between $\frac{d}{dt}\left(B_{sys}\right)$ and changes in

$$B_{CV} = \int_{CV} \beta\, dm = \int_{CV} \beta\rho\, d\forall .$$



Moving deforming CV:

$\underline{V}_r = \underline{V} - \underline{V}_s$

$\underline{V}$ = fluid velocity
$\underline{V}_s$ = CS defining CV velocity
$\underline{V}_r$ = relative velocity

} all in same coordinate system

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0} \frac{(B_{cv} + \Delta B)_{t+\Delta t} - (B_{cv} + \Delta B)_t}{\Delta t}$$

$$= \underbrace{\lim_{\Delta t \to 0} \frac{B_{cv_{t+\Delta t}} - B_{cv_t}}{\Delta t}}_{①} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta B_{t+\Delta t} - \Delta B_t}{\Delta t}}_{②}$$

1 = time rate of change of B in CV $= \frac{dB_{cv}}{dt} = \frac{d}{dt}\int_{cv} \beta\rho\, d\forall$

2 = net outflux of B from CV across CS =

$$\int_{CS} \beta\rho \underline{V}_R \cdot \underline{n}\; DA$$

$$\frac{dB_{SYS}}{dt} = \frac{d}{dt}\int_{CV} \beta\rho\, d\forall + \int_{CS} \beta\rho \underline{V}_R \cdot \underline{n}\; dA$$

$$B = \text{extensive property} = \int_{CV} \beta \mathrm{dM} = \int_{CV} \beta\rho \mathrm{d}\forall$$

$$\beta = \text{intensive property}$$

Continuity equation: $B = \text{mass}$, $\beta = 1$

$$-\frac{d}{dt}\int_{CV} \rho d\forall = \int_{CS} \rho \mathbf{V} \cdot \mathbf{dA}$$ integral form

$$\frac{D\rho}{Dt} + \rho\nabla \cdot \mathbf{V} = 0$$ differential form

i.e. $$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$