10/05/2008 Midterm 1 Review problem
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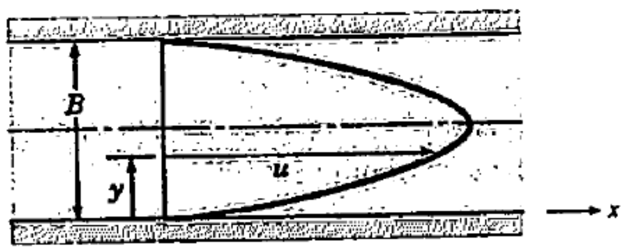
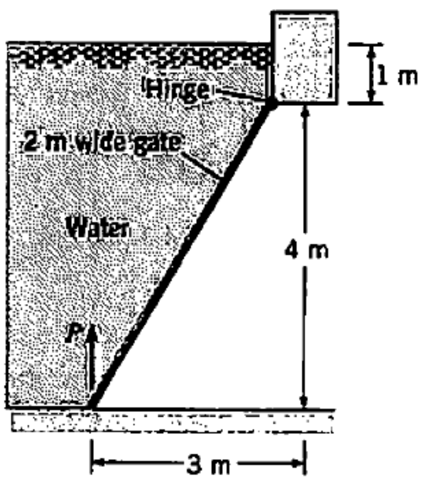
1. Shear stress

Given $u = 10y^{1/6}$, where u is the velocity of water (20°C) in meters per second and y is the distance from the boundary in mm, determine the shear stress in the water at $y = 2$ mm.



2. Hydrostatic pressure on a plane surface

Determine P to just start opening the 2-m wide gate.



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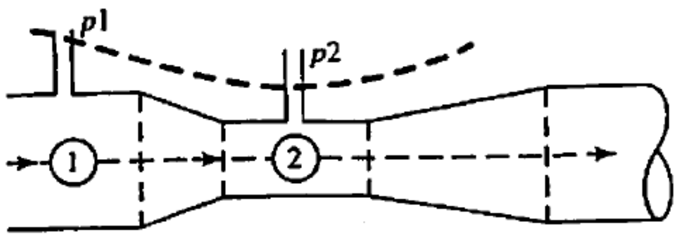
3. Bernoulli equation

A constriction in a pipe will cause the velocity to rise and the pressure to fall at section 2 in the throat. The pressure difference is a measure of the flow rate through the pipe. The smoothly necked-down system shown in Fig. E3.23 is called a *venturi tube*. Find an expression for the mass flux in the tube as a function of the pressure change.

E3.23



4. Fluid kinematics

An idealized velocity field is given by the formula

$$\mathbf{V} = 4tx\mathbf{i} - 2t^2y\mathbf{j} + 4xz\mathbf{k}$$

Is this flow field steady or unsteady? Is it two- or three-dimensional? At the point $(x, y, z) = (-1, 1, 0)$, compute the acceleration vector

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Answer

1. Shear stress

Information and assumptions

$\mu = 10^{-3}\ \mathrm{N{\cdot}s/m^2}$

Find

shear stress at $y = 2$ mm

Stress-strain relationship

$$
\begin{aligned}
\tau &= \mu du/dy \\
du/dy &= (1/6)(10)(y)^{-5/6}\ \mathrm{s}^{-1} \\
&= (1/6)(10)(.002)^{-5/6}\ \mathrm{s}^{-1} \\
&= (10/6)(177.5)\ \mathrm{s}^{-1} \\
\tau &= (10^{-3}\ \mathrm{N \cdot s/m^2})(10/6)(177.5)\ \mathrm{s}^{-1} = \underline{\underline{0.296\ \mathrm{N/m^2}}}
\end{aligned}
$$

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2. Hydrostatic pressure on a plane surface

Information and assumptions

provided in problem statement

Find

force P required to begin to open gate

Solution

The length of gate is $\sqrt{4^2 + 3^2} = 5$ m

$$\begin{aligned}
\bar{y} &= 2.5 \text{ m} + 5/4 \text{ m} \\
&= 3.75 \text{ m} \\
F &= \bar{p}A \\
&= (4/5)(3.75)(9,810)(2 \times 5) \\
&= 294,300 \text{ N} \\
&= 294.3 \text{ kN} \\
y_{cp} - \bar{y} &= I/(\bar{y}A) \\
&= 2 \times (5^3/12)/(3.75 \times 2 \times 5) \\
&= 0.555 \text{ m}
\end{aligned}$$

Sum moments about hinge:

$$\sum M_{\text{hinge}} = 0$$

$$F \times (2.5 + .555) + 3P = 0$$

$$294.3(3.055) + 3P = 0$$

$$\underline{\underline{P = 300 \text{ kN}}}$$

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3. Bernoulli equation

If the tube is horizontal, $z_1 = z_2$ and we can solve for V_2:

$$V_2^2 - V_1^2 = \frac{2\,\Delta p}{\rho} \qquad \Delta p = p_1 - p_2 \tag{1}$$

We relate the velocities from the incompressible continuity relation:

$$A_1V_1 = A_2V_2$$

or

$$V_1 = \beta^2 V_2 \qquad \beta = \frac{D_2}{D_1} \tag{2}$$

Combining (1) and (2), we obtain a formula for the velocity in the throat:

$$V_2 = \left[\frac{2\,\Delta p}{\rho(1-\beta^4)}\right]^{1/2} \tag{3}$$

The mass flux is given by

$$\dot{m} = \rho A_2 V_2 = A_2\left(\frac{2\rho\,\Delta p}{1-\beta^4}\right)^{1/2} \tag{4}$$

4. Fluid kinematics

(a) The flow is <u>unsteady</u> because time t appears explicitly in the components.

(b) The flow is <u>three-dimensional</u> because all three velocity components are nonzero.

(c) Evaluate, by laborious differentiation, the acceleration vector at (x, y, z) = (−1, +1, 0).

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 4x + 4tx(4t) - 2t^2y(0) + 4xz(0) = 4x + 16t^2x$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -4ty + 4tx(0) - 2t^2y(-2t^2) + 4xz(0) = -4ty + 4t^4y$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = 0 + 4tx(4z) - 2t^2y(0) + 4xz(4x) = 16txz + 16x^2z$$

or: $$\frac{d\mathbf{V}}{dt} = (4x + 16t^2x)\mathbf{i} + (-4ty + 4t^4y)\mathbf{j} + (16txz + 16x^2z)\mathbf{k}$$

at (x, y, z) = (−1, +1, 0), we obtain $\frac{d\mathbf{V}}{dt} = -4(1+4t^2)\mathbf{i} - 4t(1-t^3)\mathbf{j} + 0\mathbf{k}$ *Ans.* (c)