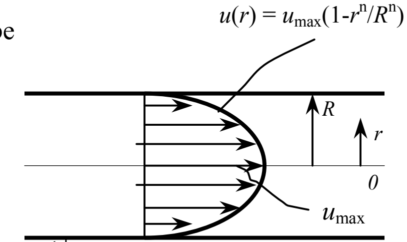
2-44 The velocity profile of a fluid flowing though a circular pipe is given. The friction drag force exerted on the pipe by the fluid in the flow direction per unit length of the pipe is to be determined.

Assumptions The viscosity of the fluid is constant.

Analysis The wall shear stress is determined from its definition to be



$$\tau_w = -\mu \left.\frac{du}{dr}\right|_{r=R} = -\mu u_{\max} \frac{d}{dr}\left(1 - \frac{r^n}{R^n}\right)_{r=R} = -\mu u_{\max} \left.\frac{-nr^{n-1}}{R^n}\right|_{r=R} = \frac{n\mu u_{\max}}{R}$$

Note that the quantity du/dr is negative in pipe flow, and the negative sign is added to the τ_w relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or, $du/dr = - du/dy$ since $y = R - r$). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F = \tau_w A_w = \frac{n\mu u_{\max}}{R}(2\pi R)L = 2n\pi\mu u_{\max} L$$

Therefore, the drag force per unit length of the pipe is

$$F / L = 2n\pi\mu u_{\max} .$$

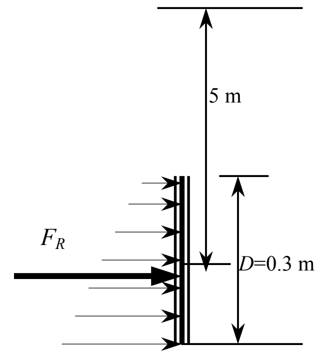
Discussion Note that the drag force acting on the pipe in this case is independent of the pipe diameter.

3-64 A room in the lower level of a cruise ship is considered. The hydrostatic force acting on the window and the pressure center are to be determined.

Assumptions The atmospheric pressure acts on both sides of the window, and thus it can be ignored in calculations for convenience.

Properties The specific gravity of sea water is given to be 1.025, and thus its density is 1025 kg/m^3.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = P_C = \rho g h_C = (1025\text{ kg/m}^3)(9.81\text{ m/s}^2)(5\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 50{,}276\text{ N/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave}A = P_{ave}[\pi D^2/4] = (50{,}276\text{ N/m}^2)[\pi(0.3\text{ m})^2/4] = \mathbf{3554\ N}$$

The line of action of the force passes through the pressure center, whose vertical distance from the free surface is determined from

$$y_P = y_C + \frac{I_{xx,C}}{y_C A} = y_C + \frac{\pi R^4/4}{y_C \pi R^2} = y_C + \frac{R^2}{4y_C} = 5 + \frac{(0.15\text{ m})^2}{4(5\text{ m})} = \mathbf{5.0011\ m}$$

Discussion Note that for small surfaces deep in a liquid, the pressure center nearly coincides with the centroid of the surface.

3-71 Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension **T** in each cable when the trough is filled to the rim is to be determined.

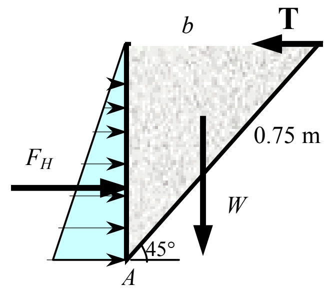
Assumptions **1** The atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height h at the midsection of the trough and width of the free surface are

$$h = L\sin\theta = (0.75\text{ m})\sin 45° = 0.530\text{ m}$$

$$b = L\cos\theta = (0.75\text{ m})\cos 45° = 0.530\text{ m}$$



The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g (h/2) A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.530/2\text{ m})(0.530\text{ m}\times 6\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 8267\text{ N}$$

The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g[w\times bh/2]$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)[(6\text{ m})(0.530\text{ m})(0.530\text{ m})/2]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 8267\text{ N}$$

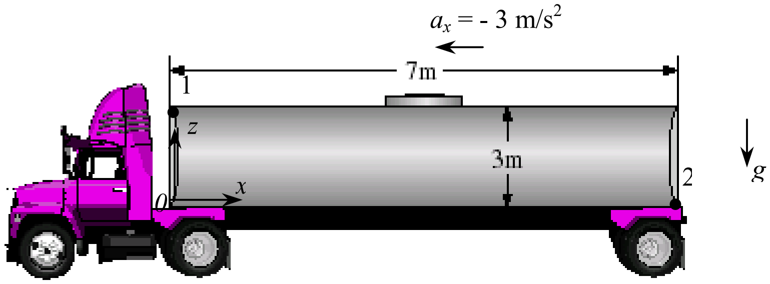
The distance of the centroid of a triangle from a side is 1/3 of the height of the triangle for that side. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad\rightarrow\quad W\frac{b}{3} + F_H\frac{h}{3} = \mathbf{T}h$$

Solving for **T** and substituting, and noting that $h = b$, the tension in the cable is determined to be

$$\mathbf{T} = \frac{F_H + W}{3} = \frac{(8267+8267)\text{ N}}{3} = \mathbf{5511\ N}$$

3-102 Milk is transported in a completely filled horizontal cylindrical tank accelerating at a specified rate. The maximum pressure difference in the tanker is to be determined. √EES



Assumptions **1** The acceleration remains constant. **2** Milk is an incompressible substance.

Properties The density of the milk is given to be 1020 kg/m^3.

Analysis We take the *x*- and *z*- axes as shown. The horizontal acceleration is in the negative *x* direction, and thus a_x is negative. Also, there is no acceleration in the vertical direction, and thus $a_z = 0$. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1)$$

The first term is due to acceleration in the horizontal direction and the resulting compression effect towards the back of the tanker, while the second term is simply the hydrostatic pressure that increases with depth. Therefore, we reason that the lowest pressure in the tank will occur at point 1 (upper front corner), and the higher pressure at point 2 (the lower rear corner). Therefore, the maximum pressure difference in the tank is

$$\Delta P_{\max} = P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1) = -[a_x (x_2 - x_1) + g(z_2 - z_1)]$$

$$= -(1020\ \text{kg/m}^3)\left[(-2.5\ \text{m/s}^2)(7\ \text{m}) + (9.81\ \text{m/s}^2)(-3\ \text{m})\right]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= (17.9 + 30.0)\ \text{kN/m}^2 = \mathbf{47.9\ \ kPa}$$

since $x_1 = 0$, $x_2 = 7$ m, $z_1 = 3$ m, and $z_2 = 0$.

Discussion Note that the variation of pressure along a horizontal line is due to acceleration in the horizontal direction while the variation of pressure in the vertical direction is due to the effects of gravity and acceleration in the vertical direction (which is zero in this case).