Problem review 1 (10/02/2006)

1.57 A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Fig. P1.57. The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of $8.0 \times 10^{-4}\ \mathrm{m^2/s}$ and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.

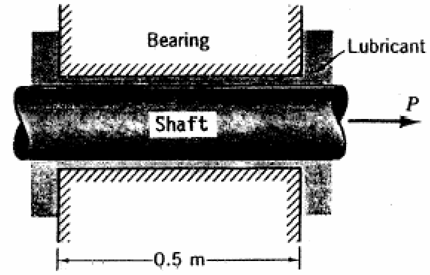
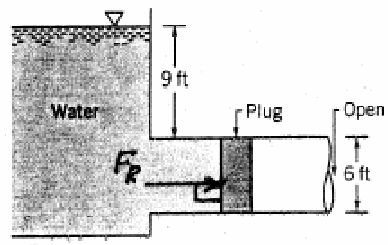


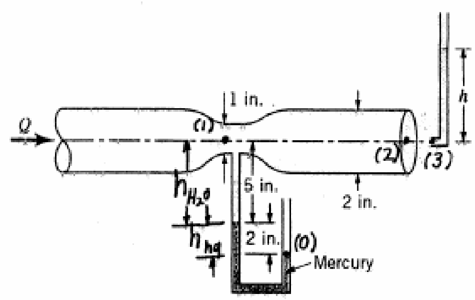
FIGURE P1.57

2.51 A large, open tank contains water and is connected to a 6-ft diameter conduit as shown in Fig. P2.51. A circular plug is used to seal the conduit. Determine the magnitude, direction, and location of the force of the water on the plug.



■ **FIGURE P2.51**

3.45 Water flows through a converging–diverging nozzle as shown in Fig. P3.45. Determine **(a)** the volumetric flowrate, Q, through the nozzle and **(b)** the height, h, of the water in the Pitot tube inserted into the free jet. Viscous effects are negligible.



■ **FIGURE P3.45**

4.23 As a valve is opened, water flows through the diffuser shown in Fig. P4.23 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})\,(1 - x/\ell)\,\hat{\mathbf{i}}$, where u_0, c, and ℓ are constants. Determine the acceleration as a function of x and t. If $V_0 = 10$ ft/s and $\ell = 5$ ft, what value of c (other than $c = 0$) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

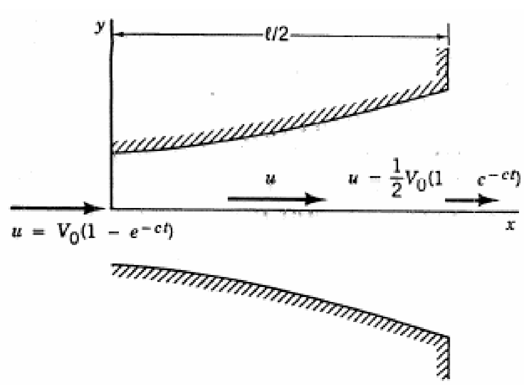


FIGURE P4.23

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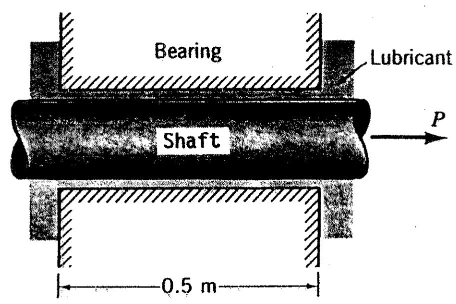
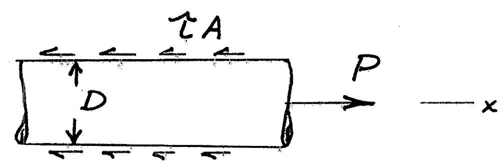


FIGURE P1.57



$$\sum F_x = 0$$

Thus, $P = \tau A$

where $A = \pi D \times$ (shaft length in bearing) $= \pi D \ell$

and $\tau = \mu \dfrac{\text{(velocity of shaft)}}{\text{(gap width)}} = \mu \dfrac{V}{b}$

so that

$$P = \left(\mu \frac{V}{b}\right)(\pi D \ell)$$

Since $\mu = \nu \rho = \nu (SG)(\rho_{H_2O@4^\circ C})$,

$$P = \frac{(8.0 \times 10^{-4} \frac{m^2}{s})(0.91 \times 10^3 \frac{kg}{m^3})(3 \frac{m}{s})(\pi)(0.025m)(0.5m)}{(0.0003m)}$$

$$= \underline{\underline{286\ N}}$$

2.51

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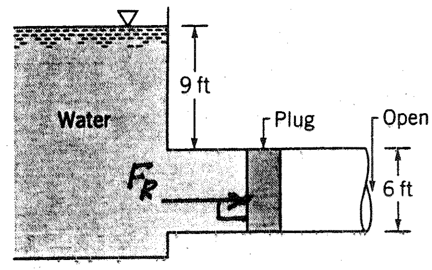


■ FIGURE P2.51

$$F_R = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right)(12\ ft)\left(\frac{\pi}{4}\right)(6\ ft)^2 = \underline{\underline{21,200\ lb}}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad I_{xc} = \frac{\pi\ (3\,ft)^4}{4} = 63.6\ ft^4$$

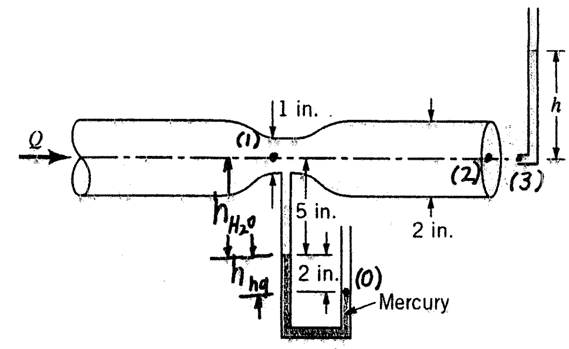
Thus,

$$y_R = \frac{\frac{\pi}{4}(3\,ft)^4}{(12\ ft)\,\pi\,(3\,ft)^2} + 12\,ft = \underline{\underline{12.19\ ft}}$$

The force of 21,200 lb acts 12.19 ft below the water surface and is perpendicular to the plug surface as shown.

3.45

3.45 Water flows through a converging–diverging nozzle as shown in Fig. P3.45. Determine (**a**) the volumetric flowrate, Q, through the nozzle and (**b**) the height, h, of the water in the Pitot tube inserted into the free jet. Viscous effects are negligible.



■ **FIGURE P3.45**

(a) From the Bernoulli equation,

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2, \text{ where } z_1 = z_2 \text{ and } p_2 = 0 \qquad (1)$$

Also, $p_1 = p_0 - \gamma_{Hg} h_{Hg} - \gamma_{H_2O} h_{H_2O}$, where $p_0 = 0$ so that

$$p_1 = -13.6\,(62.4\ \text{lb/ft}^3)(\tfrac{2}{12}\ \text{ft}) - 62.4\ \text{lb/ft}^3(\tfrac{5}{12}\ \text{ft}) = -167\ \text{lb/ft}^2$$

In addition, $A_1 V_1 = A_2 V_2$ or $V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{2\ \text{in.}}{1\ \text{in.}}\right)^2 V_2 = 4 V_2$

Hence, Eq. (1) becomes

$$-167\ \text{lb/ft}^2 + \tfrac{1}{2}(1.94\ \text{slugs/ft}^3)(4V_2)^2 = \tfrac{1}{2}(1.94\ \text{slugs/ft}^3)\,V_2^2$$

or

$$V_2 = 3.39\ \text{ft/s}$$

Thus, $Q = A_2 V_2 = \frac{\pi}{4}\left(\frac{2}{12}\ \text{ft}\right)^2 (3.39\ \text{ft/s}) = \underline{\underline{0.0739\ \text{ft}^3/\text{s}}}$

(b) From the Bernoulli equation,

$$p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2 = p_3 + \tfrac{1}{2}\rho V_3^2 + \gamma z_3, \text{ where } p_2 = 0,\ z_2 = 0,\ V_3 = 0, \text{ and } z_3 = 0$$

Thus,

$\tfrac{1}{2}\rho V_2^2 = p_3$, where $p_3 = \gamma h$ so that

$$\tfrac{1}{2}(1.94\ \text{slugs/ft}^3)(3.39\ \text{ft/s})^2 = (62.4\ \text{lb/ft}^3)\,h$$

or

$h = \underline{\underline{0.179\ \text{ft} = 2.14\ \text{in.}}}$

4.23

4.23 As a valve is opened, water flows through the diffuser shown in Fig. P4.23 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{\mathbf{i}}$, where u_0, c, and ℓ are constants. Determine the acceleration as a function of x and t. If $V_0 = 10$ ft/s and $\ell = 5$ ft, what value of c (other than $c = 0$) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

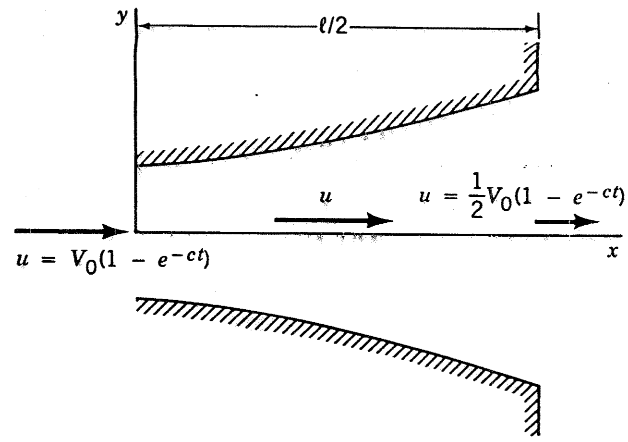


FIGURE P4.23

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x,t)$, $v = 0$, and $w = 0$

this becomes

$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\imath} = a_x\hat{\imath}$, where $u = V_0(1 - e^{-ct})(1 - \frac{x}{\ell})$

Thus,

$$a_x = V_0\left(1 - \frac{x}{\ell}\right)c\,e^{-ct} + V_0^2(1 - e^{-ct})^2\left(1 - \frac{x}{\ell}\right)\left(-\frac{1}{\ell}\right)$$

or

$$a_x = V_0\left(1 - \frac{x}{\ell}\right)\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right]$$

If $a_x = 0$ for any x at $t = 1$ s we must have

$\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right] = 0$ With $V_0 = 10$ and $\ell = 5$

$c\,e^{-c} - \frac{10}{5}(1 - e^{-c})^2 = 0$ The solution (root) of this equation is $c = 0.490\ \frac{1}{s}$

For the above conditions the local acceleration ($\frac{\partial u}{\partial t} > 0$) is precisely balanced by the convective deceleration ($u\frac{\partial u}{\partial x} < 0$). The flowrate increases with time, but the fluid flows to an area of lower velocity.