Review problems for Midterm exam1 for 057:020, Fall 2007

Chapter 1: Shear stress

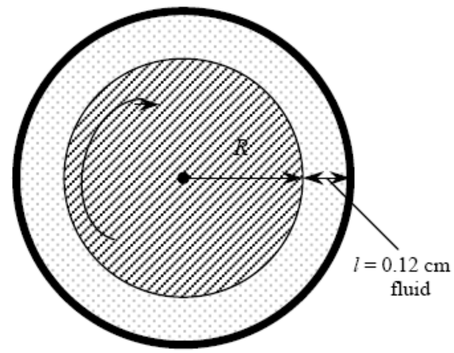
The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in oil. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is Newtonian.

Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T}\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(0.8\ \text{N}\cdot\text{m})(0.0012\ \text{m})}{4\pi^2 (0.075\ \text{m})^3 (200/60\ \text{s}^{-1})(0.75\ \text{m})} = \mathbf{0.0231\ N\cdot s/m^2}$$

Discussion This is the viscosity value at the temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.



Chapter 2: Hydrostatic forces

The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per m length are to be determined.

Assumptions 1 Friction at the hinge is negligible. **2** Atmospheric pressure acts on both sides of the gate, and thus it cancels out.

Properties We take the density of water to be 1000 kg/m³ throughout.

Analysis (*a*) We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(s + R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\ \cdot\ \text{n}}\right)$$

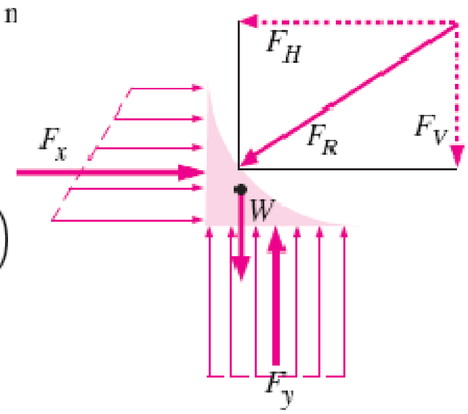
$$= \mathbf{36.1\ kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{ave}A = \rho g h_C A = \rho g h_{bottom} A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\ \cdot\ \text{m/s}^2}\right)$$

$$= 39.2\ \text{kN}$$



Weight of fluid block per m length (downward):

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(1 \text{ m})$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.8 \text{ m})^2(1 - \pi/4)(1 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 1.3 \text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9 \text{ kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = \mathbf{52.3 \text{ kN}}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 52.3 kN per m length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.4° with the horizontal.

(*b*) When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point *A* at the location of the hinge and equating it to zero gives

$$F_R R \sin\theta - W_{\text{cyl}} R = 0 \rightarrow W_{\text{cyl}} = F_R \sin\theta = (52.3 \text{ kN}) \sin 46.4° = \mathbf{37.9 \text{ kN}}$$

Discussion The weight of the cylinder per m length is determined to be 37.9 kN. It can be shown that this corresponds to a mass of 3863 kg per m length and to a density of 1921 kg/m^3 for the material of the cylinder.

Chapter 3: Bernoulli equation

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid velocity at the free surface is very low ($V_1 \cong 0$) and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad \frac{V_2^2}{2g} = \frac{P_1 - P_2}{\rho g} + z_1$$

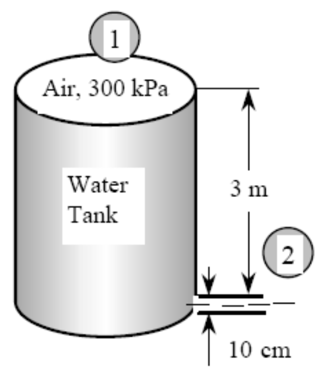
Solving for V_2 and substituting, the discharge velocity is determined to

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho} + 2gz_1} = \sqrt{\frac{2(300-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) + 2(9.81\text{ m/s}^2)(3\text{ m})}$$

$$= 21.4\text{ m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V} = A_{\text{orifice}}V_2 = \frac{\pi D^2}{4}V_2 = \frac{\pi(0.10\text{ m})^2}{4}(21.4\text{ m/s}) = \mathbf{0.168\ m^3/s}$$



Discussion Note that this is the maximum flow rate since the frictional effects are ignored. Also, the velocity and the flow rate will decrease as the water level in the tank decreases.

Solution We are to write an equation for centerline speed through a nozzle, given that the flow speed increases parabolically.

Assumptions **1** The flow is steady. **2** The flow is axisymmetric. **3** The water is incompressible.

Analysis A general equation for a parabola in the x direction is

General parabolic equation: $$u = a + b(x - c)^2 \qquad (1)$$

We have two boundary conditions, namely at $x = 0$, $u = u_{\text{entrance}}$ and at $x = L$, $u = u_{\text{exit}}$. By inspection, Eq. 1 is satisfied by setting $c = 0$, $a = u_{\text{entrance}}$ and $b = (u_{\text{exit}} - u_{\text{entrance}})/L^2$. Thus, Eq. 1 becomes

Parabolic speed: $$u = u_{\text{entrance}} + \frac{(u_{\text{exit}} - u_{\text{entrance}})}{L^2} x^2 \qquad (2)$$

To find the acceleration in the x-direction, we use the material acceleration,

Acceleration along centerline of nozzle: $$a_x = \cancel{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}} + \cancel{w\frac{\partial u}{\partial z}}$$

The first term in Eq. 2 is zero because the flow is steady. The last two terms are zero because the flow is axisymmetric, which means that along the centerline there can be no v or w velocity component. We substitute Eq. 1 for u to obtain

Acceleration along centerline of nozzle:

$$a_x = u\frac{\partial u}{\partial x} = \left(u_{\text{entrance}} + \frac{(u_{\text{exit}} - u_{\text{entrance}})}{L^2} x^2 \right)(2)\frac{(u_{\text{exit}} - u_{\text{entrance}})}{L^2} x$$

or

$$a_x = 2u_{\text{entrance}} \frac{(u_{\text{exit}} - u_{\text{entrance}})}{L^2} x + 2\frac{(u_{\text{exit}} - u_{\text{entrance}})^2}{L^4} x^3$$