Review for Exam 1, 057:020 Fall 2007

Chapter 1: Introduction and basic concepts

Properties of fluids

- SI units and BG(English) units

Primary Units	SI	BG
Mass M	kg	Slug=32.2lbm
Length L	m	ft
Time t	s	s
Temperature T	°C (°K)	°F (°R)

Secondary (derived) units	Dimension	SI	BG
velocity V	L/t	m/s	ft/s
acceleration a	L/t^2	m/s^2	ft/s^2
force F	ML/t^2	N ($kg \cdot m/s^2$)	lbf
pressure p	F/L^2	Pa (N/m^2)	lbf/ft^2
density ρ	M/L^3	kg/m^3	$slug/ft^3$
internal energy u	FL/M	J/kg (N·m/kg)	BTU/lbm

- Extensive and intensive properties

  Extensive property: Depending on total mass of system (e.g. M, W)

  Intensive property: Independent of amount of mass system (e.g. p, ρ)

- Properties involving the mass or weight of the fluid

  Specific weight $\gamma = \rho g$
  Mass density ρ=Mass/Volume
  Specific gravity $S = \gamma / \gamma_{water,\ T=4°C}$

- Variation in density: $\rho = p/RT$ for ideal gas (R: gas constant)

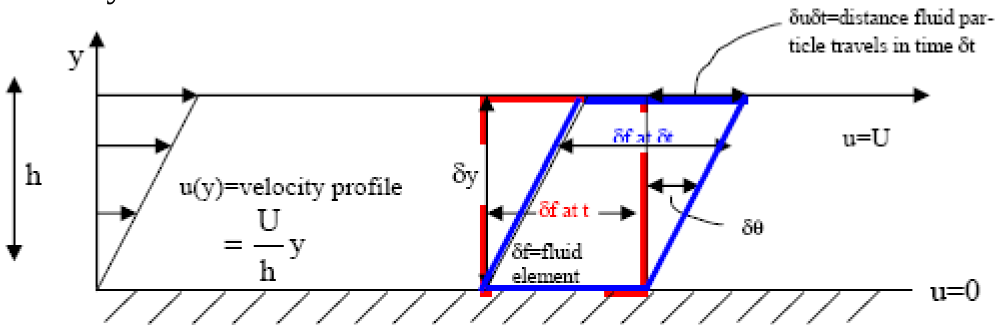
- Properties involving the flow of heat

specific heats	c_p and c_v	J/kg·°K
specific internal energy	u	J/kg
specific enthalpy	$h = u + p/\rho$	J/kg

- Compressibility:

Liquids are in general incompressible and gases are in general compressible.

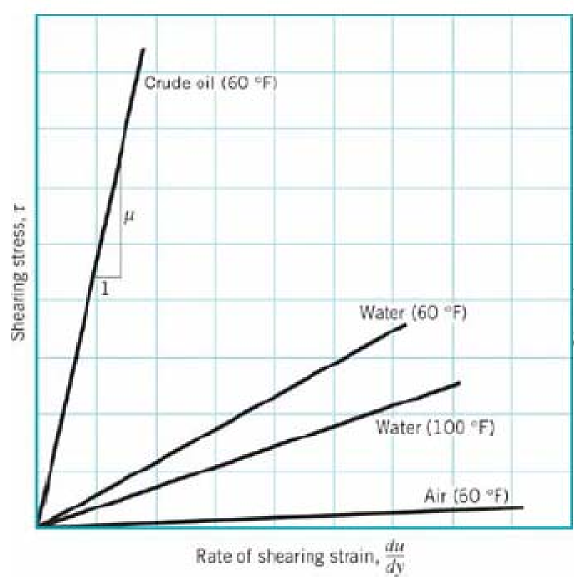
- Viscosity



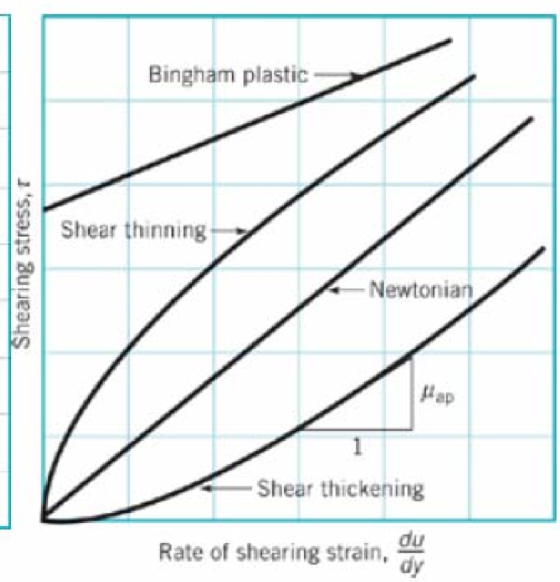
Dilatant: $\tau\uparrow \quad dV/dy\uparrow$

Newtonian: $\tau \propto dV/dy$

Pseudo plastic: $\tau\downarrow \quad dV/dy\uparrow$



$$\tau \propto dV/dy$$



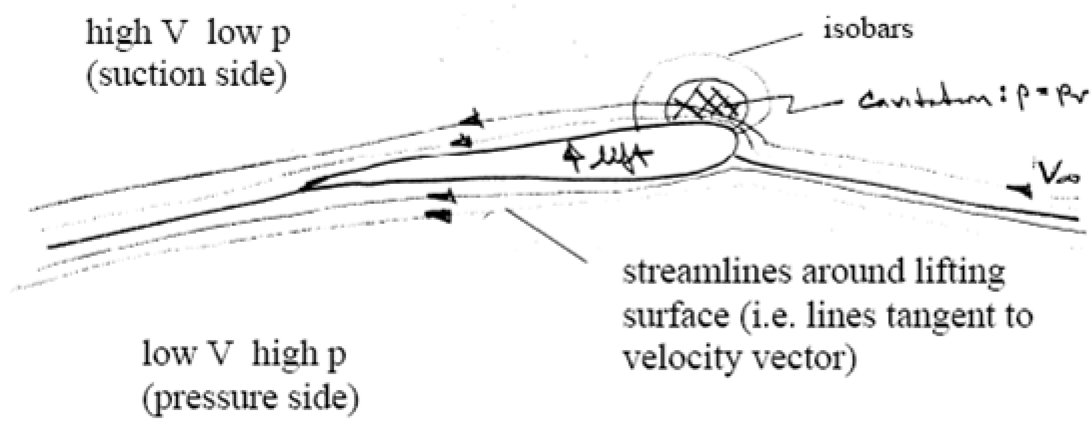
$$\tau \propto \left(dV/dy\right)^n$$

μ = coefficient of viscosity = proportionality constant for Newtonian fluid

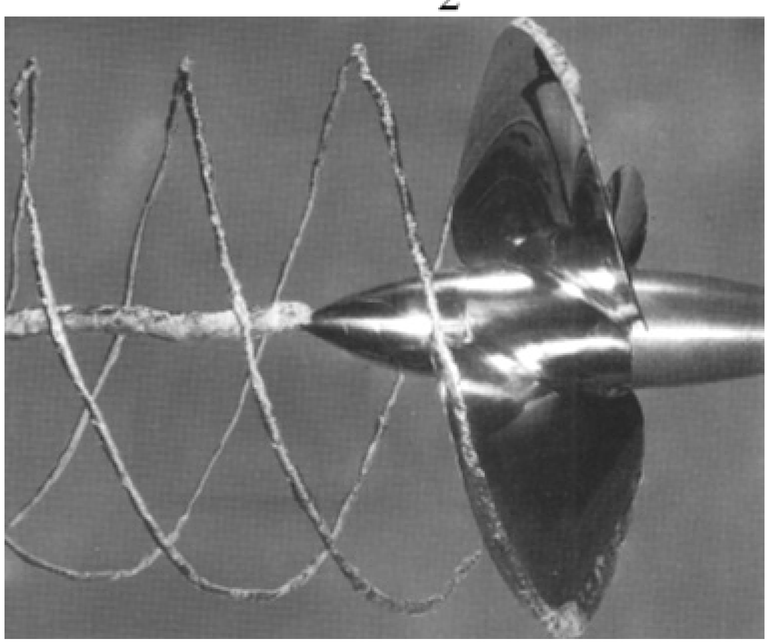
$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{N/m^2}{\frac{m}{s}/m} = \frac{Ns}{m^2}$$

$$\nu = \frac{\mu}{\rho} = \frac{m^2}{s} = \text{kinematic viscosity}$$

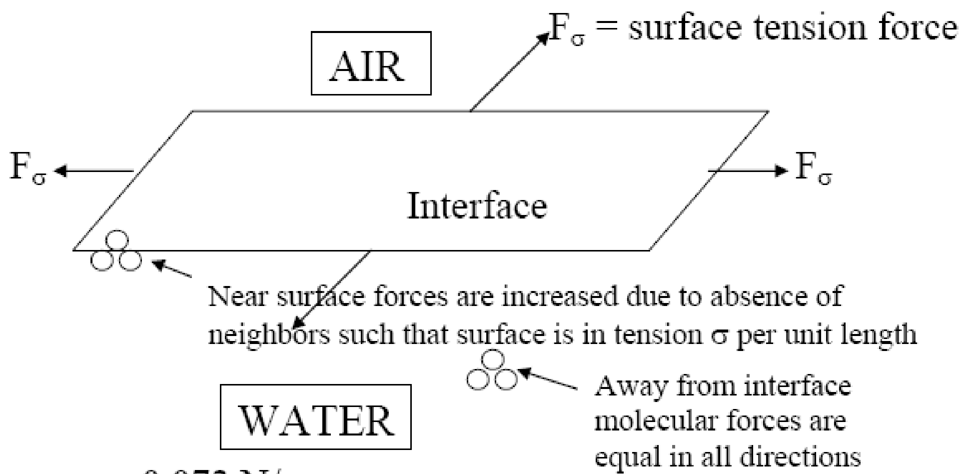
- Vapor pressure and Cavitation



$$\text{Cavitation number} = \frac{p - p_v}{\frac{1}{2}\rho V_\infty^2} < 0 \text{ implies cavitation}$$


Cavitation at ship propeller

- Surface tension and capillary effects



$\sigma_{air/water} = 0.073$ N/m

$F_\sigma = \sigma \times L =$ line force with direction normal to the cut

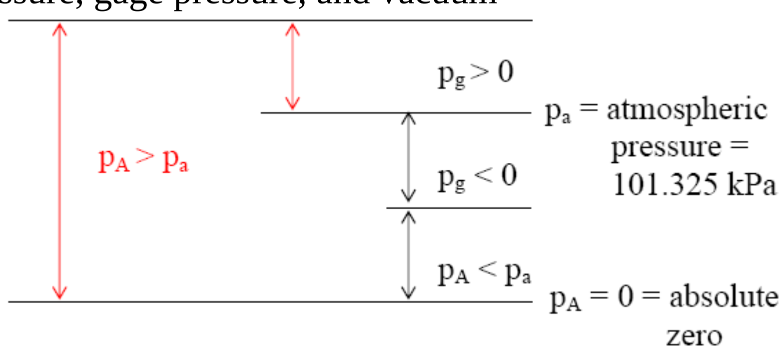
L=length of cut through the interface

Chapter 2: Pressure and Fluid Statics

- Pressure
  - For a static fluid, only stress is the normal stress since by definition a fluid subjected to a shear stress must deform and undergo motion. Normal stresses are referred to as pressure p.
  - P is isotropic, one value at a point which is independent of direction, a scalar.

$$-p = \tau_{xx} = \tau_{yy} = \tau_{zz} \quad i = j \qquad \text{normal stresses} = -p$$

- Absolute pressure, gage pressure, and vacuum

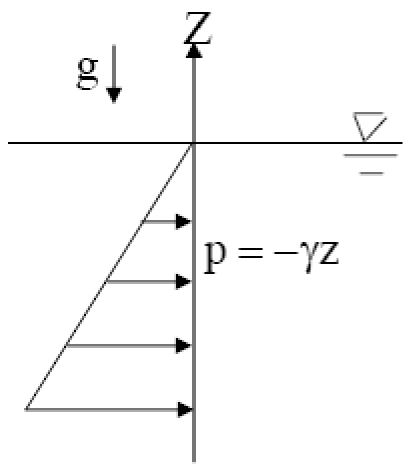


For $p_A > p_a$, $\quad p_g = p_A - p_a =$ gage pressure

For $p_A < p_a$, $\quad p_{vac} = -p_g = p_a - p_A =$ vacuum pressure

- Pressure variation

    - for a uniform-density fluid



$$p_1 + \gamma z_1 = p_2 + \gamma z_2 = \text{constant}$$

$$p + \gamma z = \text{constant} \qquad \text{piezometric pressure}$$

$$p = -\gamma z \quad \begin{array}{l}\text{increase linearly with depth}\\ \text{decrease linearly with height}\end{array}$$

    - in the troposphere

$$\frac{p}{p_o} = \left[\frac{T_o - \alpha(z - z_o)}{T_o}\right]^{g/\alpha R}$$

i.e., p decreases for increasing z

α = lapse rate = 6.5 °K/km

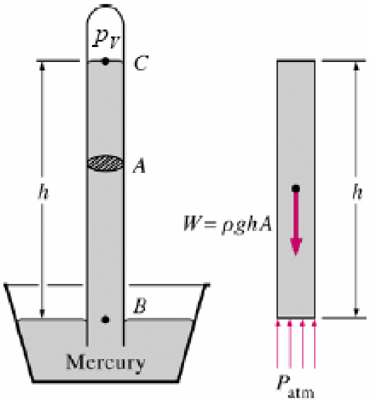
    - in the stratosphere

$$p = p_o \exp[-(z - z_o)g / RT_s]$$

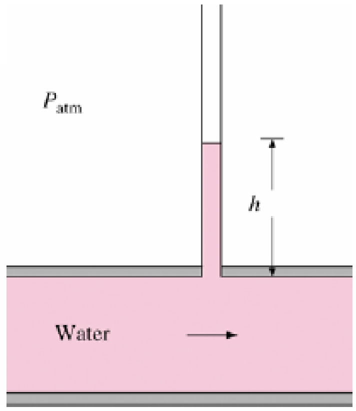
i.e., p decreases exponentially for increasing z.

- Pressure measurements

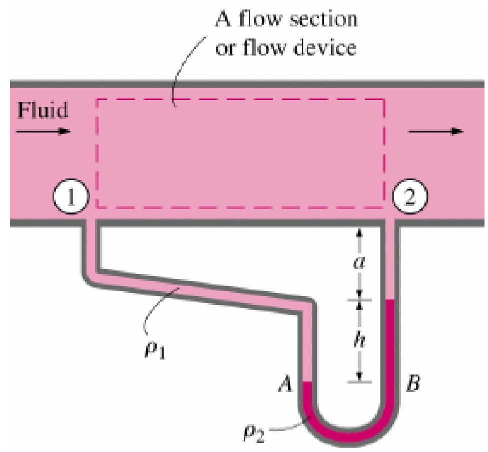
1. Barometer



2. Piezometer

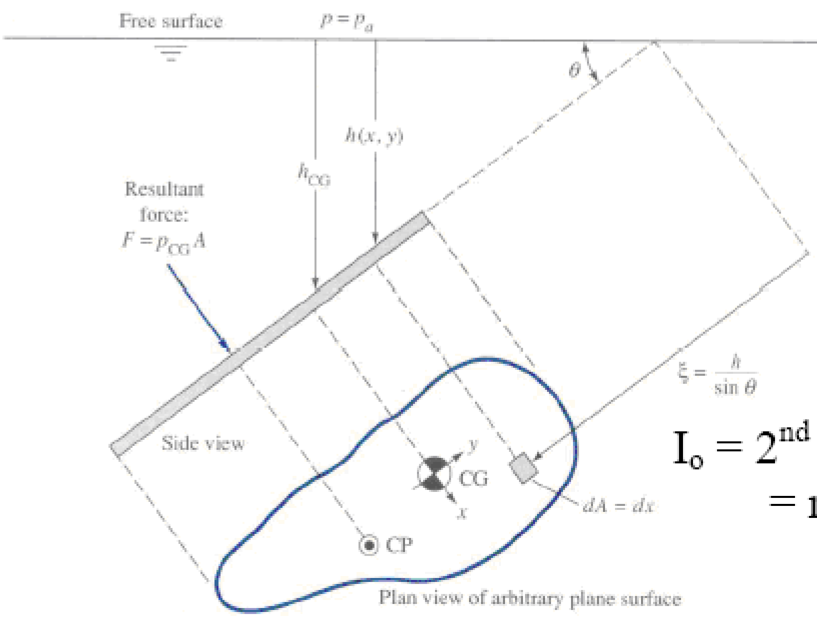


3. Differential manometer



- Hydrostatic forces on plane surfaces

For a static fluid, the shear stress is zero and the only stress is the normal stress, i.e. pressure.



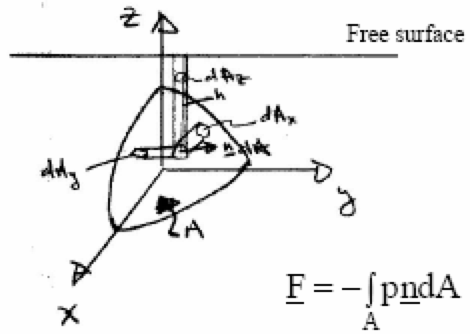
$$F = \bar{p}A$$

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$$

I_o = 2nd moment of area about 0-0
= moment of inertia

Magnitude of resultant hydrostatic force on plane surface is product of pressure at centroid of area and area of surface.

- Hydrostatic force on curved surface



$p = \gamma h$

$\underline{F} = -\int_A p\underline{n}dA$

h = distance belc
free surface

<u>Horizontal Components</u> (x and y components)

$F_x = \underline{F}\cdot\hat{i} = -\int_A p\underline{n}\cdot\hat{i}dA$

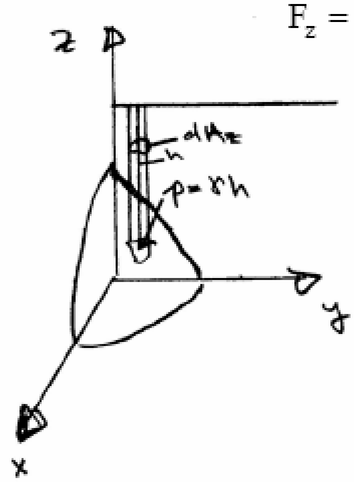
$= -\int_{A_x} pdA_x$

dA_x = projection of $\underline{n}dA$ or
plane ⊥ to x-direction

$F_y = \underline{F}\cdot\hat{j} = -\int_{A_y} pdA_y$ $dA_y = \underline{n}\cdot\hat{j}dA$

= projection $\underline{n}dA$
onto plane ⊥ to
y-direction

<u>Vertical Components</u>



$F_z = \underline{F}\cdot\hat{k} = -\int_A p\underline{n}\cdot\hat{k}dA$

$= -\int_{A_z} pdA_z$ $p = \gamma h$

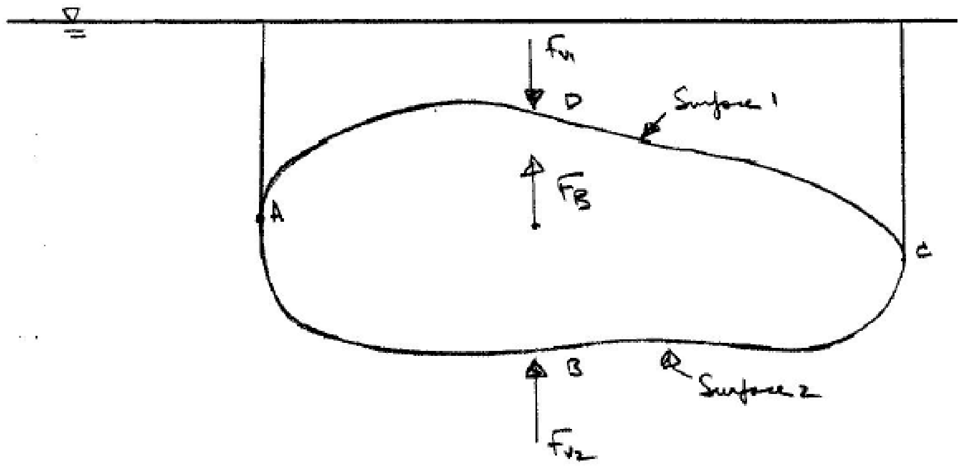
h=distance
below free
surface

$= \gamma\int_{A_z} hdA_z = \gamma V$

= weight of
fluid above
surface A

- Buoyancy

Archimedes principle



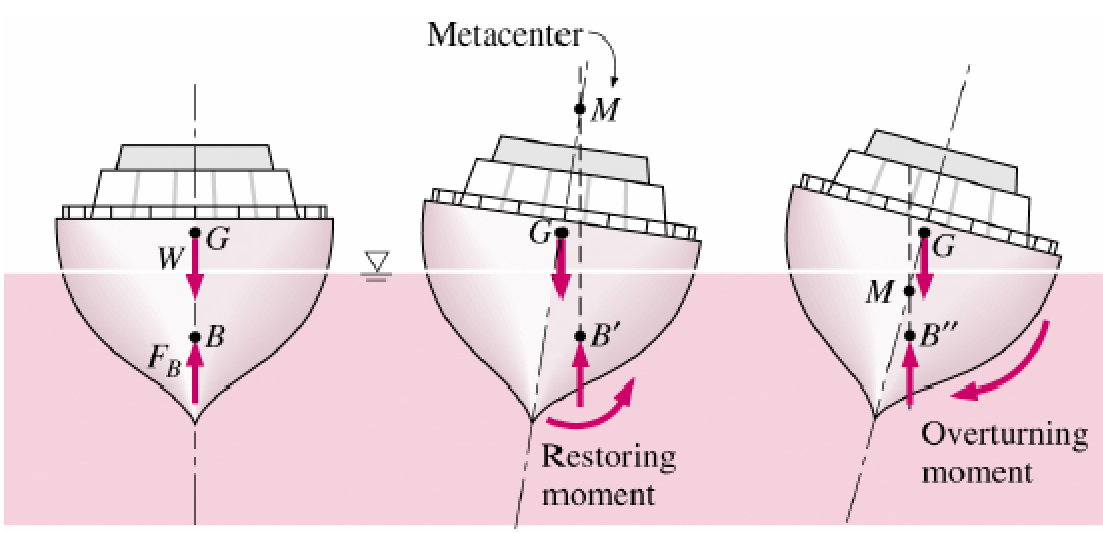
$F_B = F_{v2} - F_{v1}$

= fluid weight above Surface 2 (ABC)
– fluid weight above Surface 1 (ADC)

= fluid weight equivalent to body volume ¥

$F_B = \rho g ¥$ ¥ = submerged volume

- Stability of immersed and floating bodies



(a) Stable (b) Stable (c) Unstable

$CM = I_{OO} / \forall$

$GM = CM - CG$

$GM = \dfrac{I_{OO}}{\forall} - CG$

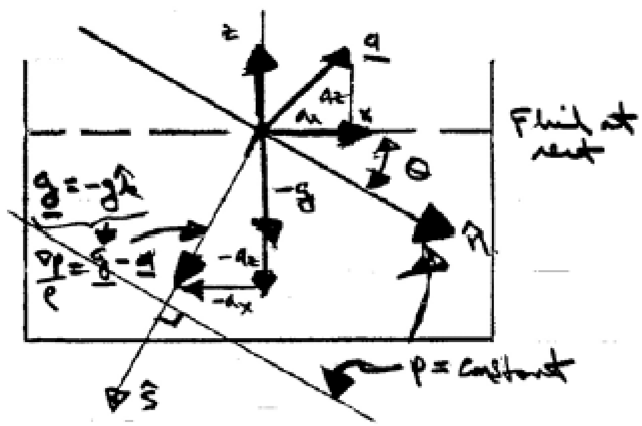
$GM > 0$ Stable

$GM < 0$ Unstable

I_{OO} = moment of inertia of waterplane area about centerplane axis

Fluids in rigid-body motion

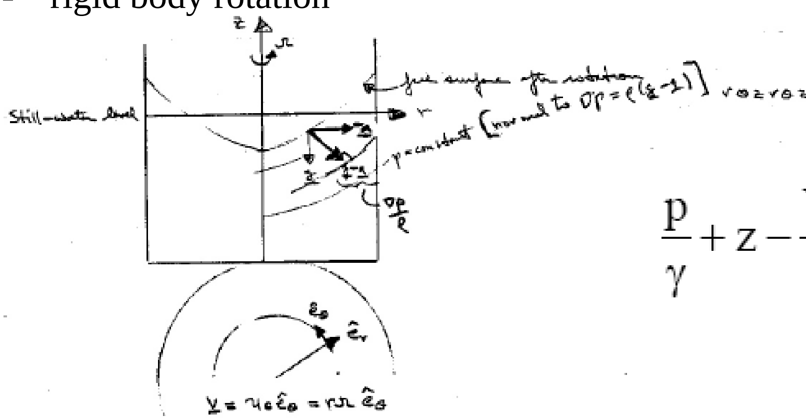
- uniform linear acceleration



$$\frac{\partial p}{\partial x} = -\rho a_x$$

1. $a_x < 0$ p increase in +x
2. $a_x > 0$ p decrease in +x
3. $a_z < 0$ and $|a_z| > g$ p increase in +z

- rigid body rotation



$$\frac{p}{\gamma} + z - \frac{V^2}{2g} = \text{constant}$$

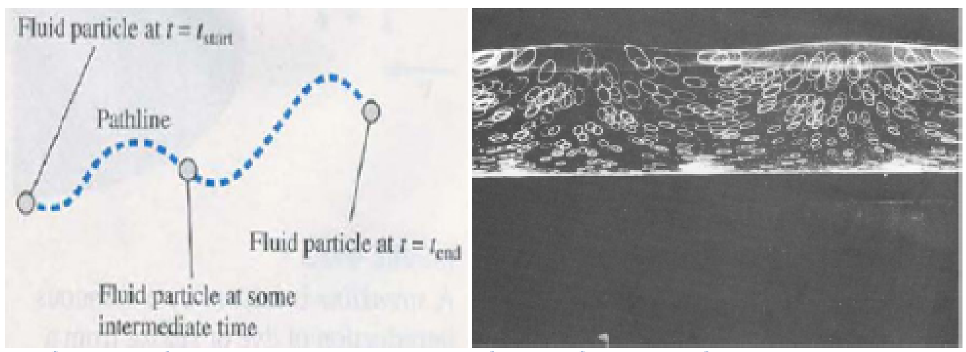
$$V = r\Omega$$

Chapter 3: Bernoulli equation

- Flow patterns
    1) A ***streamline*** $\psi(\underline{x}, t)$ is a line that is everywhere tangent to the velocity vector at a given instant.

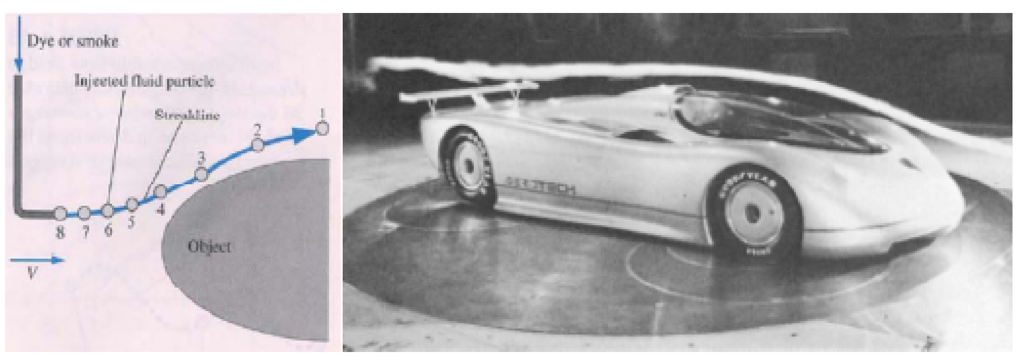
Examples of streamlines around an airfoil (left) and a car (right)

    2) A ***pathline*** is the actual path traveled by a given fluid particle.



An illustration of pathline (left) and an example of pathlines, motion of water induced by surface waves (right)

    3) A ***streakline*** is the locus of particles which have earlier passed through a particular point.



An illustration of streakline (left) and an example of streaklines, flow past a full-sized streamlined vehicle in the GM aerodynamics laboratory wind tunnel, and 18-ft by 34-ft test section facilility by a 4000-hp, 43-ft-diameter fan (right)

Note:

1. For steady flow, all 3 coincide.
2. For unsteady flow, $\psi(t)$ pattern changes with time, whereas pathlines and streaklines are generated as the passage of time

- Bernoulli equation

$$p + \frac{1}{2}\rho V^2 + \gamma z = C \quad \text{(along a streamline)}$$

- Physical interpretation: work-energy principle

An alternate but equivalent form of the Bernoulli equation is

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = constant$$

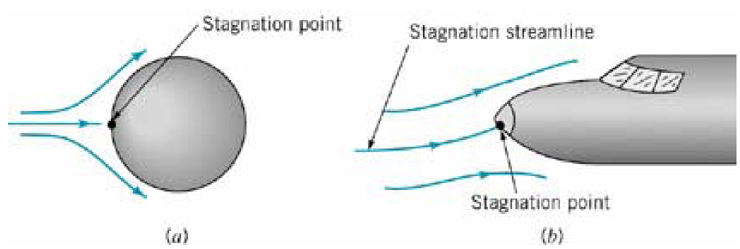
along a streamline.

Pressure head: $\frac{p}{\gamma}$ Velocity head: $\frac{V^2}{2g}$ Elevation head: z

Static pressure: p

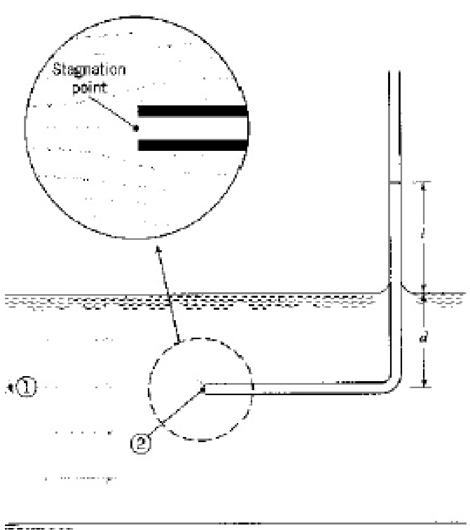
Dynamic pressure: $\frac{1}{2}\rho V^2$

Hydrostatic pressure: γz



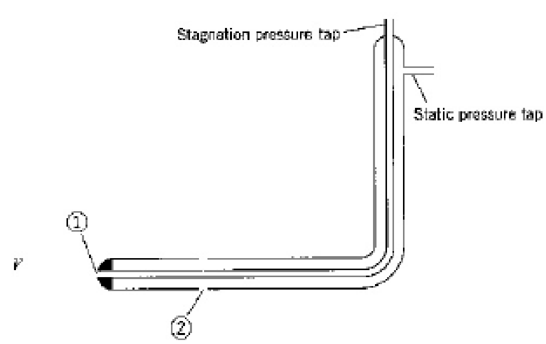
Stagnation points on bodies in flowing fluids.

Stagnation Tube

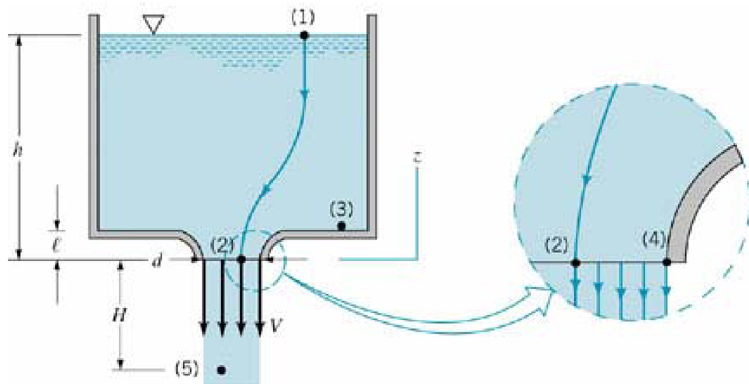


Pitot Tube

$$\frac{p_1}{\gamma} + \cancel{\frac{V_1^2}{2g}}^{\,0} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$



Free Jets

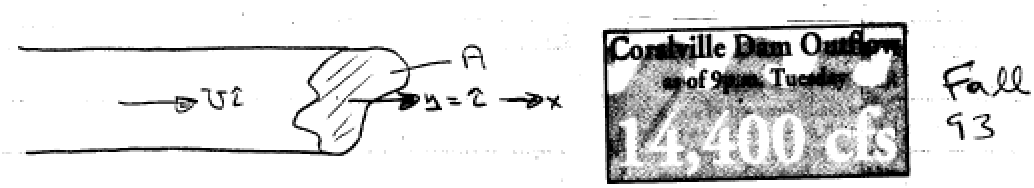


Vertical flow from a tank

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

- Volume rate of flow

cross-sectional area oriented normal to velocity vector (simple case where V ⊥ A)



U = constant: Q = volume flux = UA [m/s × m^2 = m^3/s]

U ≠ constant: $Q = \int_A U\,dA$

- Energy grade line (EGL) and hydraulic grade line (HGL)

Define $$\text{HGL} = \frac{p}{\gamma} + z$$

$$\text{EGL} = \frac{p}{\gamma} + z + \frac{V^2}{2g}$$

point-by-point application is graphically displayed

HGL corresponds to pressure tap measurement + z
EGL corresponds to stagnation tube measurement + z

$EGL_1 = EGL_2 + h_L$
for $h_p = h_t = 0$

EGL = HGL if V = 0

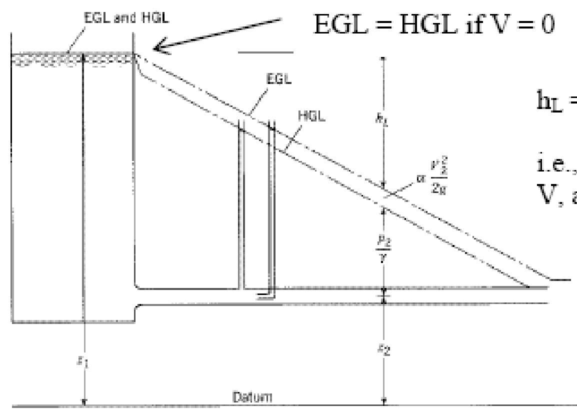


FIGURE 7.4
EGL and HGL in a straight pipe.

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

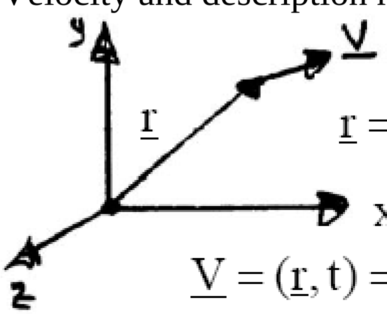
i.e., linear variation in L for D, V, and f constant

f = friction factor
f = f(Re)

- Limitations of Bernoulli equation
  (1) Inviscid
  (2) Incompressible
  (3) Steady
  (4) Conservative body force

Chapter 4: Fluids Kinematics

- Velocity and description methods



$$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\underline{V} = (\underline{r}, t) = u\hat{i} + v\hat{j} + w\hat{k}$$

Two approaches to analyzing the velocity field:

- Lagrangian: keep track of individual fluids particles (i.e., solve F = Ma for each particle): $\mathbf{V_p} = u_p\hat{i} + v_p\hat{j} + w_p\hat{k}$
- Eulerian: focus attention on a fixed point $\mathbf{x} = x\hat{i} + y\hat{j} + z\hat{k}$ in space, $\mathbf{V} = \mathbf{V}(\mathbf{x},t) = u\hat{i} + v\hat{j} + w\hat{k}$**,**
  where u = u(x,y,z,t), v = v(x,y,z,t), w = w(x,y,z,t)

- Acceleration Field and Material Derivative:

<u>Eulerian</u> approach: the velocity is a function of both space and time.

$$\mathbf{V} = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$$

Total Acceleration = Local Acceleration + Convective Acceleration

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

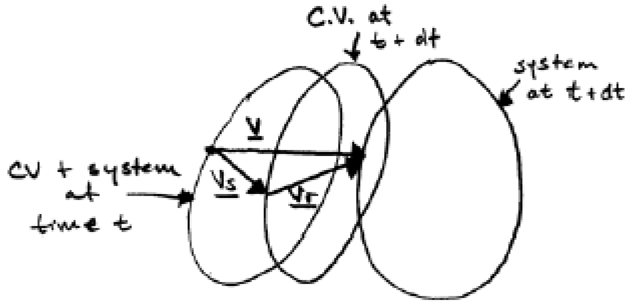
$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

- Separation, vortices, turbulence, and flow classification:

  - One-, Two-, and Three-dimensional Flow
  - Steady vs. Unsteady Flow
  - Incompressible and Compressible Flow
  - Viscous and Inviscid Flows
  - Rotational vs. Irrotational Flow
  - Laminar vs. Turbulent Viscous Flows
  - Internal vs. External Flows
  - Separated vs. Unseparated Flow

- Basic Control-Volume Approach and RTT:



Moving deforming CV:

$\underline{V}_r = \underline{V} - \underline{V}_s$

$\underline{V}$ = fluid velocity
$\underline{V}_s$ = CS defining CV velocity
$\underline{V}_r$ = relative velocity

} all in same coordinate system

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0} \frac{(B_{cv} + \Delta B)_{t+\Delta t} - (B_{cv} + \Delta B)_t}{\Delta t}$$

$$= \underbrace{\lim_{\Delta t \to 0} \frac{B_{cv_{t+\Delta t}} - B_{cv_t}}{\Delta t}}_{①} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta B_{t+\Delta t} - \Delta B_t}{\Delta t}}_{②}$$

1 = time rate of change of B in CV $= \dfrac{dB_{cv}}{dt} = \dfrac{d}{dt}\int_{cv} \beta\rho \, d\forall$

2 = net outflux of B from CV across CS $= \int_{CS} \beta\rho \underline{V}_R \cdot \underline{n} \; DA$

$$\frac{dB_{SYS}}{dt} = \frac{d}{dt}\int_{CV} \beta\rho \, d\forall + \int_{CS} \beta\rho \underline{V}_R \cdot \underline{n} \; dA$$

$B =$ extensive property $= \int_{CV} \beta \mathrm{dM} = \int_{CV} \beta\rho \mathrm{d}\forall$

$\beta =$ intensive property

Example: Continuity equation: $B = \text{mass}$, $\beta = 1$

$$-\frac{d}{dt}\int_{CV} \rho d\forall = \int_{CS} \rho \mathbf{V} \cdot \mathbf{dA}$$ integral form

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{V} = 0$$ differential form