1. Shear stress

A fluid is placed in the area between two parallel plates. The upper plate is movable and connected to a weight by a cable as shown in Figure 1.6. Calculate the velocity of the plate. Assume the fluid to be water, m = 0.002 kg, Δy = 5 mm, g =9.81 m/s^2, and the area of contact A =0.5 m^2. Assume that steady state is achieved.

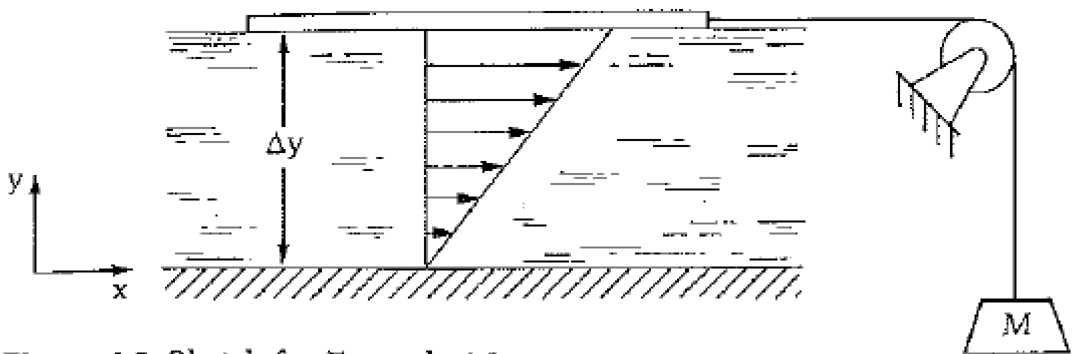
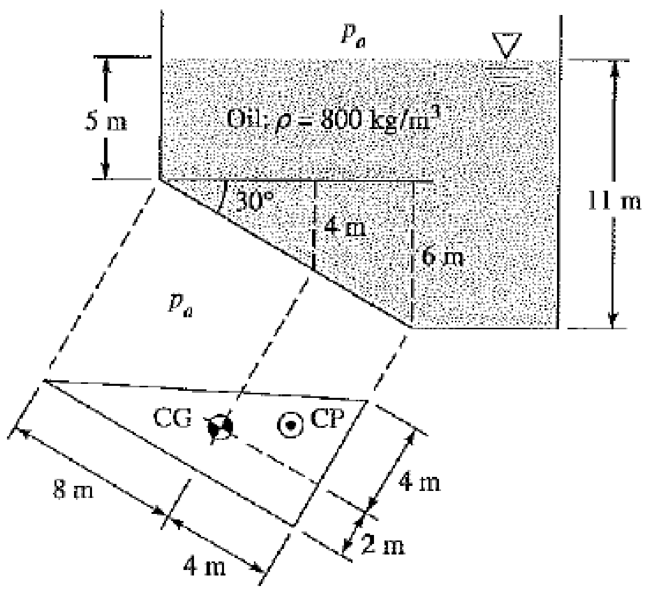


Figure 1.6. Sketch for Example 1.3.

2. Hydrostatic pressure on a plane surface

A tank of oil has a right-triangular panel near the bottom, as in Fig. E2.6. Omitting p_a, find the (a) hydrostatic force and (b) CP on the panel.



E2.6

3. Bernoulli equation

Consider water flow through an enlargement placed in a circular duct (Figure 3.22). A manometer is placed in the line and used to measure the pressure difference across the expansion. For the dimensions given, calculate the volume rate of flow in the pipe. Take the manometer fluid to be mercury.

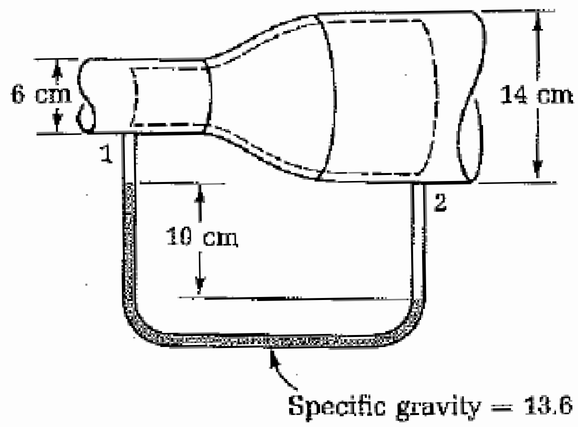


Figure 3.22. Flow through an enlargement.

4. Fluid kinematics

A two-dimensional velocity field is given by

$\mathbf{V}=(x^2-y^2+x)\mathbf{i} - (2xy+y)\mathbf{j}$

in arbitrary units. At (x, y) =(1, 2), compute (a) the accelerations a_x and a_y, (b) the velocity component in the direction θ =40˚, (c) the direction of the maximum velocity, and (d) the direction of maximum acceleration.

Solutions:

1. Shear Stress

For water, $\mu = 0.89\times10^{-3}$ N·s/m^2.

Force applied: $M = mg = 0.002\times9.81=0.0196$ N.

Shear stress: $\tau = \frac{M}{A} = 0.0196\div0.5=0.04$ N/m^2

Applying Newton's law of viscosity:

$$\tau = \mu\frac{dV}{dy} = \mu\frac{\Delta V}{\Delta y} = \mu\frac{V}{\Delta y} \implies V = \frac{\tau\Delta y}{\mu} = \frac{0.04\times0.005}{0.89\times10^{-3}} = \underline{0.225 \text{ m/s}}$$

2. Hydrostatic pressure on a plane surface

Part (a) The triangle has properties given in Fig. 2.13*c*. The centroid is one-third up (4 m) and one-third over (2 m) from the lower left corner, as shown. The area is

$$\tfrac{1}{2}(6 \text{ m})(12 \text{ m}) = 36 \text{ m}^2$$

The moments of inertia are

$$I_{xx} = \frac{bL^3}{36} = \frac{(6 \text{ m})(12 \text{ m})^3}{36} = 288 \text{ m}^4$$

and

$$I_{xy} = \frac{b(b-2s)L^2}{72} = \frac{(6 \text{ m})[6 \text{ m} - 2(6 \text{ m})](12 \text{ m})^2}{72} = -72 \text{ m}^4$$

The depth to the centroid is $h_{CG} = 5 + 4 = 9$ m; thus the hydrostatic force from Eq. (2.44) is

$$F = \rho g h_{CG} A = (800 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(9 \text{ m})(36 \text{ m}^2)$$
$$= 2.54 \times 10^6 \text{ (kg}\cdot\text{m)/s}^2 = 2.54 \times 10^6 \text{ N} = 2.54 \text{ MN} \qquad \textit{Ans. (a)}$$

Part (b) The CP position is given by Eqs. (2.44):

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(288 \text{ m}^4)(\sin 30°)}{(9 \text{ m})(36 \text{ m}^2)} = -0.444 \text{ m}$$

$$x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} = -\frac{(-72 \text{ m}^4)(\sin 30°)}{(9 \text{ m})(36 \text{ m}^2)} = +0.111 \text{ m} \qquad \textit{Ans. (b)}$$

The resultant force $F = 2.54$ MN acts through this point, which is down and to the right of the centroid, as shown in Fig. E2.6.

3. Bernoulli equation

$$Q = A_1V_1 = A_2V_2$$

and

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g_c} + \frac{gz_1}{g_c} = \frac{p_2}{\rho} + \frac{V_2^2}{2g_c} + \frac{gz_2}{g_c}$$

Rearrangement gives

$$\frac{p_2 - p_1}{\rho} = \frac{V_1^2 - V_2^2}{2g_c}$$

where $z_2 = z_1$ at the centerline for a horizontal configuration. Substitution for velocity from the continuity equation yields

$$\frac{p_2 - p_1}{\rho} = \left(\frac{Q^2}{A_1^2} - \frac{Q^2}{A_2^2}\right)\frac{1}{2g_c}$$

From the manometer reading we have

$$\frac{p_2 - p_1}{\rho_{Hg}}\frac{g_c}{g} = 0.10 \text{ m}$$

The Bernoulli equation now becomes

$$\frac{\rho_{Hg}}{\rho} 0.10 \text{ m} = \frac{Q^2}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right)$$

where $A_2 = \pi D_2^2/4 = 0.015\,4 \text{ m}^2$ and $A_1 = \pi D_1^2/4 = 0.002\,83 \text{ m}^2$. Thus

$$0.10 = \frac{Q^2}{2(9.81)}(124\,861 - 4\,216.6)\frac{\rho}{13.6\rho} = 452.1Q^2$$

$Q = 0.015 \text{ m}^3/\text{s}$

4. Fluid kinematics

Solution: (a) Do each component of acceleration:

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (x^2 - y^2 + x)(2x+1) + (-2xy - y)(-2y) = a_x$$

$$\frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (x^2 - y^2 + x)(-2y) + (-2xy - y)(-2x-1) = a_y$$

At (x, y) = (1, 2), we obtain $\mathbf{a_x} = 18\mathbf{i}$ and $\mathbf{a_y} = 26\mathbf{j}$ *Ans.* (a)

(b) At (x, y) = (1, 2), $\mathbf{V} = -2\mathbf{i} - 6\mathbf{j}$. A unit vector along a 40° line would be $\mathbf{n} = \cos 40°\mathbf{i} + \sin 40°\mathbf{j}$. Then the velocity component along a 40° line is

$$V_{40°} = \mathbf{V}\cdot\mathbf{n}_{40°} = (-2\mathbf{i} - 6\mathbf{j})\cdot(\cos 40°\mathbf{i} + \sin 40°\mathbf{j}) \approx \mathbf{5.39 \text{ units}} \quad \textit{Ans. (b)}$$

(c) The maximum acceleration is $\mathbf{a_{max}} = [18^2 + 26^2]^{1/2} = \mathbf{31.6}$ units at $\angle 55.3°$ *Ans.* (c, d)