

Chapter 1 INTRODUCTION AND BASIC CONCEPTS

1. Fluids and no-slip condition

- Fluid: a substance that deforms continuously when subjected to shear stresses
- No-slip condition: no relative motion between fluid and boundary

2. Basic units

	Dimension	SI unit	BG unit
Velocity \underline{V}	L/t	m/s	ft/s
Acceleration \underline{a}	L/t^2	m/s^2	ft/s^2
Force \underline{F}	ML/t^2	N ($Kg \cdot m/s^2$)	lbf
Pressure \underline{p}	F/L^2	Pa (N/m^2)	lbf/ft^2
Density $\underline{\rho}$	M/L^3	Kg/m^3	$slug/ft^3$
Internal energy \underline{u}	FL/M	J/Kg ($N \cdot m/kg$)	BTU/lbm

3. Weight and mass

- $\mathcal{W}^o(N) = m(Kg) \cdot g$, where $g = 9.81 \text{ m/s}^2$
- $\mathcal{W}^o(lbf) = m(slug) \cdot g$, where $g = 32.2 \text{ ft/s}^2$
- $1 \text{ N} = 1 \text{ Kg} \times 1 \text{ m/s}^2$
- $1 \text{ lbf} = 1 \text{ slug} \times 1 \text{ ft/s}^2$
- $1 \text{ slug} = 32.2 \text{ lbm}$ (weighs 32.2 lb under standard gravity)

4. Properties involving mass or weight of fluid

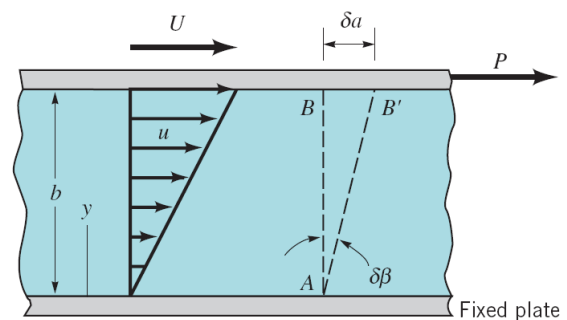
- Specific weight $\gamma = \rho g$ (N/m^3)
- Specific gravity $SG = \gamma/\gamma_{water}$

5. Viscosity

- Newtonian fluid: $\tau = \mu \dot{\theta} = \mu \frac{du}{dy}$
 - τ Shear stress (N/m^2)
 - $\dot{\theta} = \frac{\delta\theta}{\delta t} = \frac{1}{\delta t} \left(\frac{\delta u \delta t}{\delta y} \right)$
 - μ Coefficient of viscosity (Ns/m^2)
 - $\nu = \mu/\rho$ Kinematic viscosity (m^2/s)
- Non-Newtonian fluid: $\tau \propto \left(\frac{du}{dy} \right)^n$

Ex) Couette flow

$$u(y) = \frac{U}{h} y, \tau = \mu \frac{du}{dy} = \mu \frac{U}{h}$$



6. Vapor pressure and cavitation

- When the pressure of a liquid falls below the vapor pressure it evaporates, i.e., changes to a gas.
- If the pressure drop is due to fluid velocity, the process is called cavitation.
- Cavitation number

$$C_a = \frac{p - p_\infty}{1/2 \rho V_\infty^2}$$

- $C_a < 0$ implies cavitation

7. Surface tension

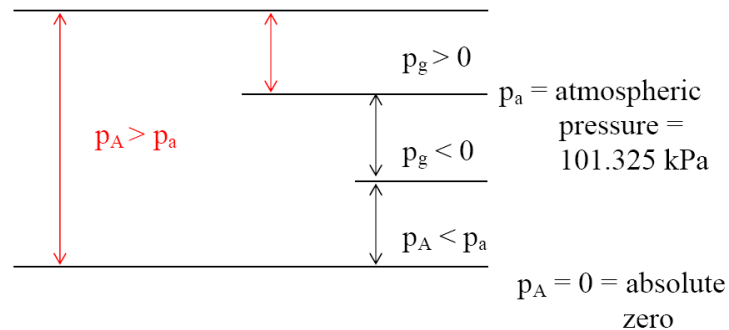
- Surface tension force

$$F_\sigma = \sigma \cdot L$$

- F_σ = line force with direction normal to the cut
- σ = surface tension [N/m]
- L = length of cut through the interface

Chapter 2 PRESSURE AND FLUID STATICS

1. Absolute pressure, Gage pressure, and Vacuum



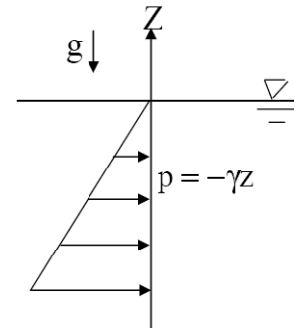
- $p_A > p_a, p_g = p_A - p_a = \text{gage pressure}$
- $p_A < p_a, p_{vac} = -p_g = p_a - p_A = \text{vacuum pressure}$

2. Pressure variation with elevation

- For a static fluid, pressure varies only with elevation z and is constant in horizontal x, y planes.

$$\frac{\partial p}{\partial x} = 0, \frac{\partial p}{\partial y} = 0, \frac{\partial p}{\partial z} = -\rho g = -\gamma$$

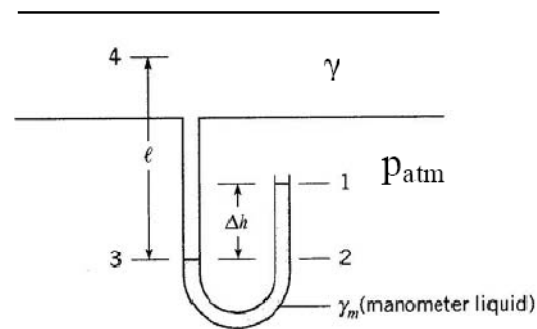
- If the density of fluid is constant,
 - $p + \gamma z = \text{constant}$ (piezometric pressure)
 - $\frac{p}{\gamma} + z = \text{constant}$ (piezometric head)
 - $p_{z=0} = 0$ gage, $p = -\gamma z$: increase linearly with depth, decrease linearly with height



3. Pressure measurements (Manometry)

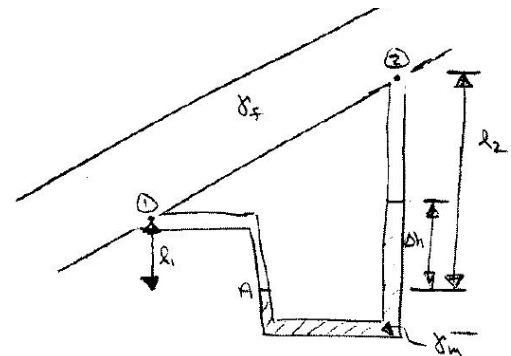
1) U-tube manometer

- $p_1 + \gamma_m \Delta h - \gamma \ell = p_4 \quad p_1 = p_{atm}$
- $p_4 = \gamma_m \Delta h - \gamma \ell$ gage
 $= \gamma_{water} (SG_m \Delta h - SG \ell)$



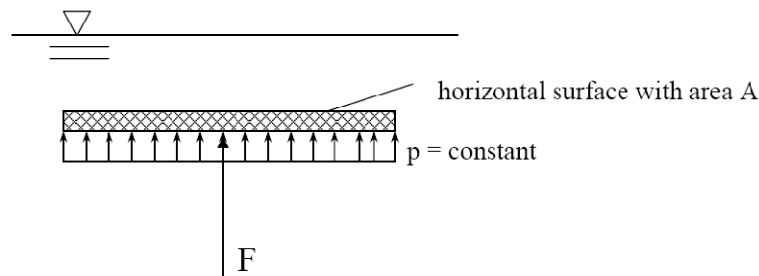
2) Differential U-tube manometer

- $p_1 + \gamma_f \ell_1 - \gamma_m \Delta h - \gamma_f (\ell_2 - \Delta h) = p_2$
- $p_1 - p_2 = \gamma_f (\ell_2 - \ell_1) + (\gamma_m - \gamma_f) \Delta h$
- $\frac{(p_1/\gamma_f + \ell_1) - (p_2/\gamma_f + \ell_2)}{\text{difference in piezometric head}} = (\gamma_m/\gamma_f - 1) \Delta h$
 - If fluid is a gas $\gamma_f \ll \gamma_m$: $p_1 - p_2 = \gamma_m \Delta h$
 - If fluid is liquid & pipe horizontal $\ell_1 = \ell_2$:
 $p_1 - p_2 = (\gamma_m - \gamma_f) \Delta h$



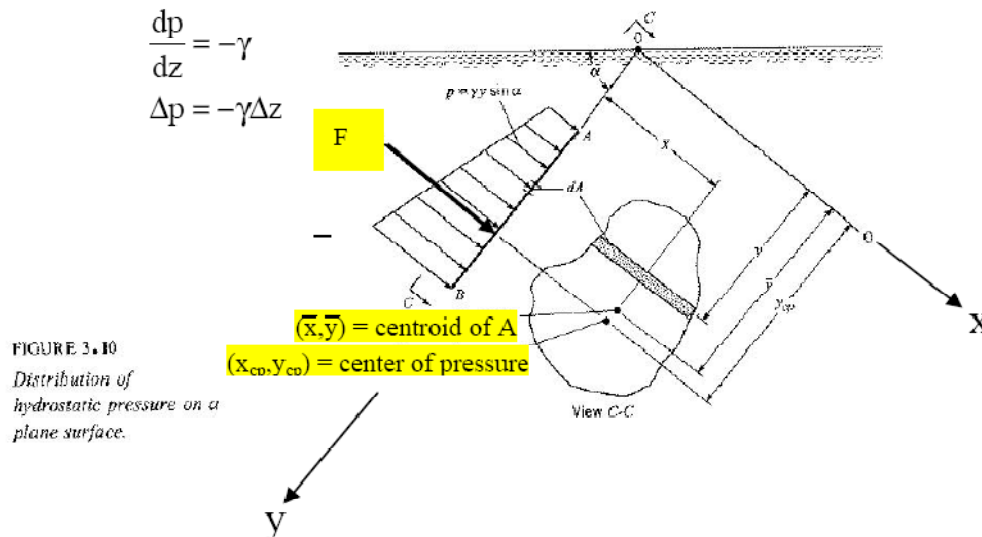
4. Hydrostatic forces on plane surfaces

1) Horizontal surfaces



- $F = pA$
- Line of action is through centroid of A, i.e., $(x_{cp}, y_{cp}) = (\bar{x}, \bar{y})$

2) Inclined surfaces

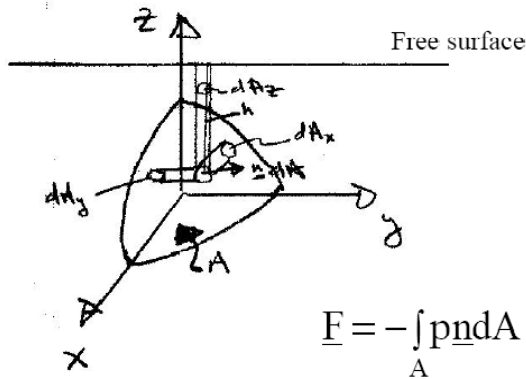


- $F = \bar{p}A$
 - $\bar{p} = \gamma \sin \alpha \bar{y}$: pressure at centroid of A
 - $\bar{y} = \frac{1}{A} \int y dA$: 1st moment of area
- Magnitude of resultant hydrostatic force on plane surface is product of pressure at centroid of area and area of surface
- Center of pressure
 - $y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$
 - $x_{cp} = \frac{\bar{I}_{xy}}{\bar{y}A} + \bar{x}$

\bar{I} : moment of inertia with respect to horizontal centeroidal axis

For plane surfaces with symmetry about an axis normal to 0-0, $\bar{I}_{xy} = 0$ and $x_{cp} = \bar{x}$

5. Hydrostatic forces on curved surfaces



$$p = \gamma h$$

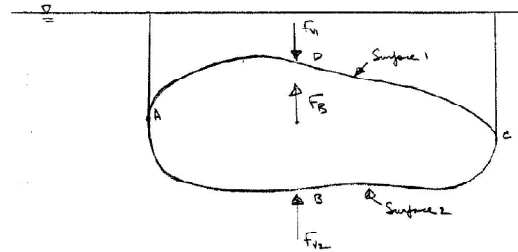
$$\underline{F} = - \int_A p \underline{n} dA$$

h = distance below free surface

- $F_x = - \int_{A_x} p dA_x$ ($dA_x = \underline{n} \cdot \hat{i} dA$: projection of $\underline{n} dA$ onto plane \perp to x -direction)
- $F_y = - \int_{A_y} p dA_y$ ($dA_y = \underline{n} \cdot \hat{j} dA$: projection of $\underline{n} dA$ onto plane \perp to y -direction)
- $F_z = - \int_{A_z} p dA_z = \gamma V = \text{weight of fluid above surface } A$

6. Buoyancy

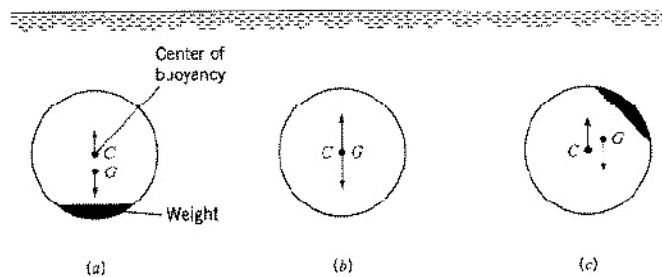
- $F_B = F_{V2} - F_{V1} = \rho g V$
- Fluid weight equivalent to body volume V
- Line of action is through centroid of V = center of buoyancy



7. Stability

1) Immersed bodies

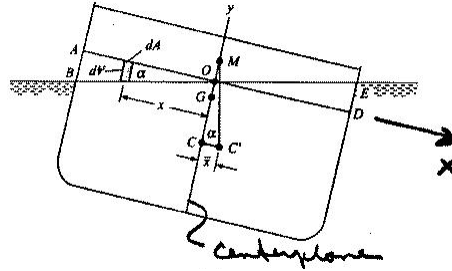
FIGURE 3.15
Conditions of stability
for immersed bodies.
(a) Stable. (b) Neutral.
(c) Unstable.



- Static equilibrium requires: $\sum F_y = 0$ and $\sum M = 0$.
- $\sum M = 0$ requires $C = G$ and the body is neutrally stable
- If C is above G : stable (righting moment when heeled)
- If G is above C : unstable (heeling moment when heeled)

2) Floating bodies

- The center of buoyancy generally shifts when the body is rotated
- Metacenter M: The point of intersection of the lines of action of the buoyant force before and after heel



- $GM = \frac{I_{00}}{V} - CG$
 - GM: metacentric height
 - I_{00} = moment of inertia of waterplane area about centerplane axis
- $GM > 0$: stable (M is above G)
- $GM < 0$: unstable (G is above M)

8. Fluids in rigid-body motion

- If no relative motion between fluid particles

$$\nabla p = \rho (\underline{g} - \underline{a})$$

- For rigid body translation: $\underline{a} = a_x \hat{i} + a_z \hat{k}$
 - $\nabla p = -\rho [a_x \hat{i} + (g + a_z) \hat{k}]$
 - $\frac{\partial p}{\partial x} = -\rho a_x$
 - $a_x < 0$, p increase in $+x$
 - $a_x > 0$, p decrease in $+x$
 - $\frac{\partial p}{\partial z} = -\rho(g + a_z)$
 - $a_z > 0$, p decrease in $+z$
 - $a_z < 0$ and $|a_z| < g$, p decrease in $+z$ but slower than g
 - $a_z < 0$ and $|a_z| > g$, p increase in $+z$
 - $p = \rho G s + \text{constant} \Rightarrow p_{gauge} = \rho G s$
 - $G = a_x^2 + (g + a_z)^2$
 - \hat{s} = unit vector in direction normal of ∇p
- For rigid body rotation: $\underline{a} = -r\Omega^2 \hat{e}_r$
 - $\nabla p = -\rho g \hat{k} + \rho r \Omega^2 \hat{e}_r$
 - $\frac{\partial p}{\partial r} = \rho r \Omega^2$ $\frac{\partial p}{\partial z} = -\rho g$ $\frac{\partial p}{\partial \theta} = 0$
 - $p = \frac{\rho}{2} r^2 \Omega^2 - \rho g z + \text{constant}$ or $\frac{p}{\gamma} + z - \frac{V^2}{2g} = \text{constant}$ ($V = r\Omega$)
 - $z = \frac{p_0 - p}{\rho g} + \frac{r^2 \Omega^2}{2g} = a + br^2$: curves of constant pressure (p_0 : pressure at $(r,z)=(0,0)$)

Chapter 3 BERNOULLI EQUATION

1. Flow patterns

- Stream line: a line that is everywhere tangent to the velocity vector at a given instant
- Pathline: the actual path traveled by a given fluid particle
- Streakline: the locus of particles which have earlier passed through a particular point

2. Streamline coordinates

- Velocity : $\underline{V}(\underline{x}, t) = v_s(\underline{x}, t)\hat{s}$
- Acceleration:

$$\underline{a} = \left(\frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial s} \right) \hat{s} + \left(\frac{\partial v_n}{\partial t} + \frac{v_s^2}{\mathfrak{R}} \right) \hat{n}$$

- $\frac{\partial v_s}{\partial t}$ = local a_s in \hat{s} direction
- $\frac{\partial v_n}{\partial t}$ = local a_n in \hat{n} direction
- $v_s \frac{\partial v_s}{\partial s}$ = convective a_s due to spatial gradient of \underline{V}
- $\frac{v_s^2}{\mathfrak{R}}$ = convective a_n due to curvature ψ : centrifugal acceleration
- \mathfrak{R} : the radius of curvature of the streamline

3. Bernoulli equation

- Euler equation: $\rho \underline{a} = \nabla(p + \gamma z)$
- Along streamline

$$\frac{v_s^2}{2} + \frac{p}{\rho} + gz = \text{constant}$$

or

$$\frac{v_{s1}^2}{2} + \frac{p_1}{\rho} + gz_1 = \frac{v_{s2}^2}{2} + \frac{p_2}{\rho} + gz_2$$

- Across streamline

$$\int \frac{v_s^2}{\mathfrak{R}} dn + \frac{p}{\rho} + gz = \text{constant}$$

- Assumptions
 - Inviscid flow
 - Steady flow
 - Incompressible flow
 - Flow along a streamline

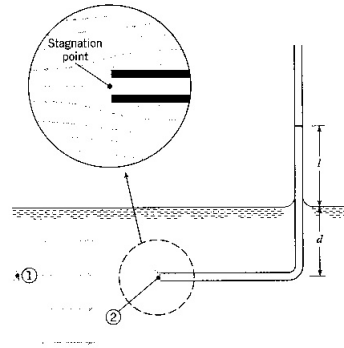
4. Applications of Bernoulli equation

1) Stagnation tube

$$p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

$$z_1 = z_2, p_1 = \gamma d, V_2 = 0, p_2 = \gamma(l + d)$$

$$V_1 = \sqrt{\frac{2}{\rho}(p_2 - p_1)} = \sqrt{\frac{2}{\rho}\gamma l} = \sqrt{2gl}$$



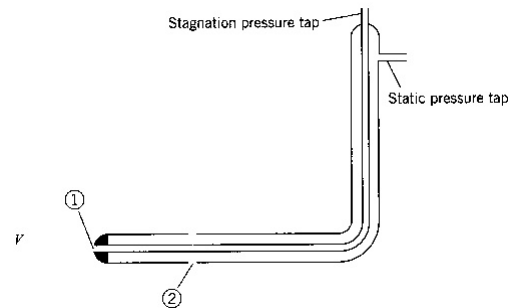
2) Pitot tube

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$V_1 = 0, h = \frac{p}{\gamma} + z$$

$$V_2 = \sqrt{2g(h_1 - h_2)}$$

$h_1 - h_2$ from manometer or pressure gage



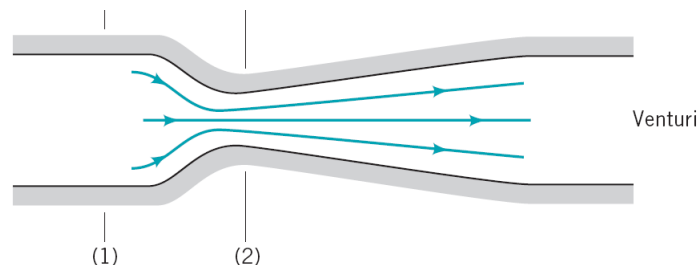
3) Simplified continuity equation

- Volume flow rate: $Q = VA$
- Mass flow rate: $\dot{m} = \rho Q = \rho VA$
- Conservation of mass: $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$
- For incompressible flow ($\rho = \text{constant}$): $V_1 A_1 = V_2 A_2$ or $Q_1 = Q_2$

4) Flow rate measurement

- If the flow is horizontal ($z_1 = z_2$), steady, inviscid, and incompressible, $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$
- If velocity profiles are uniform at sections (1) and (2), $Q = V_1 A_1 = V_2 A_2$
- Flow rate is, $Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$

Ex) Venturi meter



Chapter 4 FLUIDS KINEMATICS

1. Velocity and description Methods

- Lagrangian: keep track of individual fluids particles

$$\underline{V}_p = u_p \hat{i} + v_p \hat{j} + w_p \hat{k}$$

- Eulerian: focus attention on a fixed point in space

$$\underline{V} = \underline{V}(\underline{x}, t) = u \hat{i} + v \hat{j} + w \hat{k}$$

2. Acceleration and material derivatives

- Lagrangian:

$$\underline{a}_p = \frac{d\underline{V}_p}{dt} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$a_x = \frac{du_p}{dt} \quad a_y = \frac{dv_p}{dt} \quad a_z = \frac{dw_p}{dt}$$

- Eulerian:

$$\underline{a} = \frac{D\underline{V}}{Dt} = \frac{\partial \underline{V}}{\partial t} + (\underline{V} \cdot \nabla) \underline{V} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

where,

$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} : \text{ gradient operator}$$

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

- $\frac{\partial \underline{V}}{\partial t}$ = local or temporal acceleration. Velocity changes with respect to time at a given point.
- $(\underline{V} \cdot \nabla) \underline{V}$ = convective acceleration. Spatial gradients of velocity
- Material (substantial) derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

3. Flow classification

- One-, Two-, and Three-dimensional flow
- Steady vs. Unsteady flow
- Incompressible and Compressible flow
- Viscous and Inviscid flow
- Rotational vs. Irrotational flow
- Laminar vs. Turbulent viscous flow
- Internal vs. External flow
- Separated vs. Unseparated flow

4. Reynolds Transport Theorem (RTT)

$$\frac{dB_{sys}}{dt} = \frac{\partial}{\partial t} \int_{CV} \beta \rho dV + \int_{CS} \beta \rho \underline{V}_R \cdot \underline{n} dA$$

Special Cases:

- Non-deforming CV moving at constant velocity: $\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t} (\beta \rho) dV + \int_{CS} \beta \rho \underline{V}_R \cdot \underline{n} dA$
- Fixed CV: $\frac{dB_{sys}}{dt} = \int_{CV} \left[\frac{\partial}{\partial t} (\beta \rho) + \nabla \cdot (\beta \rho \underline{V}) \right] dV$
- Steady flow: $\frac{\partial}{\partial t} = 0$
- Uniform flow across discrete CS (steady or unsteady):

$$\int_{CS} \beta \rho \underline{V} \cdot \underline{n} dA = \sum_{CS} \beta \rho \underline{V} \cdot \underline{n} dA_{(-inlet, +outlet)}$$

5. Continuity equation

$$\frac{dM}{dt} = 0 = \frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \underline{V}_R \cdot \underline{n} dA$$

Simplifications:

- Steady flow: $-\frac{\partial}{\partial t} \int_{CV} \rho dV = 0$
- $\underline{V} = \text{constant}$ over discrete dA (flow sections): $\int_{CS} \rho \underline{V} \cdot \underline{n} dA = \sum_{CS} \rho \underline{V} \cdot \underline{A}$
- Incompressible fluid ($\rho = \text{constant}$): $\int_{CS} \rho \underline{V} \cdot \underline{A} = -\frac{\partial}{\partial t} \int_{CV} dV$ (conservation of volume)
- Steady One-dimensional flow in a conduit: $\sum_{CS} \rho \underline{V} \cdot \underline{A} = 0$, $-\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0$, for $\rho = \text{const}$ $Q_1 = Q_2$