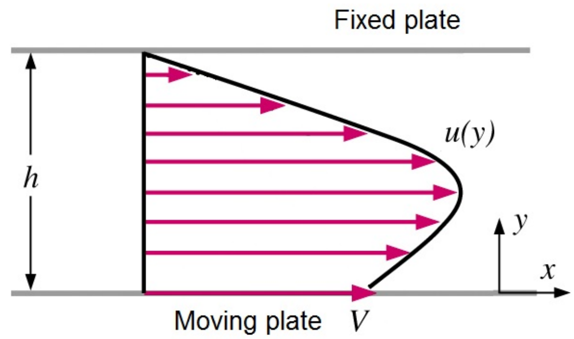
EXAM1 Solutions

Problem 1: Shear stress (Chapter 1)



Information and assumptions

- $v = 1.28 \times 10^{-2}\ ft^2/s$
- $\rho_{water} = 1.94\ slugs/ft^3$
- $SG = 1.26$
- $u(y) = \frac{B}{2\mu}(y^2 - hy) + V\left(1 - \frac{y}{h}\right)$
- $V = 0.02\ ft/s$
- $h = 1.0\ in$
- $B = -0.334\ lb/ft^3$
- $A = 100\ ft^2$

Find

- Find (a) shear stress on the plate, (b) required force, and (c) power to pull the plate

Solution

(a) Calculate dynamic viscosity

$$\mu = \nu \cdot \rho = \nu \cdot (SG \cdot \rho_{water}) = (1.28 \times 10^{-2})(1.26 \times 1.94) = 3.13 \times 10^{-2}\ \mathrm{lb \cdot s/ft^2} \quad +1$$

Shear stress

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu\left(\frac{B}{2\mu}(2y - h) - \frac{V}{h}\right)$$

Shear stress at the wall (at y=0)

$$\therefore \tau = (3.13 \times 10^{-2})\left(\frac{(-0.334)}{(2)(3.13 \times 10^{-2})}\left(2 \times 0 - \frac{1}{12}\right) - \frac{(0.02)}{(1/12)}\right) = \mathbf{0.0064\ lb/ft^2} \quad +7$$

(b) Friction force

$$F = \tau \cdot A = (0.0064)(100) = \mathbf{0.64\ lb} \quad +1$$

(c) Power

$$P = F \cdot V = (0.64)(0.02) = \mathbf{0.0128\ lb \cdot ft/s} \quad +1$$

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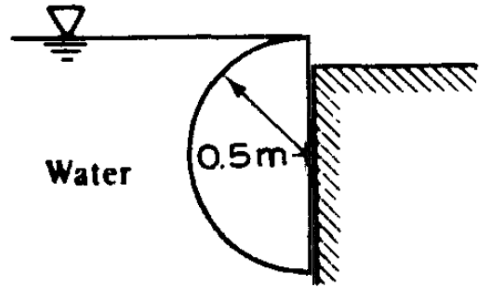
Problem 2: Hydrostatic force (Chapter 2)



Information and assumptions

- Length of barrier 8 m
- $\gamma = 9.80\ kN/m^3$

Find

- Find (a) the horizontal force (magnitude F_H and location y_cp) and vertical force (magnitude F_V) exerted on the curved surface of the barrier and (c)the barrier weight W

Solution

(a) Horizontal pressure force

$$F_H = \gamma h_c A$$

$$F_H = (9.80)(0.5)(8 \times 1) = \mathbf{39.2\ kN} \quad +3$$

Location of the horizontal force

$$y_{cp} = \bar{y} + \frac{I}{\bar{y}A}$$

$$y_{cp} = 0.5 + \frac{(8)(1)^3/12}{(0.5)(8 \times 1)} = 0.5 + 0.167 = \mathbf{0.667\ m} \quad +3$$

(b) Vertical pressure force

$$F_V = \gamma \forall$$

$$F_V = (9.80)(\pi)(0.5)^2(8)\left(\frac{1}{2}\right) = \mathbf{30.8\ kN} \quad +3$$

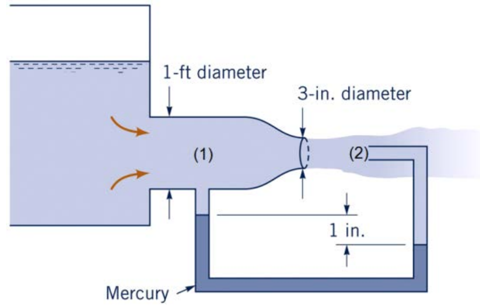
(c) Since the contact is frictionless the barrier weight must balance the vertical hydrostatic force,

$$\sum F = F_V - W = 0$$

$$\therefore W = \mathbf{30.8\ kN} \quad +1$$

EXAM1 Solutions

Problem 3: Bernoulli equation (Chapter 3)



Information and assumptions

- $\gamma = 62.4\ lb/ft^3$
- Ignore friction loss
- SG = 13.6 for the manometer fluid
- $g = 32.2\ ft/s^2$

Find

- Determine (a) the volume flow rate Q and (b) the water-jet velocity V_j leaving nozzle

Solution

(a) Bernoulli equation

$$p_1 + \frac{1}{2}\rho V_1^2 + z_1 = p_2 + \frac{1}{2}\rho V_2^2 + z_2$$

Applying assumptions $z_1 = z_2$ and $V_2 = 0$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 \quad +5$$

Also, for the manometer,

$$p_2 - p_1 = \gamma(SG - 1)(1\text{ in.}) = (62.4)(13.56 - 1)\left(\frac{1}{12}\right) = 65.3\ \text{lb/ft}^2 \quad +2$$

By solving the Bernoulli equation for V_1,

$$V_1 = \sqrt{2\left(\frac{p_2 - p_1}{\rho}\right)} = \sqrt{(2)\frac{(65.3)}{62.4/32.2}} = 8.21\ \text{ft/s}$$

Thus,

$$Q = V_1 A_1 = (8.21)\left(\frac{\pi}{4}\right)(1)^2 = \mathbf{6.45\ ft^3/s} \quad +2$$

(b) From continuity

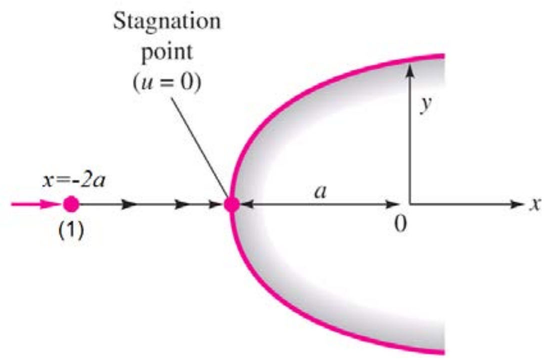
$$V_j = \frac{Q}{A_j} = \frac{6.45}{\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2} = \mathbf{131.4\ ft/s} \quad +1$$

EXAM1 Solutions

Problem 4: Acceleration and Euler equation (Chapter 4)

Information and assumptions

- Two dimensional flow
- $\underline{V} = U(1 - a^2/x^2)\hat{\boldsymbol{\imath}}$
- $\rho a_x = -dp/dx$
- $\rho = 917\ kg/m^3$
- $U = 2\ m/s$
- $a = 6\ cm$



Find

- Calculate (a) the fluid velocity u, (b) the acceleration $\boldsymbol{a_x}$, and (c) the pressure gradient dp/dx at point 1

Solution

(a) Velocity

$$u = U\left(1 - \frac{a^2}{x^2}\right)_{x=-2a} = (2)\left(1 - \frac{a^2}{(2a)^2}\right) = (2)\left(1 - \frac{1}{4}\right) = \mathbf{1.5\ m/s} \qquad +2$$

(b) Acceleration

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

Since u is a function of x only

$$a_x = u\frac{\partial u}{\partial x}$$

$$a_x = U\left(1 - \frac{a^2}{x^2}\right)U\left(\frac{2a^2}{x^3}\right) = \frac{2U^2}{a}\left(\frac{a^3}{x^3} - \frac{a^5}{x^5}\right)$$

At x = -2a,

$$\therefore a_x = \frac{2U^2}{a}\left(\frac{a^3}{(-2a)^3} - \frac{a^5}{(-2a)^5}\right) = -\frac{3U^2}{16a} = -\frac{(3)(2)^2}{(16)(0.06)} = \mathbf{-12.5\ m/s^2} \qquad +6$$

(c) Pressure gradient

$$\frac{dp}{dx} = -\rho a_x = -(917)(-12.5) = 11462.5\ \mathrm{N/m^3} = \mathbf{11.5\ kPa/m} \qquad +2$$