EXAM1 Solutions

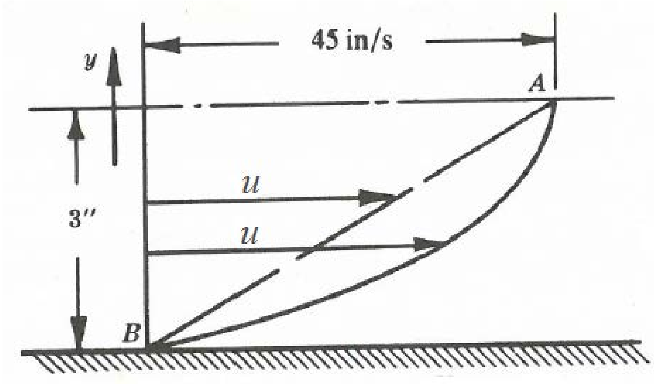
Problem 1: Shear stress (Chapter 1)

Information and assumptions

- μ = 0.001 lb·s/ft^2
- (a) $u = 15y$
- (b) $u = 45 - 5(3 - y)^2$

Find

- Find the shear stress at the boundary (y=0)and 3" from the boundary (y=3")



Solution

$$\tau = \mu \frac{du}{dy} \qquad \text{(+5 points)}$$

(a) Straight-line

$$\frac{du}{dy} = \frac{d}{dy}(15y) = 15$$

and

$$\left.\frac{du}{dy}\right)_{y=0} = \left.\frac{du}{dy}\right)_{y=3} = 15\ \mathrm{s}^{-1} \qquad \text{(+1 point)}$$

Thus,

$$\tau_{y=0} = \tau_{y=3} = (0.001\ \mathrm{lb\cdot s/ft^2})(15\ \mathrm{s}^{-1}) = \mathbf{0.015\ lb/ft^2} \qquad \text{(+1 point)}$$

(b) Parabolic

$$\frac{du}{dy} = 10(3 - y) \qquad \text{(+1 point)}$$

and

$$\left.\frac{du}{dy}\right)_{y=0} = 10(3 - 0) = 30\ \mathrm{s}^{-1} \qquad \text{(+0.5 point)}$$

$$\left.\frac{du}{dy}\right)_{y=3} = 10(3 - 3) = 0\ \mathrm{s}^{-1} \qquad \text{(+0.5 point)}$$

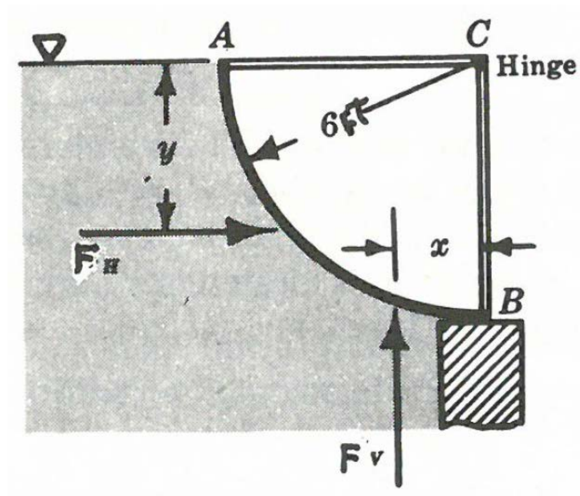
Thus,

$$\tau_{y=0} = (0.001\ \mathrm{lb\cdot s/ft^2})(30\ \mathrm{s}^{-1}) = \mathbf{0.03\ lb/ft^2} \qquad \text{(+0.5 point)}$$

$$\tau_{y=3} = (0.001\ \mathrm{lb\cdot s/ft^2})(0\ \mathrm{s}^{-1}) = \mathbf{0\ lb/ft^2} \qquad \text{(+0.5 point)}$$

EXAM1 Solutions

Problem 2: Hydrostatic force (Chapter 2)



Information and assumptions

- Use gauge pressure.
- γ_{water} = 62.4 lb/ft^3

Find

- Determine the magnitude and location of the vertical and horizontal components of the hydrostatic force

Solution

(a) Pressure force

$$F_H = \gamma h_c A = (62.4)\left(\frac{6}{2}\right)(6 \times 10) = \mathbf{11,232\ lb}$$ (+2.5 points)

$$F_V = \gamma \forall = (62.4)\left(\frac{(\pi)(6)^2}{4}\right)(10) = \mathbf{17,643\ lb}$$ (+2.5 points)

(b) Pressure center

F_H is located at

$$y = y_c + \frac{I_{xc}}{y_c A} = \left(\frac{6}{2}\right) + \frac{(6)^3(10)/12}{(6/2)(6)(10)} = \mathbf{4\ ft}$$ (+2.5 points)

F_V is located at the center of gravity of area *ABC*, or

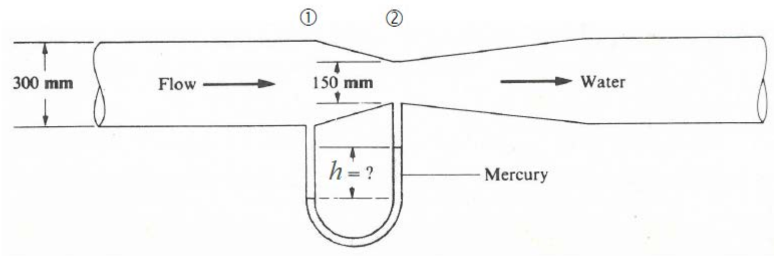
$$x = \frac{4R}{3\pi} = \frac{(4)(6)}{3\pi} = \mathbf{2.55\ ft}$$ (+2.5 points)

EXAM1 Solutions

Problem 3: Bernoulli equation (Chapter 3)

Information and assumptions

- Q = 0.142 kg/m^3
- Ignore friction loss
- SG = 13.6 for the manometer fluid
- γ_{water} = 9,780 N/m^3



Find

- Determine the h

Solution

Continuity equation

$$Q = AV \qquad \text{(+1 point)}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.142}{\pi(0.3)^2/4} = 2\ m/s \qquad \text{(+0.5 point)}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.142}{\pi(0.15)^2/4} = 8\ m/s \qquad \text{(+0.5 point)}$$

Bernoulli equation,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad \text{(+4 points)}$$

Since $V_1 = 2$ m/s, $V_2 = 8$ m/s, and $z_1 = z_2$,

$$p_1 - p_2 = \frac{\gamma}{2g}(V_2^2 - V_1^2) = \left(\frac{9{,}780}{2 \times 9.81}\right)[(8)^2 - (2)^2] = 29{,}908\ \text{Pa} \qquad \text{(+1 points)}$$

Manometer equation,

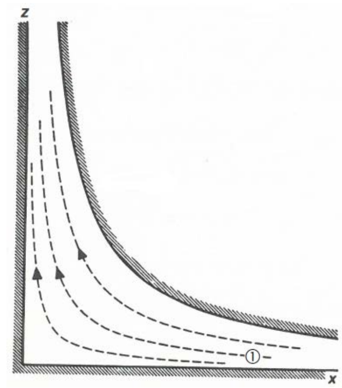
$$p_1 - p_2 = h(\gamma_m - \gamma_w) \qquad \text{(+2 points)}$$

or

$$h = \frac{1}{\gamma_m - \gamma_w}(p_1 - p_2) = \frac{1}{(13.6 - 1)(9780)} \times 29{,}908 = \mathbf{0.243\ m} \qquad \text{(+1 point)}$$

EXAM1 Solutions

Problem 4: Acceleration and Euler equation (Chapter 4)



Information and assumptions

- Two dimensional flow
- $\boldsymbol{V} = -x\,\boldsymbol{i} + z\,\boldsymbol{j}$
- Viscosity effects are negligible
- $\nabla p = -\rho\left(\underline{a} - \underline{g}\right)$
- ρ = 1.23 kg/m^3

Find

- Acceleration a_x and a_y at x=0.5m, z=0.1m
- Pressure gradients $\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial z}\right)$ at x=0.5m, z=0.1m

Solution

(a) Acceleration,

$$a_x = \frac{\partial u}{\partial t} + \left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = 0 + (-x)(-1) + (z)(0) = x \qquad \text{(+3 points)}$$

$$a_z = \frac{\partial w}{\partial t} + \left(u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right) = 0 + (-x)(0) + (z)(1) = z \qquad \text{(+3 points)}$$

At point 1,

$$\boldsymbol{a_x = 0.5\ \mathrm{m/s^2}} \qquad \text{(+0.5 point)}$$

$$\boldsymbol{a_z = 0.1\ \mathrm{m/s^2}} \qquad \text{(+0.5 point)}$$

(b) Pressure gradient,

$$\frac{\partial p}{\partial x} = -\rho(a_x) \qquad \text{(+1 points)}$$

$$\frac{\partial p}{\partial z} = -\rho(a_z + g) \qquad \text{(+1 points)}$$

At point 1,

$$\frac{\partial p}{\partial x} = -(1.23)(0.5 - 0) = \boldsymbol{-0.615\ \mathrm{Pa/m}} \qquad \text{(+0.5 point)}$$

$$\frac{\partial p}{\partial z} = -(1.23)(0.1 + 9.81) = \boldsymbol{-12.19\ \mathrm{Pa/m}} \qquad \text{(+0.5 point)}$$