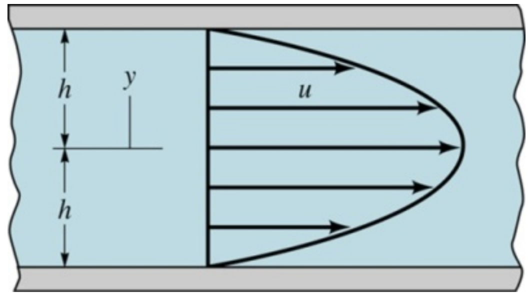
EXAM1 Solutions

Problem 1: Shear stress (Chapter 1)

Information and assumptions

- μ = 0.04 lb·s/ft^2
- V = 2 ft/s
- h = 0.2 in
- *bottom wall area A* = 2 ft^2



Find

- Find the shear stress at the bottom plane, y=-h
- Find the shear-force acting on the bottom wall

Solution

$$\tau = \mu \frac{du}{dy}$$

(+4 points)

(a) At the bottom wall where $y = -h$

$$\frac{du}{dy} = -\frac{3Vy}{h^2}\bigg)_{y=-h} = \frac{3V}{h}$$

(+2 points)

Thus,

$$\tau = \mu\left(\frac{3V}{h}\right) = \frac{(0.04\ \text{lb}\cdot\text{s/ft}^2)(3)(2\ \text{ft/s})}{(0.2\ \text{in.})(1\ \text{ft}/12\ \text{in.})} = 14.4\ \text{lb/ft}^2$$

(+2 points)

(b) Then, the shear-force is

$$F = \tau \cdot A = (14.4\ \text{lb/ft}^2)(2\ \text{ft}^2) = 28.8\ \text{lb}$$

(+2 points)

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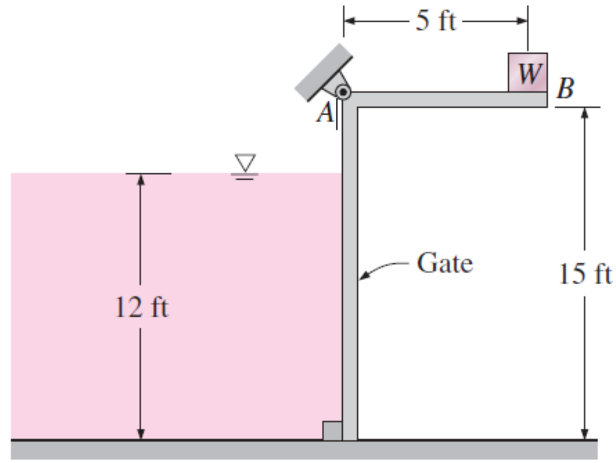
Problem 2: Hydrostatic force (Chapter 2)



Information and assumptions

- Ignore atmospheric pressure.
- The weight of the gate is negligible.
- $\rho = 62.4\ \text{lbm/ft}^3$
- $g = 32.2\ \text{ft/s}^2$
- unit conversion, $1\ \text{lbf} = 32.2\ \text{lbm ft/s}^2$

Find

- Determine the mass of the required weight W to keep the L-shaped gate closed.

Solution

The average pressure on a surface

$$\text{P} = \rho g h_c = \rho\, g \left(\frac{h}{2}\right)$$

(+2 points)

Thus,

$$\text{P} = \left(62.4\frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2\frac{ft}{s^2}\right)\left(\frac{12}{2}\,ft\right)\left(\frac{1\ lbf}{32.2\ lbm.ft/s^2}\right) = 374.4\ lbf/ft^2$$

(+1 points)

Resultant force acting on the gate

$$\text{F} = \text{P A} = \left(374.4\frac{\text{lbf}}{\text{ft}^2}\right)(12\ ft \times 5\ ft) = 22464\ lbf$$

(+1 points)

The line of action of the force passes through the pressure center below the water surface , y_P, is

$$y_P = \frac{2h}{3} = 2 \times \frac{12\ ft}{3} = 8\ ft$$

(+2 points)

For equilibrium

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$$\sum \mathrm{M_A} = (22464\ lbf) \times (8ft + 3ft) - W \times (5\ ft) = 0$$

(+3 points)

Thus,

$$\mathrm{W = 30900\ lbf}$$

(+1 points)

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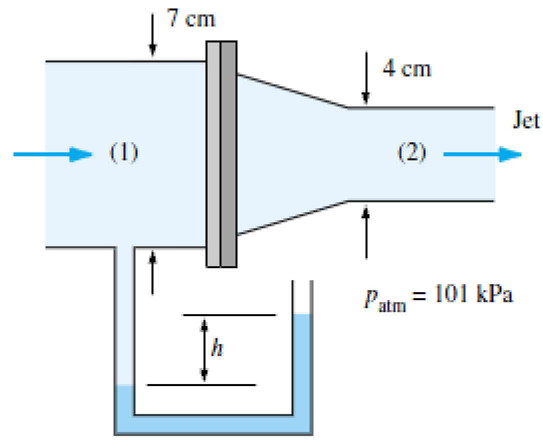
Problem 3: Bernoulli equation (Chapter 3)



Information and assumptions

- $\gamma_w = 9.80 \times 10^3 N/m^3$
- $g = 9.80\ m/s^2$
- $D_1 = 7\ cm$
- $D_2 = 4\ cm$
- Ignore friction loss
- SG of manometer fluid is 1.6
- h=66 cm

Find

- Determine the gage pressure at section (1) using manometer reading.
- Find the relation of the velocities at section (1) and (2) using volume flux Q.
- Determine the velocity at section (2).

Solution

(a) Pressure at section (1) is

$$p_1 - p_2 = (SG - 1)\gamma_w\, h$$

(+3 points)

Thus,

$$p_1 - p_2 = (1.6 - 1)\left(9.80 \times \frac{10^3 N}{m^3}\right)(0.66\ m) = 3880\,\frac{N}{m^2} = 3880\ Pa$$

(+1 points)

(b) From the continuity

$$Q = V_1 A_1 = V_2 A_2$$

Thus,

$$V_1 = V_2 \frac{A_2}{A_1} = V_2 \frac{\frac{\pi}{4} D_2^2}{\frac{\pi}{4} D_1^2} = V_2 \frac{D_2^2}{D_1^2} = 0.327\ V_2$$

(+2 points)

(c) Using Bernoulli's equation,

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$$\frac{p_1}{\gamma_w}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma_w}+\frac{V_2^2}{2g}+z_2$$

(+3 points)

Since $z_1 = z_2$, the above equation reduces to

$$p_1-p_2=\frac{\gamma_w}{2g}\left(V_2^2-V_1^2\right)$$

Using the relation between V_1 and V_2

$$p_1-p_2=\frac{\gamma_w}{2g}\left(1-\frac{D_2^4}{D_1^4}\right)V_2^2$$

Therefore,

$$V_2=\sqrt{(p_1-p_2)/\left(\frac{\gamma_w}{2g}\left(1-\frac{D_2^4}{D_1^4}\right)\right)}$$

$$V_2=\sqrt{(3880\ Pa)\Bigg/\left(\frac{9.8\times\frac{10^3 N}{m^3}}{2\left(9.8\frac{m}{s^2}\right)}\left(1-\frac{(0.04\ m)^4}{(0.07\ m)^4}\right)\right)}=2.94\ m/s$$

(+1 points)

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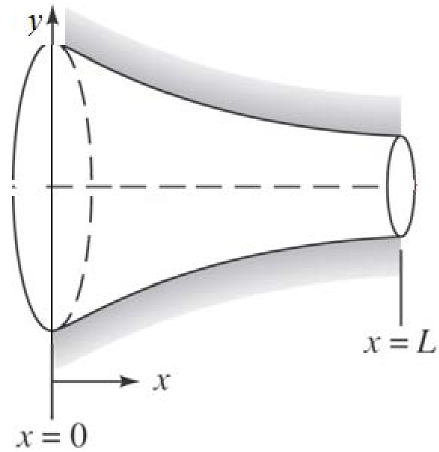
Problem 4: Acceleration and Euler equation (Chapter 4)

Information and assumptions

- One-dimensional flow
- Steady flow through a nozzle
- $u = U_0\left(1 + 2\frac{x}{L}\right)$
- Viscosity effects are negligible
- $U_0 = 6\ m/s$
- $\rho = 1.23\ \mathrm{kg/m^3}$
- $L = 0.1m$
- $p_0 = 177\ Pa\ (gage)$



Find

- Acceleration a_x at x=L
- Pressure p at x = L

Solution

(a) Accelerations are

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = (0) + U_0\left(1 + 2\frac{x}{L}\right)\left(2\frac{U_0}{L}\right) = \frac{2U_0^2}{L}\left(1 + 2\frac{x}{L}\right)$$

(+6 ponits)

$$= \frac{(2)\left(6\frac{m}{s}\right)^2}{0.1\ m}\left(1 + 2\frac{0.1\ m}{0.1\ m}\right) = 2160\ m/s^2$$

(+1 points)

(b)

$$p = -\rho\int a_x\, dx = -\rho\frac{2U_0^2}{L}x\left(1 + \frac{x}{L}\right) + C$$

where $C = p_0$ as $p = p_0$ at $x = 0$.

(+2 points)

At x = L

$$p = -\rho\frac{2U_0^2}{L}L\left(1 + \frac{L}{L}\right) + p_0 = -4\rho U_0^2 + p_0$$

$$= -(4)\left(1.23\frac{kg}{m^3}\right)\left(6\frac{m}{s}\right)^2 + 177\ Pa = 0\ (gage)$$

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(+1 points)