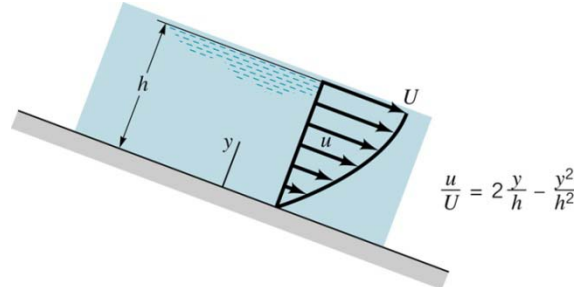


EXAM1 Solutions

Problem 1: Shear stress (Chapter 1)

Information and assumptions

- $\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2}$
- $\mu = 1.12 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2$
- $U = 2 \text{ m/s}$
- $h = 0.1 \text{ m}$



Find

- shearing stress that the water exerts on the fixed surface

Solution

$$\tau = \mu \frac{du}{dy} \quad (+4 \text{ points})$$

where

$$\frac{du}{dy} = \frac{2U}{h} \left(1 - \frac{y}{h}\right) \quad (+3 \text{ points})$$

At the fixed surface ($y = 0$)

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{h} \quad (+1 \text{ point})$$

So that

$$\tau = \mu \left(\frac{2U}{h}\right) = \left(1.12 \times 10^{-3} \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right) \frac{(2) \left(2 \frac{\text{m}}{\text{s}}\right)}{(0.1 \text{ m})} = 4.48 \times 10^{-2} \text{ N/m}^2 \quad (+2 \text{ points})$$

EXAM1 Solutions

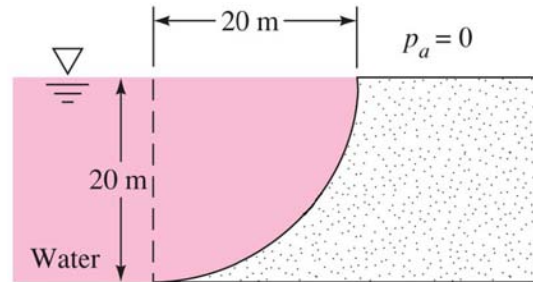
Problem 2: Hydrostatic force (Chapter 2)

Information and assumptions

- $\gamma = 9790 \text{ N/m}^3$
- A quarter-circle 50 m wide dam
- Water depth = 20 m

Find

- the horizontal and vertical components of hydrostatic force against the dam and the angle of the resultant force from the vertical



Solution

The horizontal force acts as if the dam were vertical and 20 m high:

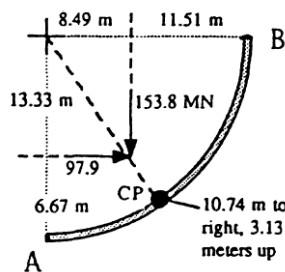
$$F_H = \gamma h_c A = \left(9790 \frac{\text{N}}{\text{m}^3}\right) (10 \text{ m})(20 \times 50 \text{ m}^2) = \mathbf{97.9 \text{ MN}} \quad (+4 \text{ points})$$

The vertical force is the weight of the fluid above the dam:

$$F_V = \gamma V = \left(9790 \frac{\text{N}}{\text{m}^3}\right) \left(\frac{\pi}{4}\right) (20 \text{ m})^2 (50 \text{ m}) = \mathbf{153.8 \text{ MN}} \quad (+4 \text{ points})$$

The angle of the resultant force from the vertical is

$$\theta = \tan^{-1} \left(\frac{F_H}{F_V}\right) = \mathbf{32.5^\circ} \quad (+2 \text{ points})$$

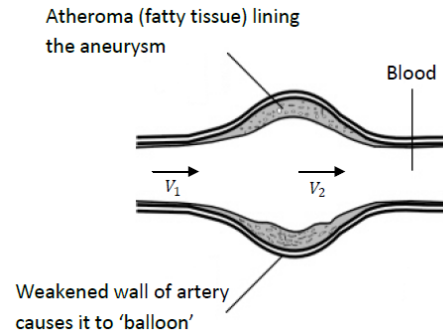


EXAM1 Solutions

Problem 3: Bernoulli equation (Chapter 3)

Information and assumptions

- Blood (SG = 1, $\rho_{water} = 999 \text{ Kg/m}^3$)
- $V_1 = 0.5 \text{ m/s}$
- $A_2 = 1.8A_1$
- Steady and inviscid flow



Find

- pressure difference between the blood in the aneurysm and that in the artery

Solution

From Bernoulli equation,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

where $z_1 = z_2$. Or,

$$p_2 - p_1 = \frac{1}{2}\rho(V_1^2 - V_2^2) \quad (+4 \text{ points})$$

However,

$$\rho = \rho_{water} \cdot \text{SG} = (999 \text{ kg/m}^3)(1) = 999 \text{ kg/m}^3$$

and from continuity,

$$V_1 A_1 = V_2 A_2$$

or

$$V_2 = \frac{A_1}{A_2} V_1 = \left(\frac{1}{1.8}\right) V_1 \quad (+4 \text{ points})$$

Thus,

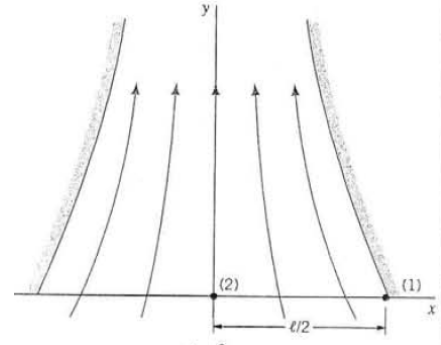
$$p_2 - p_1 = \frac{1}{2} \left(999 \frac{\text{kg}}{\text{m}^3} \right) \left[\left(0.5 \frac{\text{m}}{\text{s}} \right)^2 - \left(\frac{1}{1.8} \right)^2 \left(0.5 \frac{\text{m}}{\text{s}} \right)^2 \right] = \mathbf{86.3 \text{ Pa}} \quad (+2 \text{ points})$$

EXAM1 Solutions**Problem 4: Acceleration (Chapter 4)****Information and assumptions**

- Steady flow through a nozzle
- $u = -V_0 x/\ell$ and $v = V_0[1 + (y/\ell)]$, where V_0 and ℓ are constants
- water with $V_0 = 1 \text{ m/s}$ and $\ell = 0.1 \text{ m}$
- Neglect viscous effects and use the Euler equation

Find

- pressure gradient along the y -axis at point (2), or at $(x,y) = (0 \text{ m}, 0 \text{ m})$

**Solution**

Pressure gradient along the y -axis by using the Euler equation can be written as

$$\frac{\partial p}{\partial y} = -\rho(a_y + g) \quad (+2 \text{ points})$$

Where,

$$\begin{aligned} a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= 0 + \left(-V_0 \frac{x}{\ell}\right)(0) + V_0 \left(1 + \frac{y}{\ell}\right) \left(\frac{V_0}{\ell}\right) = \frac{V_0^2}{\ell} \left(1 + \frac{y}{\ell}\right) \end{aligned} \quad (+5 \text{ points})$$

Thus,

$$\frac{\partial p}{\partial y} = -\rho \left[\frac{V_0^2}{\ell} \left(1 + \frac{y}{\ell}\right) + g \right] \quad (+2 \text{ points})$$

At $(x,y) = (0 \text{ m}, 0 \text{ m})$

$$\frac{\partial p}{\partial y} = -\left(999 \frac{\text{kg}}{\text{m}^3}\right) \left[\frac{\left(1 \frac{\text{m}}{\text{s}}\right)^2}{0.1 \text{ m}} \left(1 + \frac{0 \text{ m}}{0.1 \text{ m}}\right) + 9.81 \frac{\text{m}}{\text{s}^2} \right] = -19,790 \text{ Pa/m} \quad (+1 \text{ point})$$