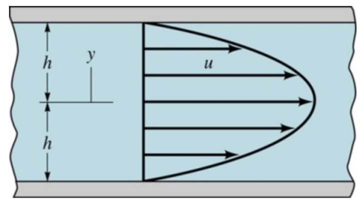
EXAM1 Solutions

Problem 1: Shear stress (Chapter 1)

Information and assumptions



- $u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$
- μ = 0.04 lb·s/ft^2
- V = 2 ft/s
- h = 0.2 in
- A = 2 ft^2

Find

- The shear stress at the midplane and the shear-force acting on the bottom wall

Solution

$$\tau = \mu \frac{du}{dy}$$

(+4 points)

(a) Along the midplane where y = 0,

$$\frac{du}{dy} = -\left.\frac{3Vy}{h^2}\right)_{y=0} = 0$$

Thus,

$$\tau = 0$$

(+2 points)

(b) At the bottom wall where y = -h

$$\frac{du}{dy} = -\left.\frac{3Vy}{h^2}\right)_{y=-h} = \frac{3V}{h}$$

Thus,

$$\tau = \mu\left(\frac{3V}{h}\right) = (0.04\ \text{lb} \cdot \text{s/ft}^2)\frac{(3)(2\ \text{ft/s})}{(0.2\ \text{in.})(1\ \text{ft}/12\ \text{in.})} = 14.4\ \text{lb/ft}^2$$

(+2 points)

Then, the shear-force is

$$F = \tau \cdot A = (14.4\ \text{lb/ft}^2)(2\ \text{ft}^2) = 28.8\ \text{lb}$$

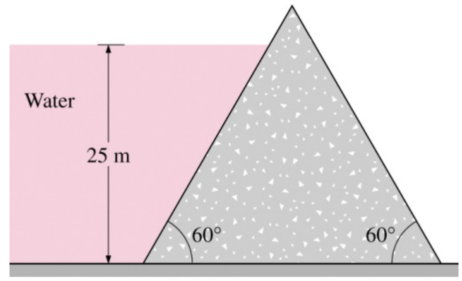
(+2 points)

EXAM1 Solutions

Problem 2: Hydrostatic force (Chapter 2)

Information and assumptions

- γ = 9.81 kN/m^3
- Wall width a = 150 m
- Water depth = 25 m
- Wall slant angle = 60°
- $I_{xc} = \frac{ab^3}{12}$



Find

- The force acting on the wall and its line of action

Solution

Let $a = 150$ m and $b = 25\text{ m}/\sin 60° = 28.87$ m be the width and the length of the wall surface underwater, respectively.

(a)

$$F_R = \gamma h_c A$$
$$= (9.81\ \text{kN/m}^3)\left(\frac{25\text{ m}}{2}\right)(150\text{ m} \times 28.87\text{ m})$$
$$= 5.31 \times 10^8\text{ N}$$

(+ 4 points)

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

where,

$$I_{xc} = \frac{ab^3}{12} = \frac{(150\text{ m})(28.87\text{ m})^3}{12} = 300780.98\text{ m}^4$$
$$y_c = \frac{b}{2} = \frac{28.87\text{ m}}{2} = 14.435\text{ m}$$
$$A = ab = 150\text{ m} \times 28.87\text{ m} = 4330.5\text{ m}^2$$

Thus,

$$y_R = \frac{300780.98\ m^4}{(14.435\text{ m})(4330.5\text{ m}^2)} + 14.435\text{ m} = 19.25\text{ m}$$

(+5 points)

(b)

$$F_H = F_R \cdot \sin 60° = 4.60 \times 10^8\text{ N}$$

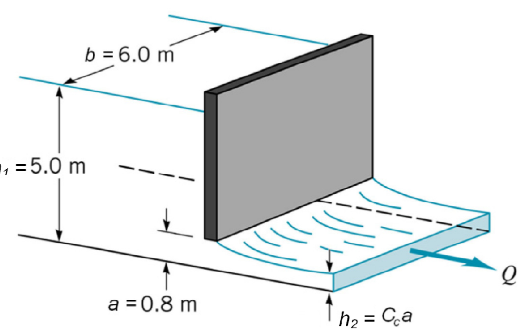
(+1 point)

EXAM1 Solutions

Problem 3: Bernoulli equation (Chapter 3)

Information and assumptions

- Width b = 6.0 m
- Height h = 5.0 m
- Gate opening height a = 0.8 m
- Contraction coefficient Cc = 0.61



Find

- Flow rate Q

Solution

From Bernoulli equation,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where, $p_1 = p_2 = 0$, $z_1 = h = 5\text{ m}$, and $z_2 = C_c a = 0.61 \times 0.8\text{ m} = 0.488\text{ m}$.

(+4 points)

From the continuity equation,

$$A_1 V_1 = A_2 V_2$$

$$\therefore V_2 = \frac{bh}{bC_c a} V_1 = \frac{5\text{ m}}{0.488\text{ m}} V_1 = 10.25\, V_1$$

(+2 points)

Then, the Bernoulli equation becomes,

$$\frac{V_1^2}{2(9.81\text{ m/s}^2)} + 5\text{ m} = \frac{(10.25\, V_1)^2}{2(9.81\text{ m/s}^2)} + 0.488\text{ m}$$

or,

$$V_1 = 0.922\text{ m/s}$$

(+3 points)

Hence

$$Q = A_1 V_1 = (6\text{ m})(5\text{ m})(0.922\text{ m}/s) = 27.66\text{ m}^3/s$$

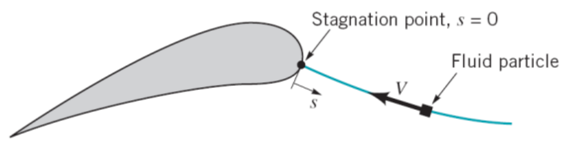
(+1 point)

EXAM1 Solutions

Problem 4: Acceleration (Chapter 4)

Information and assumptions

- Streamline coordinate
- $s = 0.6e^{-0.5t}$
- Particle velocity $V_P(t) = \frac{ds}{dt}$
- Fluid velocity $V(s) = -0.5s$



Find

- $V_P(t)$ at t = 1 sec
- $V(s)$ at s = 1 ft
- Fluid acceleration a_s at s = 1 ft

Solution

(a)

$$V_P(t) = \left.\frac{ds}{dt}\right)_{t=1} = -0.3e^{-0.5\times 1} = -0.182 \text{ ft/s}$$

(+3 points)

(b)

$$V(s)|_{s=1} = -0.5 \times 1 = -0.5 \text{ ft/s}$$

(+2 points)

(c)

$$a_s = V\frac{\partial V}{\partial s}$$

$$a_s = V\left.\frac{\partial V}{\partial s}\right)_{s=1} = (-0.5\times 1)(-0.5) = 0.25\,\text{ft/s}^2$$

(+5 points)