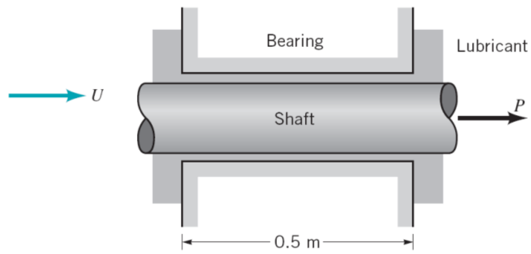
# EXAM1 Solutions

## Problem 1: Shear stress (Chapter 1)

### Information and assumptions

- $SG$ = 0.91, $\rho_{H_2O@4°C}$ = 1000 kg/m$^3$
- $\nu$ = 8 × 10$^{-4}$ m$^2$/s
- $U$ = 3 m/s, $\delta$ = 0.3 mm = 3 × 10$^{-4}$ m
- $D$ = 25 mm = 0.025 m, $L$ = 0.5 m
- $u = \frac{U}{\delta} y$
- $P = \tau A$




### Find

- The force $P$ required to pull the shaft at a velocity of 3 m/s.

### Solution

$$P = \tau \cdot A$$

Shear stress $\tau$

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{d}{dy}\left(\frac{U}{\delta} y\right) = \mu \frac{U}{\delta}$$ (+6 point)

where, $\mu = \nu\rho = \mu \cdot SG \cdot \rho_{H_2O@4°C}$

$$= (8 \times 10^{-4}\ \mathrm{m^2/s})(0.91)(1000\ \mathrm{Kg/m^3}) = 0.728\ \mathrm{N \cdot s/m^2}$$ (+2 point)

$$\therefore \tau = (0.728\ \mathrm{N \cdot s/m^2})\left(\frac{3\ \mathrm{m/s}}{3 \times 10^{-4}\ \mathrm{m}}\right) = 7280\ \mathrm{N/m^2}$$

Contacting area $A$

$$A = \pi D L = (\pi)(0.025\ \mathrm{m})(0.5\ \mathrm{m}) = 0.0393\ \mathrm{m^2}$$

Pulling force

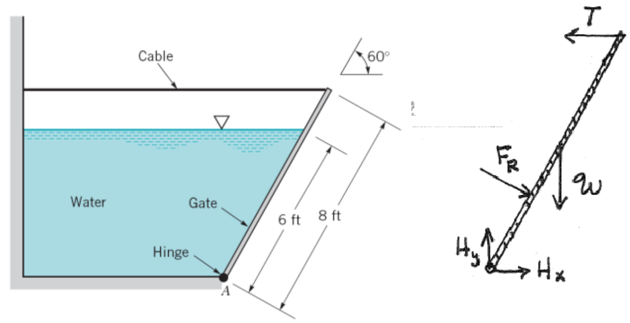
$$P = \tau \cdot A = (7280\ \mathrm{N/m^2})(0.0393\ \mathrm{m^2}) = \mathbf{286\ N}$$ (+2 point)

# EXAM1 Solutions

## Problem 2: Hydrostatic force (Chapter 2)

### Information and assumptions

- $\gamma$ = 62.4 lb/ft$^3$
- Gate weight $W$ = 800 lb
- panel width $a$ = 4 ft
- wetted panel height $b$ = 6 ft
- Slant angle $\theta$ = 60°
- $I_{xx} = \frac{ab^3}{12}$




### Find

- The tension in the cable $T$

### Solution

Water force $F_R$

$$F_R = \bar{p}A = \gamma h_c A$$

$$\text{where, } h_c = \frac{1}{2} \times 6 \text{ ft} \times \sin 60° = 2.598 \text{ ft}$$

$$A = 6 \text{ ft} \times 4 \text{ ft} = 24 \text{ ft}^2$$

$$\therefore F_R = (62.4 \text{ lb/ft}^3)(2.598 \text{ ft})(24 \text{ ft}^2) = 3890 \text{ lb} \qquad \text{(+4 point)}$$

Center of pressure

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$\text{where, } I_{xc} = \frac{ab^3}{12} = \frac{(4 \text{ ft})(6 \text{ ft})^3}{12} = 72 \text{ ft}^4$$

$$A = ab = (4 \text{ ft})(6 \text{ ft}) = 24 \text{ ft}^2$$

$$y_c = 3 \text{ ft}$$

$$\therefore \; y_R = \frac{72 \text{ ft}^4}{(3 \text{ ft})(24 \text{ ft}^2)} + 3 \text{ ft} = 4 \text{ ft} \qquad \text{(+4 point)}$$

For equilibrium ($\sum M_H = 0$)

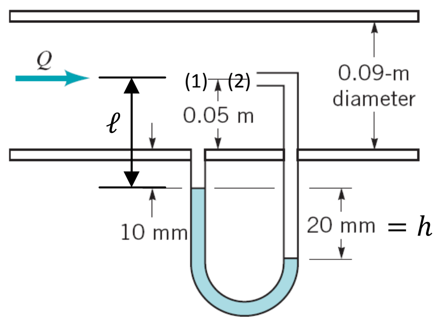
$$T(8 \text{ ft})(\sin 60°) = W(4 \text{ ft})(\cos 60°) + F_R(2 \text{ ft})$$

$$\therefore T = \frac{(800 \text{ lb})(4 \text{ ft})(\cos 60°) + (3890 \text{ lb})(2 \text{ ft})}{(8 \text{ ft})(\sin 60°)} = \mathbf{1350 \text{ lb}} \qquad \text{(+2 point)}$$

# EXAM1 Solutions

## Problem 3: Bernoulli equation and manometer (Chapter 3)

### Information and assumptions




- $\gamma_{water} = 9.80\,\text{kN/m}^3$
- $\gamma = 6.67\,\text{kN/m}^3$ for gasolin
- $SG$ = 1.07 for manometer fluid
- Losses in the manometer are negligible
- Steady, inviscid, incompressible flow

### Find

- Flow rate $Q$

### Solution

Manometer

$$p_1 + \gamma\ell + \gamma_m h = p_2 + \gamma(\ell + h)$$

$$p_2 - p_1 = (\gamma_m - \gamma)h$$

$$= (9.80\ \text{kN/m}^3 \times 1.07 - 6.67\,\text{kN/m}^3)(0.02\ \text{m}) = 76.32\,\text{N/m}^2$$ (+4 points)

Bernoulli equation

$$p_1 + \frac{\rho}{2}V_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2}V_2^2 + \rho g z_2$$

Since $V_2 = 0$ and $z_1 = z_2$, (+3 points)

$$V_1 = \sqrt{2\frac{(p_2 - p_1)}{\rho}} = \sqrt{2g\frac{(p_2 - p_1)}{\gamma}}$$

$$= \sqrt{2\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left(\frac{76.32\ \text{N/m}^2}{6.67\times10^3\ \text{N/m}^3}\right)} = 0.474\,\frac{\text{m}}{\text{s}}$$ (+2 point)

Flow rate

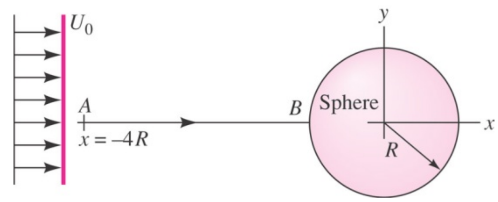
$$Q = V_1 A_1 = \frac{\pi}{4}D^2 V_1$$

$$= \left(\frac{\pi}{4}\right)(0.09\ \text{m})^2\left(0.474\,\frac{\text{m}}{\text{s}}\right) = \mathbf{3.02 \times 10^{-3}\,\frac{m^3}{s}}$$ (+1 point)

# EXAM1 Solutions

## Problem 4: Acceleration and Euler equation (Chapter 4)

### Information and assumptions




- One-dimensional flow
- $u = U_0\left(1 + \frac{R^3}{x^3}\right)$
- $U_0 = 1\ \text{m/s}, R = 1\ \text{m}$

### Find

- Fluid acceleration at $x = -2R$

### Solution

Acceleration

$$a_x = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} \qquad \text{(+5 points)}$$

$$= U_0\left(1 + \frac{R^3}{x^3}\right)\left(-3U_0\frac{R^3}{x^4}\right) = -\frac{3U_0^2}{R}\left(\left(\frac{R}{x}\right)^4 + \left(\frac{R}{x}\right)^7\right) \qquad \text{(+3 points)}$$

At $x = -2R$,

$$a_x = -\frac{(3)(1\ m/s)^2}{1\ m}\left(\left(\frac{R}{-2R}\right)^4 + \left(\frac{R}{-2R}\right)^7\right) = \boldsymbol{-0.164\ \frac{m}{s^2}} \qquad \text{(+2 point)}$$