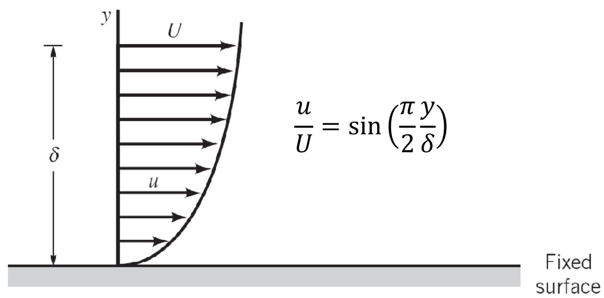
EXAM1 Solutions

Problem 1: Shear stress (Chapter 1)

Information and assumptions

- Newtonian fluid
- SG = 0.92, $\rho_{H_2O@4°C}$ = 1000 kg/m^3
- ν = 4 × 10^{-4} m^2/s
- U = 1 m/s, δ = 2 mm = 0.002 m
- $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$



Find

- The magnitude and direction of the shearing stress developed on the plate

Solution

(a) Shear stress

$$\tau = \mu \frac{du}{dy}\bigg|_{y=0} \quad \text{(+6 points)}$$

$$= \mu \frac{d}{dy}\left(U \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right)\bigg|_{y=0}$$

$$= \mu \frac{\pi}{2}\frac{U}{\delta}\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\bigg|_{y=0} = \frac{\pi}{2}\frac{U}{\delta}\mu = \frac{\pi}{2}\frac{U}{\delta}\left(SG \cdot \rho_{H_2O} \cdot \nu\right) \quad \text{(+3 points)}$$

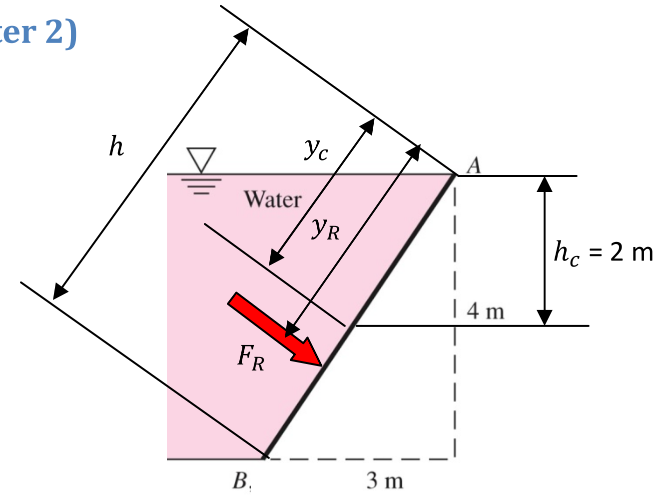
$$= \frac{\pi}{2} \times \frac{1 \text{ m/s}}{0.002 \text{ m}} \times (0.92 \times 1000 \text{ kg/m}^3 \times 4 \times 10^{-4} \text{ m}^2/\text{s})$$

= **289.0 N/m^2** (+0.5 point)

(b) Direction: acting to **right** on plate (+0.5 point)

EXAM1 Solutions

Problem 2: Hydrostatic force (Chapter 2)



Information and assumptions

- γ_{water} = 9790 N/m^3
- panel width b = 2 m
- Slant angle: tan θ = 4/3
- $I_{xx} = \frac{bh^3}{12}$

Find

- Water force acting on the panel
- Line of action (center of pressure)

Solution

(a) Water force F_R

$$F_R = \bar{p}A = \gamma h_c A = \gamma h_c (bh)$$ (+3.5 points)

$$= 9790\ \mathrm{N/m^3} \times 2\ \mathrm{m} \times \left(2\ \mathrm{m} \times \sqrt{3^2+4^2}\ \mathrm{m}\right)$$

$$= 195{,}800\ \mathrm{N} = \mathbf{196\ kN}$$ (+0.5 point)

(b) Center of pressure

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{bh^3/12}{(h/2)(bh)} + \frac{h}{2}$$ (+5.5 points)

$$= \frac{2\ \mathrm{m} \times (5\ \mathrm{m})^3/12}{(5/2\ \mathrm{m}) \times 10\ \mathrm{m}^2} + \frac{5}{2}\ \mathrm{m}$$

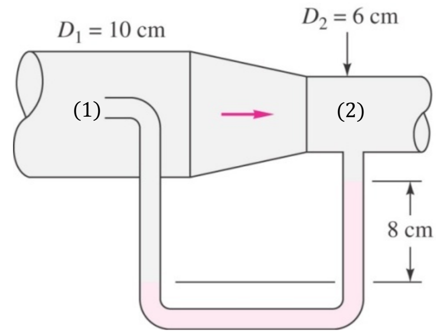
$$= \mathbf{3.33\ m}$$ (+0.5 point)

EXAM1 Solutions

Problem 3: Bernoulli equation and manometer (Chapter 3)

Information and assumptions

- $\rho_{H_2O} = 998.2\ \mathrm{kg/m^3}$ at 20°C
- $\rho_M = 1360\ \mathrm{kg/m^3}$ at 20°C
- $p_1 = 170\ \mathrm{kPa}$
- Losses in the manometer are negligible
- Steady, inviscid, incompressible flow



Find

- p_2 and flow rate Q

Solution

Manometer

$$p_2 = p_1 - (\rho_M - \rho_{H_2O})gh$$ (+1.5 points)

$$= 170{,}000\ \mathrm{Pa} - (1360 - 998.2)\ \mathrm{Kg/m^3} \times 9.81\ \mathrm{m/s^2} \times 0.08\ \mathrm{m}$$

$$= \mathbf{169,716\ Pa}$$ (+0.5 points)

Bernoulli equation

$$p_1 + \frac{\rho}{2}V_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2}V_2^2 + \rho g z_2$$

Since $V_1 = 0$ and $z_1 = z_2$,

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho_{H_2O}}}$$ (+6 points)

Flow rate

$$Q = V_2 A_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho_{H_2O}}} \times \frac{\pi}{4} D_2^2$$ (+1.5 points)

$$= \sqrt{\frac{2 \times (170{,}000 - 169{,}716)\ \mathrm{Pa}}{998.2\ \mathrm{Kg/m^3}}} \times \frac{\pi}{4} \times (0.06\ \mathrm{m})^2$$

$$= 0.0021\ \mathrm{m^3/s}$$

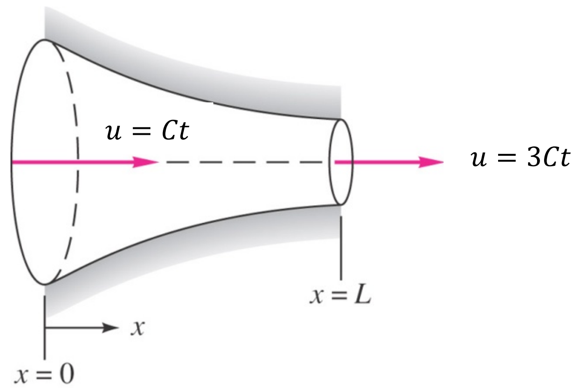
$$= \mathbf{7.6\ m^3/hr}$$ (+0.5 point)

EXAM1 Solutions

Problem 4: Acceleration and Euler equation (Chapter 4)

Information and assumptions

- One-dimensional flow
- $u = C\left(1+\frac{2x}{L}\right)t$, $C = 1\ \text{ft/s}^2$, $L = 6$ in
- Viscosity effects are negligible
- $\rho = 1.0\ \text{lbm/ft}^3$



Find

- Local and convective accelerations and pressure gradient at $x = \frac{L}{2}$ and $t = 1$ s

Solution

(a) Local acceleration

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left[C\left(1+\frac{2x}{L}\right)t\right] = C\left(1+\frac{2x}{L}\right)$$ (+3.5 points)

$$\left.\frac{\partial u}{\partial t}\right|_{x=\frac{L}{2},t=1} = 1\ \frac{\text{ft}}{\text{s}^2}\times\left(1+\frac{2\times L/2}{L}\right) = \mathbf{2\ ft/s^2}$$ (+0.5 point)

(b) Convective acceleration

$$u\frac{\partial u}{\partial x} = \left[C\left(1+\frac{2x}{L}\right)t\right]\times\frac{\partial}{\partial x}\left[C\left(1+\frac{2x}{L}\right)t\right] = \frac{2C^2t^2}{L}\left(1+\frac{2x}{L}\right)$$ (+3.5 points)

$$u\left.\frac{\partial u}{\partial x}\right|_{x=\frac{L}{2},t=1} = \frac{2\times(1\ \text{ft/s}^2)^2\times(1\ \text{s})^2}{1/2\ \text{ft}}\times\left(1+\frac{2\times L/2}{L}\right) = \mathbf{8\ ft/s^2}$$ (+0.5 points)

(c) Pressure gradient

$$\frac{dp}{dx} = -\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)$$

$$= -\rho\left[C\left(1+\frac{2x}{L}\right)+\frac{2C^2t^2}{L}\left(1+\frac{2x}{L}\right)\right]$$

$$= -\rho C\left(1+\frac{2x}{L}\right)\left(1+\frac{2Ct^2}{L}\right)$$ (+1.5 points)

$$\left.\frac{dp}{dx}\right|_{x=\frac{L}{2},t=1} = -(1\ \text{lbm/ft}^3\times 1\ \text{ft/s}^2)\times\left(1+\frac{2\times L/2}{L}\right)\times\left(1+\frac{2\times 1\text{ft/s}^2\times(1\ \text{s})^2}{1/2\ \text{ft}}\right)$$

$$= \mathbf{-10\ lbm/ft^2\cdot s^2}$$ (+0.5 point)