057:20 TEST # 1 Fall 2004

Prob. 1

Information and assumptions

Provided in problem statement

Find

Shear stress at walls and at centerline

Solution

$$u = 100y(0.1 - y) = 10y - 100y^2$$

$$\frac{du}{dy} = 10 - 200y$$

$$\tau = \mu \frac{du}{dy} = \mu(10 - 200y)$$

(Eqn. = 7)

Shear stress at $y = 0\,ft$

$$\left.\frac{du}{dy}\right|_{y=0} = 10$$

$$\tau_0 = \mu \times 10 = (8 \times 10^{-5}) \times 10 = 8 \times 10^{-4}\ lbf/ft^2$$

(Inter. + Ans. = 1)

Shear stress at $y = 0.05\,ft$

$$\left.\frac{du}{dy}\right|_{y=0.05} = 0$$

$$\tau_{0.05} = \mu \times 0 = 0$$

(Inter. + Ans. = 1)

Shear stress at $y = 0.1\,ft$

$$\left.\frac{du}{dy}\right|_{y=0.1} = -10$$

$$\tau_{0.1} = \mu \times (-10) = -8 \times 10^{-4}\ lbf/ft^2$$

(Inter. + Ans. = 1)

057:20 TEST # 1 Fall 2004

Prob. 2

Information and assumptions

Flow in a nozzle, details are provided in problem statement

Find

Local acceleration a_l

Convective acceleration a_c

Solution

$$a_l = \frac{\partial V}{\partial t}$$ (Eqn. = 2)

$$= \frac{\partial}{\partial t}\left(\frac{2t}{(1-0.5x/L)^2}\right) = \frac{2}{(1-0.5x/L)^2} = \frac{2}{(1-0.5\times 0.5L/L)^2}$$

$$= 3.56\ ft/s^2$$ (Inter. + Ans. = 1)

$$a_c = V\frac{\partial V}{\partial x}$$ (Eqn. = 5)

$$= \frac{2t}{(1-0.5x/L)^2}\frac{\partial}{\partial x}\left(\frac{2t}{(1-0.5x/L)^2}\right)$$

$$= \frac{4t^2}{(1-0.5x/L)^5 L} = \frac{4\times 3^2}{(1-0.5\times 0.5L/L)^5 \times 4}$$

$$= 37.9\ ft/s^2$$ (Inter. + Ans. = 2)

057:20 TEST # 1 Fall 2004

Prob. 3

Information and assumptions

Provided in problem statement

Find

Hydrostatic force acting on the gate.
Gate is stable or unstable

Solution 1

$$F = \overline{p}A$$ (Eqn. = 4)

$$= 9810 \times 4 \times \left(\frac{8}{\sin 45^o} \right) \times 1$$

$$= 443,950 N$$ (Inter. + Ans. = 1)

$$\overline{y} = \frac{1}{2} \left(\frac{8}{\sin 45^o} \right) = 5.657 m$$

$$y_{cp} = \overline{y} + \overline{I} / \overline{y} A$$ (Eqn. = 3)

$$= 5.657 + \frac{1}{12} w \left(\frac{8}{\sin 45^o} \right)^3 \Bigg/ \left(5.657 \times w \times \frac{8}{\sin 45^o} \right)$$

$$= 7.54 m$$ (Inter. + Ans. = 1)

Point B is

$$\frac{8m}{\sin 45^o} - 3.5m = 7.81m > 7.54m$$

The gate is unstable (Inter. + Ans. = 1)

Solution 2

$$F = \overline{p}A$$ (Eqn. = 4)

$$= 9810 \times 4 \times \left(\frac{8}{\sin 45^o} \right) \times 1$$

$$= 443{,}950\,N$$ (Inter. + Ans. = 1)

$$y_{cp} = \frac{2}{3}\frac{8}{\sin 45^o} = 7.54m$$ (Eqn. + Inter. = 4)

Point B is

$$\frac{8m}{\sin 45^o} - 3.5m = 7.81m > 7.54m$$

The gate is unstable. (Inter. + Ans. = 1)

057:20 TEST # 1 Fall 2004

Prob. 4

Information and assumptions

Provided in problem statement

Find

Estimate the time required to fill the remainder of the tank

Solution 1

Continuity equation

$$\sum_{cs} \rho \mathbf{V} \cdot \mathbf{A} = -\frac{d}{dt}\int_{cv} \rho d\forall$$ (Eqn. = 7)

$$\left(\mathbf{V}\cdot\mathbf{A}\right)_{out} - \left(\mathbf{V}\cdot\mathbf{A}\right)_{in} = -\frac{d}{dt}\int_{cv} d\forall$$

$$\left(\mathbf{V}\cdot\mathbf{A}\right)_{in} - \left(\mathbf{V}\cdot\mathbf{A}\right)_{out} = \frac{\forall_2 - \forall_1}{t}$$

$$\frac{1}{4}\pi d^2 V_1 - \frac{1}{4}\pi d^2 V_2 = \frac{\frac{1}{4}\pi D^2 h_2 - \frac{1}{4}\pi D^2 h_1}{t}$$

$$d^2\left(V_1 - V_2\right) = \frac{D^2\left(h_2 - h_1\right)}{t}$$

$$t = \frac{D^2\left(h_2 - h_1\right)}{d^2\left(V_1 - V_2\right)} = \frac{0.75^2\left(1 - 0.3\right)}{0.12^2\left(2.5 - 1.9\right)} = 45.57s$$

(Inter. + Ans. = 3)

057:20 TEST # 1 Fall 2004

Solution 2

$$Inflow \quad Q_1 = 2.5 \times \frac{1}{4} \times \pi \times 0.12^2 = 0.028274\, m^3/s$$

(Eqn. + Inter. + Ans. = 1)

$$Outflow \quad Q_2 = 1.9 \times \frac{1}{4} \times \pi \times 0.12^2 = 0.021488\, m^3/s$$

(Eqn. + Inter. + Ans. = 1)

Continuity equation:
Net flow rate into the tank

$$Q_3 = Q_1 - Q_2 = 0.028274 - 0.021488 = 0.006786\, m^3/s$$

(Eqn. + Inter. + Ans. = 7)

Volume of the remainder of the tank at t = 0:

$$\forall = (1 - 0.3) \times \frac{1}{4} \times \pi \times 0.75^2 = 0.30925 m^3$$

Time required to fill the remainder of the tank:

$$t = \frac{\forall}{Q} = \frac{0.30925}{0.006786} = 45.57 s$$

(Inter. + Ans. = 1)
