

8.61

8.61 To conserve water and energy, a "flow reducer" is installed in the shower head as shown in Fig. P8.61. If the pressure at point (1) remains constant and all losses except for that in the "flow reducer" are neglected, determine the value of the loss coefficient (based on the velocity in the pipe) of the "flow reducer" if its presence is to reduce the flowrate by a factor of 2. Neglect gravity.

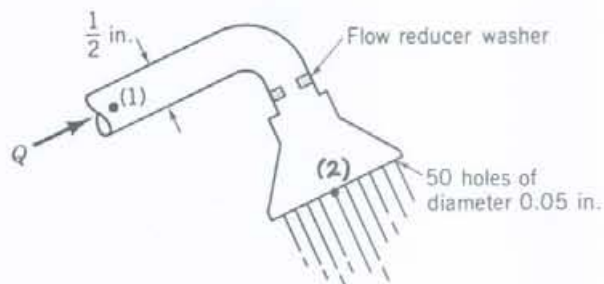


FIGURE P8.61

Without the reducer $\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + z_2$, where $p_2 = 0$, $z_1 = z_2$
and

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4Q}{\pi \left(\frac{0.5 \text{ ft}}{12}\right)^2} = 733Q$$

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{50\pi \left(\frac{0.05 \text{ ft}}{12}\right)^2} = 1467Q \quad (V_1 \text{ and } V_2 \sim \frac{\text{ft}}{\text{s}} \text{ and } Q \sim \frac{\text{ft}^3}{\text{s}})$$

$$\text{Thus, } p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2}\rho(1467^2 Q^2 - 733^2 Q^2) = 8.07 \times 10^5 \rho Q^2 \frac{\text{lb}}{\text{ft}^2}, \quad \text{where } \rho \sim \frac{\text{slug}}{\text{ft}^3}, Q \sim \frac{\text{ft}^3}{\text{s}} \quad (1)$$

With the flow reducer the flowrate is reduced by a factor of two.

$$\text{Thus, } V_1 = \frac{1}{2}(733Q) \text{ and } V_2 = \frac{1}{2}(1467Q) \text{ with} \quad (2)$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + K_L \frac{V_1^2}{2g} \text{ or } p_1 = \frac{1}{2}\rho(V_2^2 + (K_L - 1)V_1^2) \quad (3)$$

Hence, by combining Eqs. (1), (2), and (3) we obtain

$$8.07 \times 10^5 \rho Q^2 = \frac{1}{2}\rho \left[\left(\frac{1467}{2}Q\right)^2 + (K_L - 1)\left(\frac{733}{2}Q\right)^2 \right]$$

or

$$\underline{\underline{K_L = 9.00}}$$