

8.34

8.34 Show that the power law approximation for the velocity profile in turbulent pipe flow (Eq. 8.31) cannot be accurate at the centerline or at the pipe wall because the velocity gradients at these locations are not correct. Explain.

$$\text{If } \bar{u} = V_0 \left[1 - \frac{r}{R}\right]^{\frac{1}{n}}, \text{ then } \frac{d\bar{u}}{dr} = \frac{V_0}{n} \left[1 - \frac{r}{R}\right]^{\left(\frac{1}{n}-1\right)} \left(-\frac{1}{R}\right)$$

$$\text{or } \frac{d\bar{u}}{dr} = -\frac{V_0}{nR} \left[1 - \frac{r}{R}\right]^{\left(\frac{1-n}{n}\right)} \quad \text{Thus, } \left. \frac{d\bar{u}}{dr} \right|_{r=0} = -\frac{V_0}{nR}, \text{ but by symmetry it must be zero.}$$

$$\text{Also, } \left. \frac{d\bar{u}}{dr} \right|_{r=R} = -\frac{V_0}{nR} \left[1-1\right]^{\left(\frac{1-n}{n}\right)} = -\infty \text{ since } \left(\frac{1-n}{n}\right) < 0 \text{ for } n > 1$$

Physically, the velocity gradient must be finite.