7.21

7.21 A cone and plate viscometer consists of a cone with a very small angle α which rotates above a flat surface as shown in Fig. P7.21. The torque, $\mathcal{T}$, required to rotate the cone at an angular velocity, ω, is a function of the radius, R, the cone angle, α, and the fluid viscosity, μ, in addition to ω. With the aid of dimensional analysis, determine how the torque will change if both the viscosity and angular velocity are doubled.

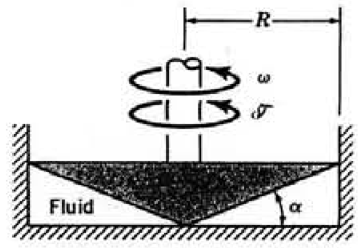

FIGURE P7.21

$$\mathcal{T} = f(R, \alpha, \mu, \omega)$$

$\mathcal{T} \doteq FL \qquad R \doteq L \qquad \alpha \doteq F^0L^0T^0 \qquad \mu \doteq FL^{-2}T \qquad \omega \doteq T^{-1}$

From the pi theorem, $5-3 = 2$ pi terms required.

By inspection, for Π_1 (containing $\mathcal{T}$):

$$\Pi_1 = \frac{\mathcal{T}}{\mu\omega R^3} \doteq \frac{FL}{(FL^{-2}T)(T^{-1})(L)^3} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\mathcal{T}}{\mu\omega R^3} \doteq \frac{ML^2T^{-2}}{(ML^{-1}T^{-1})(T^{-1})(L)^3} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

The angle, α, can be used as Π_2 since it is dimensionless. Thus,

$$\frac{\mathcal{T}}{\mu\omega R^3} = \phi(\alpha)$$

or

$$\mathcal{T} = \mu\omega R^3\phi(\alpha)$$

It follows that if both μ and ω are doubled $\mathcal{T}$ will increase by a factor of 4.